ORAL REFERENCE: ICF100825OR

MATERIAL RESPONSE WITH THE CELL METHOD

F. Cosmi

Department of Energetics, University of Trieste,
via A. Valerio 10, 34127 Trieste, IT

ABSTRACT

A new numerical model estimating the effect of randomly located micro-cracks on the structural response of a material is presented. Both heterogeneities of the structure and stress concentrations are taken into account by the model, that makes use of an homogeneous matrix in which randomly distributed cracks are present. The Cell Method is a recently developed numerical method that can be used to solve such a model. Both elastic and elastic-plastic behaviours can be included in the model. The Cell Method is presented and results of simulations from tests in both the elastic and plastic fields are discussed.

KEYWORDS

Damage, structural response, numerical method, cells.

INTRODUCTION

A lot of work has been done in the past in order to estimate the effect on the material properties of a low concentration of elliptical and circular voids. A self-consistent approximation of a material with identical polygonal randomly oriented voids has been proposed in [1]. In the crossover regime beam lattice models [2] along with interpolation functions [3] have been proposed. Finite element models have also been used to estimate the stiffness of a 2D initially elastic continuum containing square perforations [4]. A new model for the investigation of the effect of randomly located voids on the structural response of a material is presented in this paper. The discrete model makes use of a homogeneous matrix, in which randomly distributed cracks are present, as shown in Figure 1.

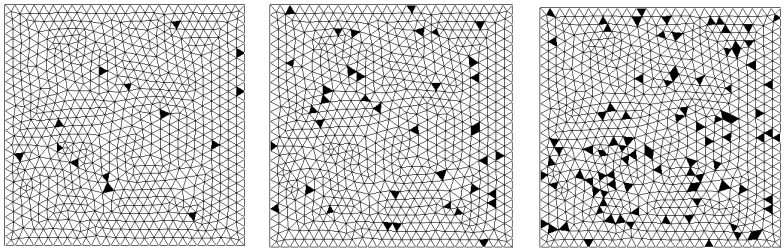

Figure 1: Randomly distributed void cells in homogeneous matrixes.

The Cell Method concept is deeply different from that of other widely used numerical methods such as FEM, and brings some advantages with it. As will be shown, CM is a "real" discrete method, in the sense that field equations are directly written for a whole region and no differentiation is needed to form the field equations. CM is thus applicable in all those cases in which variables cannot be differentiated, for example when the displacement field undergoes large variations, i.e. when the size of the heterogeneities is the same scale of that of the mesh [5]. A model such as the mentioned homogeneous matrix with distributed heterogeneities can be then solved with the Cell Method leading to consistent results.

It must be also mentioned that some aspects of the Cell Method may recall the Finite Volumes Method or the so-called *direct* – or *physical* – approach [6]. These methods did not have much success, maybe due to an intrinsic difficulty to develop higher order elements. On the contrary, higher order interpolation functions are easily implemented in CM.

When compared, CM results agree with those obtainable with other numerical methods, although convergence and accuracy are better than those obtained with FEM with the same interpolation order [7].

The present research makes uses only of linear interpolation functions and focuses on the other mentioned peculiar aspect of the Cell Method: the possibility to have heterogeneities the same size of that of the mesh.

THE CELL METHOD FOR PLANE ELASTICITY

The Cell Method has been recently introduced by E. Tonti [8, 9] and is currently being applied in several fields, such as thermal conduction, electromagnetism, mechanics of porous materials, and fracture mechanics. Application of the method to high porosity materials such as sintered alloys has been discussed in [10], where the Young modulus of four sintered alloys was computed and simulation showed a good agreement with experimental results.

In order to describe the Cell Method for plane elasticity, let us consider a body of constant thickness t, loaded in a plane. The variables used in the problem can be classified in

- *Configuration variables*, that is kinematic variables such as nodal coordinates and displacements, strain tensor, etc.,
- *Source variables*, that is static and dynamic variables such as forces, torques, momenta, etc.
- *Energy variables*, which result from the product of a configuration and a source variable, and which we are not going to use in the following.

According to this classification, the method implies the use of two staggered meshes: configuration variables are to be associated with a primal complex of cells, in this case a Delaunay triangles complex, while source variables are associated with a dual complex, the Voronoi tessellation associated with the primal mesh. Other choices for the dual complex are possible. Each node of the primal cell will be surrounded by a dual cell, which can be regarded as an influence region for the inner node.

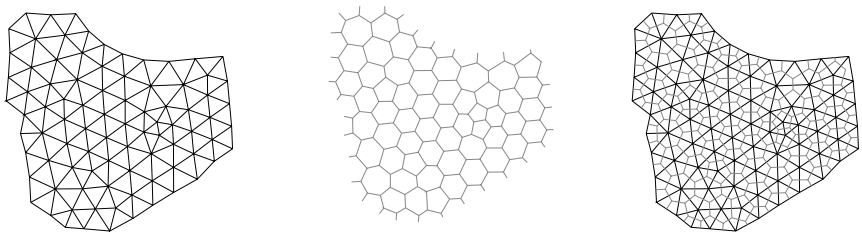

Figure 2: Primal (Delaunay), dual (Voronoi) complexes of cells and their ensemble.

If a Delaunay 3-nodes primal cell is used, the displacement field over the primal cell will be interpolated by a linear function. Strain components in a point inside a cell are given by

$$\{\varepsilon\}_c = [B]_c \{u\}_c$$

where

$$\{\varepsilon\}_c = \{\varepsilon_x \quad \varepsilon_y \quad \gamma_y\}^T, \quad [B]_c = -\frac{1}{2A_c t}\begin{bmatrix} A_{1x} & 0 & A_{2x} & 0 & A_{3x} & 0 \\ 0 & A_{1y} & 0 & A_{2y} & 0 & A_{3y} \\ A_{1y} & A_{1x} & A_{2y} & A_{2x} & A_{3y} & A_{3x} \end{bmatrix}, \quad \{u\}_c = \{u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3\}^T,$$

t is the thickness of the sample, A_c the area of the cell, and the meaning of A_{ij} is shown in Figure 3.

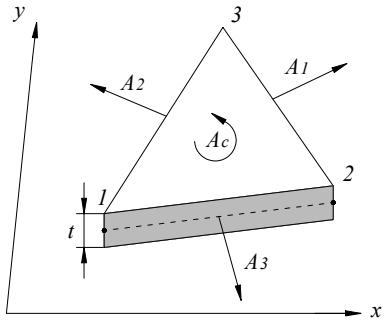

Figure 3: Geometrical quantities.

Introducing the constitutive matrix for the primal cell $[D]_c$ we may write the constitutive equation in the usual form

$$\{\sigma\}_c = [D]_c \{\varepsilon\}_c = [D]_c [B]_c \{u\}_c$$

In order to write equilibrium equations for the influence region of the node, we shall now need to express the forces acting through each side of the dual polyhedron surrounding the node.

As expected using a linear interpolation of the displacement field, strain and stress tensor components are constant within each primal cell. As a consequence, the surface force $\boldsymbol{T}$ (see Figure 4) will be given by

$$\begin{Bmatrix} T_{1x} \\ T_{1y} \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} A_{1x} & 0 & A_{1y} \\ 0 & A_{1y} & A_{1x} \end{bmatrix}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

and for the three nodes of a cell

$$\{T\}_c = -tA_c [B]_c^T [D]_c [B]_c \{u\}_c = -[K]_c \{u\}_c \tag{1}$$

where $\{T\}_c = \{T_{1x} \quad T_{1y} \quad T_{2x} \quad T_{2y} \quad T_{3x} \quad T_{3y}\}^T$ and $[K]$ represents the 6x6 stiffness matrix for the cell.

We shall call $\tilde{U}_h$ the dual cell surrounding node h (see Figure 5), $\boldsymbol{T}_h$ the total force acting on the boundary of $\tilde{U}_h$ - due to all the cells that surround node h-, $\boldsymbol{F}_h$ the resultant of volume forces and of external forces acting on $\tilde{U}_h$ through the boundary cells:

$$\boldsymbol{T}_h = \sum_c \boldsymbol{T}_h^c, \quad \boldsymbol{F}_h = \sum_c \boldsymbol{F}_h^c + \sum_c \boldsymbol{B}_h^c.$$

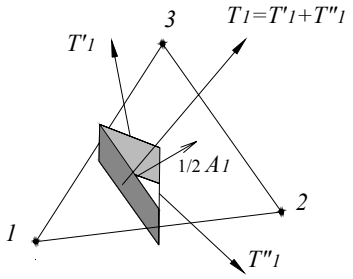

Figure 4: Forces acting through the sides of the dual cell of node *1*.

Equilibrium can be then written for region $\tilde{U}_h$:

$$\boldsymbol{T_h} + \boldsymbol{F_h} = 0$$

that is a set of $2n$ linear equations in the $2n$ unknowns u_{ix}, u_{iy} (i=1,…,n) which can be also written as

$$[K]\{u\} = \{F\} \tag{2}$$

and solved with the usual methods.

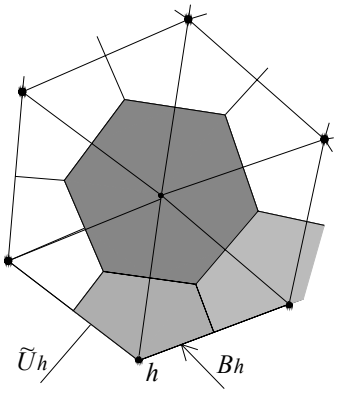

Figure 5: Equilibrium of the dual cell of node *h*.

SIMULATIONS AND DISCUSSIONS

It can be easily seen that the constitutive matrix may vary freely from one cell to the neighbour. As a consequence, the size of the heterogeneities can be the size of the mesh.

In a first set of simulations a 1498 cells matrix was tested in the elastic field. A number n of cells (see Figure 1) was set to void in each simulation and a tensile test was performed imposing a displacement on the right end of the sample, while the left end was constrained, though allowing lateral contractions of the sample. Figure 6 shows the ratio between computed Young modulus and the unimpaired one E^*/E vs. f, where $f=n/N$, n= number of void cells, N=total number of cells, E=210 GPa. For each void density 5 simulations were run. Porosity being equal, fluctuations of Young modulus can be observed in the graph, due to different distributions of the void cells, whose location varies randomly from one simulation to another. The plotted line corresponds to the linear fit with slope –2.47. The decrease in stiffness is similar to that reported in [1, pp.360-361] for FEM simulations with square perforations, although less pronounced. Figure 7 shows the stress concentrations near the void cells (black) in one of the simulations.

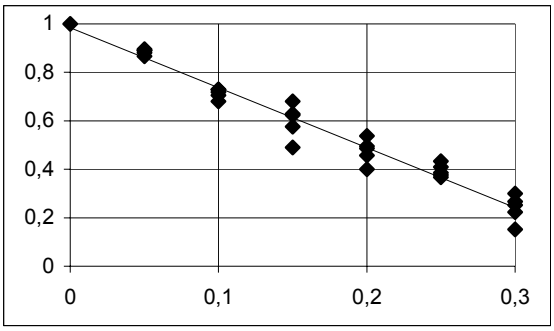

Figure 6: Cell method estimates of stiffness *E*/E* vs. *f*.

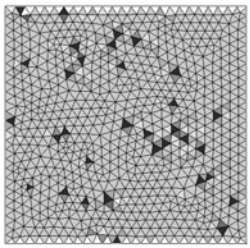

Figure 7: Stress concentrations near the void cells (black) in one of the simulations.

The ratio *E*/E* vs. *f* with low void concentrations is shown in Figure 8 for 25 simulations on a 1498 cells matrix. Small fluctuations of Young modulus can be observed in the graph. The plotted line corresponds to the self-consistent model proposed in [2], which assumes a quicker decrease of stiffness (slope –4.2) than the one computed with the proposed method.

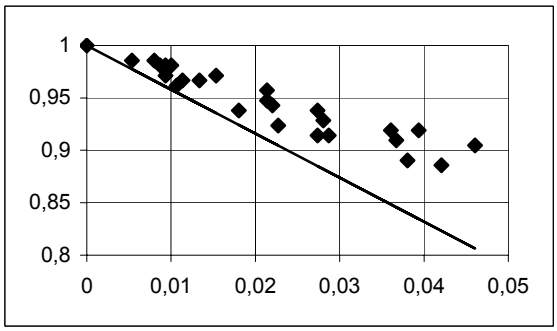

Figure 8: Young modulus *E*/E* vs. *f*.

PLASTICITY

Plasticity may also be implemented in the framework of the Cell Method. An elastic perfectly plastic material and von Mises yield condition have been assumed. For a primal cell

$$\{\sigma\}_c = [D]_c[\{\varepsilon\}_c - \{\lambda\}_c]$$

where $\{\lambda\}_c$ represents the homogeneous inelastic strain tensor for cell *c*. Equation (1) becomes now

$$\{T\}_c = -[K]_c\{u\}_c + \frac{1}{2}[A]_c^T[D]_c\{\lambda\}_c = -[K]_c\{u\}_c + [L]_c\{\lambda\}_c \tag{3}$$

and the fundamental equation (2) in incremental terms is

$$[K]\{\Delta u\} = \{\Delta F\} + [L]\{\Delta\lambda\} \tag{4}$$

The non-linear incremental problem is solved dividing the load history into steps, at the beginning of which displacements and internal stresses are known, and considering a backward difference integration scheme. Equation (4) can be solved for $\{\Delta u\}$, the second invariant of the deviatoric stress computed, and von Mises condition used to update $\{\Delta\lambda\}$. The process is repeated until convergence is obtained, then a new step is considered. Figure 9 shows the progressive plasticization of a 1498 cells matrix with 33 void cells, yield strength Ry = 430 MPa. From simulations, E^* = 195 GPa in the initial, elastic, part of the diagram.

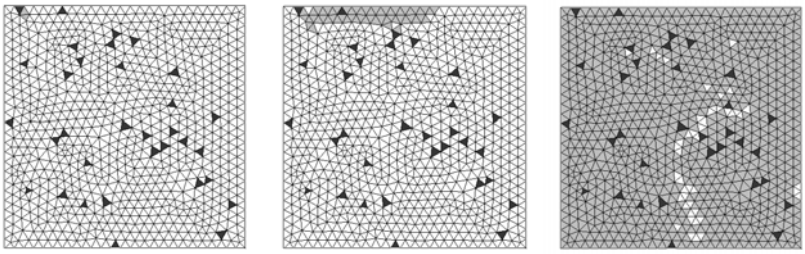

Figure 9: The model with void cells (black), initial and progressive plasticization (grey).

The result of the simulation is reported in Figure 10 in the form of a stress-strain plot. Figure 10 shows also the stress-strain curve obtained from simulation running an unimpaired matrix (E = 210 GPa).

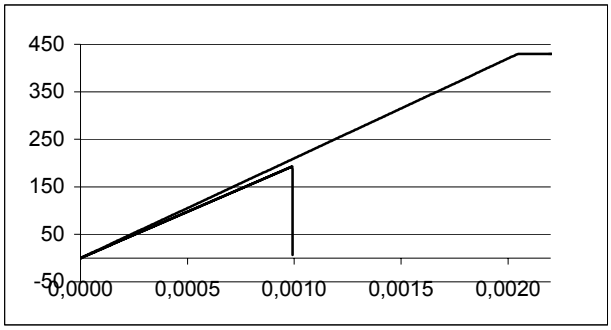

Figure 10: Stress-strain curve of 4198 cells matrix, with 33 and without void cells.

CONCLUSIONS

A new numerical model, consisting of a homogeneous matrix in which randomly distributed cracks are present, has been employed to estimate the effect of randomly located micro-cracks on the structural response of a material. The model considers both heterogeneities of the structure and stress concentrations, and may be solved with the Cell Method. Both elastic and elastic-plastic behaviours have been included in the model, and results of simulations from tests in both the elastic and plastic fields have been discussed.

REFERENCES

1. Jasiuk, I., Chen, J. and Thorpe, M.F. (1994), *Appl. Mech. Rev*., Vol.47, pp. S18-S28
2. Day, A.R., Snyder, K.A., Garboczi, E.J. and Thorpe, M.F. (1992), *J. Mech. Phys. Solids*, Vol. 40, pp. 1031-1051
3. Thorpe, M.F., Jasiuk, I., (1992), *Proc. Royal Soc. London*, Vol. A 438, pp.531-544
4. Krajcinovic, D. (1996) *Damage Mechanics*, Elsevier, North Holland
5. Roux, S. (1990). In *Statistical Models for the Fracture of Disordered Media*, pp. 109-113, Herrmann H.J., and Roux, S. (Eds). Elsevier, North Holland
6. Huebner, K.H. (1995) *The finite element method for Engineers*, John Wiley & Sons, pp. 39-59
7. Cosmi, F. *Computer Modeling in Engineering and Sciences*, to be published
8. Tonti, E., (2000) In *Atti del XIII Convegno Italiano di Meccanica Computazionale*, pp. 1-41, Brescia (IT).
9. Tonti, E., (2001) *Computer Modeling in Engineering and Sciences*, Vol.2, N.2
10. Cosmi, F., Di Marino, F. (2000) In *Proc. of 17th Danubia Adria Symposium on Experimental Methods in Solid Mechanics*, pp. 75-78, Prague, Czech Republic