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LRM CONSTITUTIVE MODEL FOR DESCRIBING FUNCTIONALLY GRADED MATERIAL WITH FAST PROPAGATING CRACK

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ABSTRACT

This paper treats a computational procedure to analyse a fast crack propagating transversely in a heat shield type functionally graded material(FGM). The constituents of the FGM vary from a 100 percent zirconia to a 100 percent ceramic. To best describe the severity of the propagating crack in such an inhomogeneously varying material, the T* crack-tip integral parameter is used. The FGM is assumed to be a particle dispersed composite material in elasto-plastic regime. The material constitutive modeling for the composite is performed by using authors' Localized Rigidity Model(LRM) on the basis of Self-consistent Compliance(SCC) scheme. Also a Neuro-Fuzzy technique is used to describe a constitutive law for the FGM in the same regime. The constitutive models are incorporated in a dynamic finite element code to study the crack propagation in the FGM with various grading patterns across the cracked plate. A fuzzy technique is utilized to perform a T* controlled crack propagation, thus predicted crack speed during the natural propagation in the varying toughness medium.

KEYWORDS

Functionally Graded Material, Self-consistent Scheme, Equivalent Inclusion Method, Localized Rigidity Model, Fracture, T*-integral

INTRODUCTION

In meso-mechanical point of view, the Functionally Graded Material(FGM) is a composite, in which the material composition continuously varies with location. A heat shield type FGM employs a ceramic material in combination with some metals for its counterpart. It is designated to have high resistance to very high temperature and also to have effective removal of the transmitted heat on the other side. If the composite is thin, a very high temperature gradient will occur accompanying extensive thermal stress causing meso-scopic damage as well as macroscopic defects. Assuming the most probable mode of the macro-failure for the FGM will be of fracture, the fracture resistibility is requested as another important function for the FGM. Cracks may emanate from the high temperature side surface in the ceramic, followed by possible propagation into the depth of the material. Hopefully, relatively high toughness of the metal material as counterpart may prevent the crack propagating into the depth, where the fracture toughness would continuously vary with the variation of the volume fraction of the mixed materials. Thus predicting the crack propagation speed in the FGM is important for estimating the performance of the fracture resistibility.

In the present, a theoretical/computational methodology for the analysis of the FGM is introduced, and its application is made to a dynamically propagating crack running transversely in the FGM, where the intensity of the estimated crack-tip severity is managed to keep in valance with the graded material toughness in the FGM during the propagation. To detect the crack-tip severity, an integral fracture parameter, T*[1], is used. Emphasis is placed on the use of a fuzzy inference algorithm to control the crack speed deduced from the T* values preceding the current crack position.

As to describing the constitutive rule for the FGM, spherical particles of arbitrary size in mesoscale are supposed to be randomly dispersed in the matrix medium. By assuming that the volume fraction of the inclusion is continuously varied, the grading is modeled. Authors have attempted modeling the constitutive law for the graded particle dispersed composite media of thermo-elastoplasticity by neural network scheme[2]. In the present effort, a closed form constitutive model[3] describing the nonlinear material mechanics of the particle dispersed medium is employed.

MODELING

Functionally Graded Material

One of the manufacturing process of the FGM is sintering the powder constituents in the hot isostatic pressure condition. Consider for simplicity the case when the FGM consists of the material of two phases. A schematic representation of such may be depicted in Figure 1(a), where two materials grade from 0 to 100 percent from one to another. On one side, the inclusions of the meso-structure are isolated with each other and suspended in the matrix material. On the other side, the same situation holds, but the constituent materials are reversed. Regarding the inclusion suspended medium as a particle dispersed composite material, the suspended spherical grain model by Kerner[4] was used to describe the constitutive law to evaluate elastic crack in the FGM under a quasi-static thermal shock load[5]. Treating a thermo-elastoplastic FGM, Nakagaki et al[6] devised a neural network procedure to model the constitutive law.

Since the dissemination of Eshelby's equivalent inclusion theory[7] with eigen strain concept and the mean field theory by Mori-Tanaka[8] opened the way to the recent developments of particle dispersed composites. Those are the works represented by Tandon-Weng[9]. Calling their model by EMT, it hypothesize that the constitutive relation between the average stress and average strain in phase is same as that of the monolith material. However, it is found that when the plastic deformation occurs, average stress-strain behavior in the phase is greatly influenced by the volume fraction of particles. This is due to the interaction between particles, which causes localized plastic deformations even at a low global load level. This nonlinearity in the microscopic level is directly reflected to the macroscopic stress and strain behavior of the composite. Thus, the averaged stress and strain in each phase is not constituted by the plastic state of the monolith material.

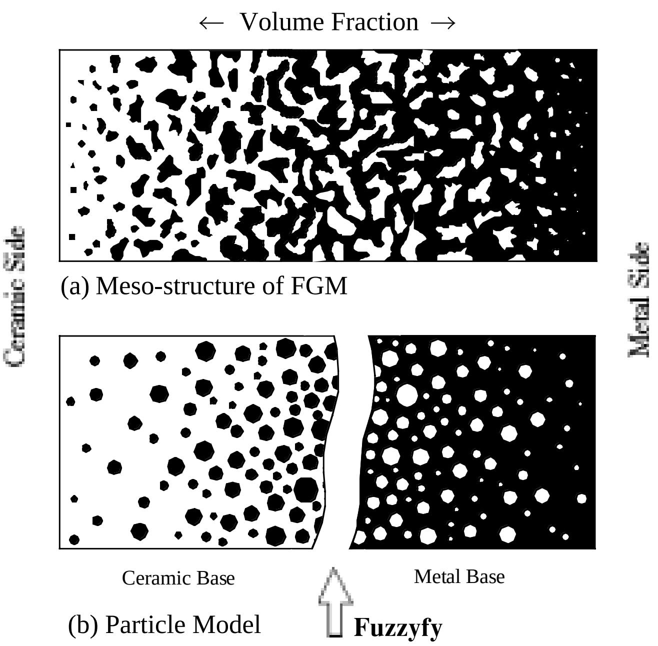


Figure 1: Schematics of (a)FGM and (b)Modeled particle domain

In this paper, we use the Localized Rigidity Method (LRM), in which meso-scopically localized plasticity due to interaction between particles is treated by means of a statistical scheme. Description of the present LRM model with elliptical equivalent inclusions and the pertinent mathematical treatment is given with the use of Gaussian distribution function. Upon relating the stress and the strain at the distributed meso-local level in each phase, the state variables and the constitutive law for the composite is derived.

Localized Rigidity Modeling(LRM)

In order to define precisely the distributed local stress, the entire range of the distributed stress over the phases is divided into a number of segments, in each of which an appropriate elastoplastic constitutive law can be used. Denote the stress in a segment in the matrix phase by $i, (i = 1, 2 \cdots)$, and for the inclusion by $j, (j = 1, 2 \cdots)$. Define the distribution functions over the normalized stress ρ_1^i and ρ_2^j, by $P_1(\rho_1^i)$ and $P_2(\rho_2^j)$ respectively. The stress segment corresponds to some number of small domains in which the stress is $\underset{\sim}{\sigma}_1^i$ or $\underset{\sim}{\sigma}_2^j$. The volume fractions of the area and can be represented in terms of the distribution function.

$$\Delta c_i = (1 - f)\, P_1(\rho_1^i)\, \Delta\rho_1^i \tag{1}$$

$$\Delta c_j = f\, P_2(\rho_2^j)\, \Delta\rho_2^j \tag{2}$$

The sum of the volume fraction of the segments in the matrix and the particles should be equal to their total volume fraction of the matrix and the inclusions. The distribution function will be defined such that the sum of the segments are unity. Therefore we have,

$$\sum_{i=1}^{n} \Delta c_i = 1 - f \tag{3}$$

$$\sum_{j=1}^{m} \Delta c_j = f \tag{4}$$

Since the inclusions are thought to be randomly located in the medium, a Gaussian type regular distribution function is assumed for the stress in the matrix

$$P_1(\rho_1^i) = \frac{1}{\sqrt{2\pi}\phi} e^{-\frac{1}{2\phi^2}(\rho_1^i - 1)^2} \tag{5}$$

where, ϕ is the variance. According to the theory of statistics, 99.7% of the stress distribution between the maximum and the minimum in the matrix will fall within the limits such that,

$$(1 - 3\phi) \le \rho_1^i \le (1 + 3\phi) \tag{6}$$

Therefore, an approximation is made to define the normalized minimum and the maximum stresses such that

$$\rho_1^{\min} = (1 - 3\phi) \qquad \text{and} \qquad \rho_1^{\max} = (1 + 3\phi) \tag{7}$$

Since a stress concentration often occurs at the interface between the matrix and the inclusions, the stress distribution along the surface of the inclusion must be known. Utilizing the works by Mura-Cheng[10] and Tandon-Weng[9] for an ellipsoidal inclusion under the uni-axial stretch, the variance in the present model can be defined.

$$\phi = \frac{f\left|g_1 - g_2\right|}{6} \tag{8}$$

where, and are the parameters of the minimum and the maximum stresses such that,

$$g_1 = \sqrt{\frac{1}{2}[(q_1 - \frac{p_1 + \hat{\gamma} p_2}{1 - \hat{\gamma}^2} - q_2)^2 + (q_1 - \frac{p_1 + \hat{\gamma} p_2}{1 - \hat{\gamma}^2})^2 + (q_2)^2]} \tag{9}$$

$$g_2 = \sqrt{\frac{1}{2}[(q_1 - \frac{p_2}{1-\hat{\gamma}} - q_2)^2 + (q_1)^2 + (\frac{p_2}{1-\hat{\gamma}} + q_2)^2]} \tag{10}$$

and further where, p_1, p_2, p_2 and q_2 are the parameters given in Tandon-Weng's.

At the local stress segment level, it is assumed that the stress and the strain are incrementally related in the homogeneous material fashion, so that the stress and the strain increments are related such that,

$$d\underset{\sim}{\varepsilon}_1^i = \underset{\sim}{\Gamma}_1^i : d\underset{\sim}{\sigma}_1^i \qquad \text{and} \qquad d\underset{\sim}{\varepsilon}_2^j = \underset{\sim}{\Gamma}_2^j : d\underset{\sim}{\sigma}_2^j \tag{11}$$

The average strain increment over the macro composite will be

$$d\underset{\sim}{\varepsilon}_0 = \sum_{i=1}^{n} \Delta c_i \underset{\sim}{\Gamma}_1^i : d\underset{\sim}{\sigma}_1^i + \sum_{j=1}^{m} \Delta c_j \underset{\sim}{\Gamma}_2^j : d\underset{\sim}{\sigma}_2^j \tag{12}$$

On the basis of the self-consistency concept, the final incremental form of the macro average constitutive relation for the nonlinear composite will be given as in the following:

$$d\underset{\sim}{\varepsilon}_0 = \hat{\underset{\sim}{\Gamma}} : d\underset{\sim}{\sigma}_0 \tag{13}$$

where,

$$\hat{\underset{\sim}{\Gamma}} = \sum_{i=1}^{n} \Delta c_i \underset{\sim}{\Gamma}_1^i : \underset{\sim}{C}_0^i + \sum_{j=1}^{m} \Delta c_j \underset{\sim}{\Gamma}_2^j : (\underset{\sim}{I} \otimes \underset{\sim}{I} + \underset{\sim}{A}_0^j : \hat{\underset{\sim}{E}}_0) \tag{14}$$

Since the quantities with hat in both the hands of the above are the global quantities and are unknown, the equations must be solved by some numerical scheme. Detail of the LRM constitutive model is found in Ref. [11].

Fuzzy Treatment

As it was mentioned earlier, the constituent material of the FGM's meso-scopic phase structure reverses from one end to the other, as depicted in Figure 1(a). For the heat shield type FGM, candidate constituents are a ceramic and a metal. On one end, the metal particles are suspended in the ceramic medium, and vice versa on the other end. In between, a type of bone mellow structure, which dominates the most of the domain, exists, and it is difficult to say which constituent takes the role of the matrix or the inclusion. Thus, suggested in Figure 1(b) shows a meso-structure model for the FGM, in which two sets of particle dispersed domains are considered, one with the ceramic as matrix and one with the metal as matrix. In both the cases, the inclusions are of spherical or ellipsoidal geometry so that the LRM equivalent inclusion model can be utilized. Between these two cases, considerable discrepancy will be seen in the material nonlinear stress-strain behaviors even when the distribution pattern of the volume fraction of the constituents are identical each other. In order for modeling the FGM composites of such an ambiguous meso-structural media, the present paper suggests the use of a fuzzy algorithm as one of an appropriate mean.

In the real FGM, the dominant volume fraction of, say, the ceramic does not warrant that the ceramic should takes the role of the matrix. Thus, the crisp data of the volume fraction are selected to be an input in the present fuzzy scheme, to be processed to determine its fuzzy membership grade via a membership function such that,

$$F(x) = \exp\,[-\frac{1}{a^2}(x-b)^2] \tag{15}$$

The membership grade of the counterpart is also calculated to be added to the former by Max-min scheme. The results are de-fuzzified to estimate a weight between the two estimates by the LRM constitutive law.

In the present work, the fuzzy scheme is used in twofold. One is for the aforementioned FGM grade modeling. Another is for controlling the fast crack speed running transversely in the FGM plate, where the material composition as well as the fracture toughness are varying. The mixing rule for the fracture toughness has not been established. Hence, known values for the toughness of each constituent were mixed in the linear mixing rule along the volume fraction of the constituents to be converted to obtain the critical . For the latter, the fuzzy is used to minimize the gap between the material parameter and the calculated T* from the previous load step to predict the most probable time increment at the current step. Thus the scheme for the T* controlled crack propagation in the FGM is established.

RESULTS

An edge cracked plate of a transversely graded FGM at 800ºC is under tension on the top and the bottom edges of the plate. The FGM is assumed to be composed of a ceramic and a titanium alloy. Figure 2(a) depicts the plate geometry and the loading condition. The metal is thermo-elastoplastic while the ceramic is considered to behave merely elastic. The plate was statically loaded until the crack-tip attain the criticality.
Analysis of the problem was performed with the use of a finite element method, where the constitutive law for the FGM is mathematically described by the LRM model and the fuzzy inference scheme as shown in the above.
To evaluate the crack severity, the crack-tip parameter, T* contour integral, is employed such that,

$$T^* = \int [(W+T)n_1 - n_j\sigma_{ij}u_{i,1}]ds - \int_{V-V_\rho} [W_{,1} + \gamma \dot{u}_i \dot{u}_{i,1} (G_i - \gamma \ddot{u}_i) u_{i,1} - \sigma_{ij}\varepsilon_{ij,1}]dV \tag{16}$$

At the moment when the estimated T* reaches the critical value, the crack was set to dynamically propagate across the plate. During the propagation, the crack running speed was controlled by the fuzzy algorithm so that the T* keeps agreement with its varying critical values during the propagation described in the above.

The results shown in the rest are those for a linearly graded FGM plate. Figure 2(b) shows the calculated T* values versus crack depth during the crack propagation compared with the critical , which is a unique material property graded along the material design for the FGM plate. Reasonable agreements between the two in average over the entire propagation indicate a successful simulation of the present T* based natural crack propagation in the graded FGM with the fuzzy procedure. Some difficulty was encountered at the incipient crack propagation resulting over estimate of T*. It was because the crack speed was zero in the preceding time step. It may not be difficult to find some remedial procedure to fix the misfit.

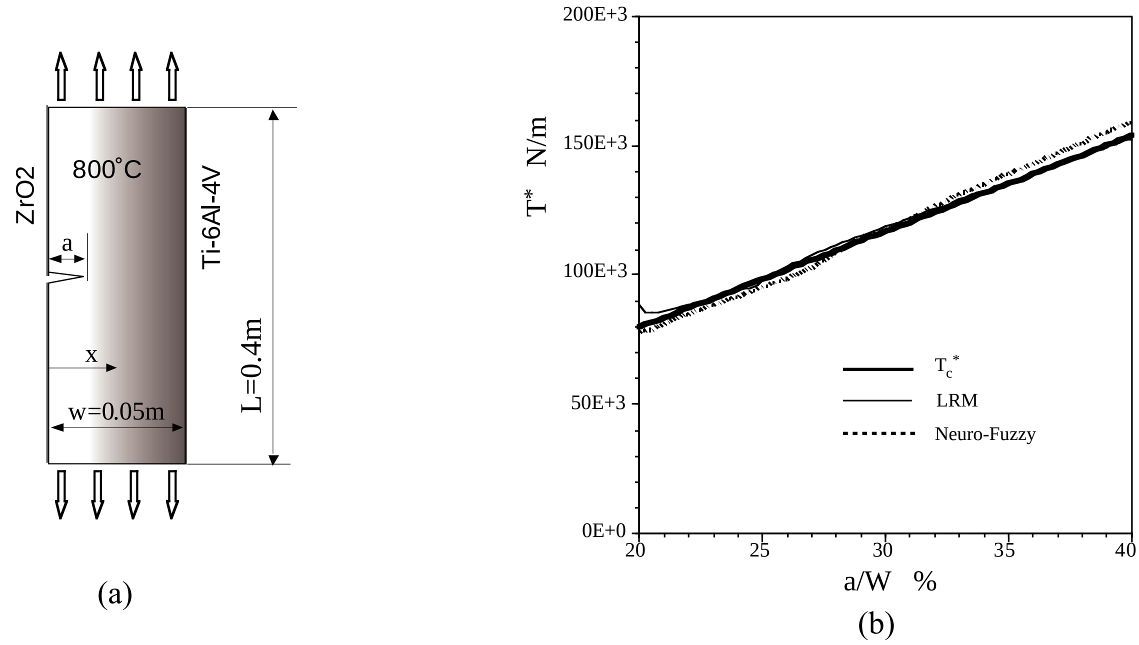
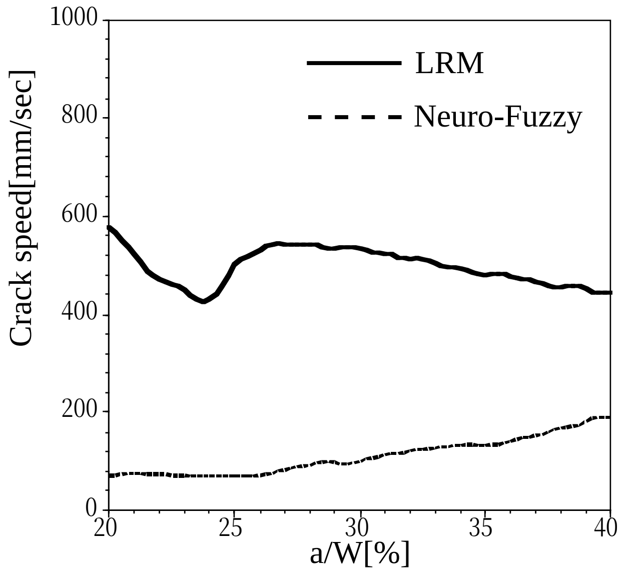


Figure 2: (a) Cracked FGM plate under tension and (b) Fuzzy controlled T* results

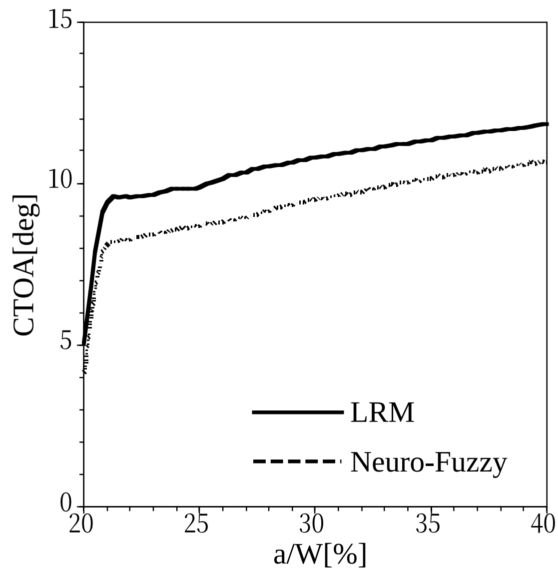
Shown together in the figure is the result for the same problem with the same procedure but calculated using the constitutive model obtained by a neural network scheme[2]. In this case also a reasonable simulation is seen to be performed.

Figure 3(a) shows the predicted crack speeds during the T* controlled crack propagation in the FGM plate thus simulated. In this result, the starting speed with LRM constitutive law much differs from that by the Neuro-Fuzzy calculation.

Shown together in the figure is another estimate of the crack speed where the FGM constitutive law was modeled by a neural network scheme instead of the closed form LRM. Although the incipient misfit in T* might cause in the LRM result the unreasonably high value of predicted crack speed, overall discrepancy between the two results are quite different. Far field quantities such as the remote stress at the plate ends do not observe a quite discrepancy between the two cases, but the propagation speed is sensitive and reflects the material description model. Therefore the present study warns the significance of the proper description of the constitutive law especially predicting the possible crack arrest in the very inhomogeneous material such as FGM.



(a)



(b)

Figure 3: (a) Predicted crack speed and (b) Crack-tip opening angle during T* controlled propagation

During the crack propagation, the crack-tip opening angle is calculated and shown in Figure 3(b) for the two procedure. Continuous increase of CTOA indicates possible ductile fracture in the area.

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