# INTERFACE CRACK IN PERIODICALLY LAYERED BIMATERIAL COMPOSITE

M. Ryvkin and L. Kucherov

Department of Solid Mechanics, Materials and Systems,
Tel Aviv University, Ramat Aviv 69978, Israel

**ABSTRACT**

An interface crack in a periodically layered bimaterial composite plane is considered. The solution method is based on the combined using of the representative cell and the dislocation approaches. First, after applying the discrete Fourier transform, the analytical expression of the Green function for a single interface dislocation in a periodically layered media without the crack is derived. Using this result the problem is then reduced to the system of two singular integral equations which are solved numerically. The parametric study of the stress intensity factor and energy release rate revealed new phenomena unlike to the ones observed for the more simple three layered sandwich models of multilayered bodies. It is found that for the case of sufficiently large mismatch in the geometric and elastic characteristics of the layers the stress intensity factor approaches some maximum value which can essentially exceed the corresponding one for a crack on a homogeneous plane.

**KEYWORDS:** periodically layered composite, interface crack, discrete Fourier transform

## 1 Introduction

Fracture behavior of periodically layered composite materials for the crack parallel to the interfaces was intensively investigated recently for the cases of antiplane deformation [1], [2], mode I [3], and mixed mode cracks [4]), [5]). The study of the interface crack related to the delamination phenomenon is of special interest. Kaczynsky and Matysiak [6], [7] considered this problem using the non-standard homogenization approach. This approach includes some approximating assumptions and does not give the precise stress distribution required for the fracture analysis. The mathematical technique employed in the present paper hinges on the discrete and integral Fourier transforms and it is free of any homogenization procedure. Hence, it enables to determine the exact values of the stress intensity factor and energy release rate and to carry out the complete parametric study. One of the goals of this study is to compare the results with the ones obtained for the more simple case of an interface crack in a three layered sandwich composite. The latter problem is well investigated, so one will obtain the important information regarding the limits of using sandwich approximation for predicting the fracture properties of a periodically layered composite.

## 2 Analysis

The plane deformation of a composite consisting of two types of layers arranged periodically is considered (see insert in Fig. 2). The layers of each type are defined by the thickness $h_r$, shear modulus of the material $\mu_r$ and Poisson ratio $\nu_r$ $(r = 1, 2)$. The translational symmetry of the composite is violated by an interface crack of length $2a$ subjected to uniform loading

$$\sigma_y + i\tau_{xy} = \sigma_0 + i\tau_0 \; , \qquad |x| < a, \quad y = 0 \; . \tag{1}$$

The solution method is based on the dislocation approach developed by Erdogan and Gupta [8] applied in combination with the representative cell method [9], [1]. The crack is viewed as distributed dislocations with an unknown amplitude $f = f_1 + if_2$

$$\frac{\partial}{\partial x}[\Delta u(x,0) + i\Delta v(x,0)] = (f_1 + if_2)\,\delta(x-t) \; , \tag{2}$$

where $\Delta u(x,0)$ and $\Delta v(x,0)$ are the displacement jumps across the interface in the $x-$ and $y-$directions respectively, and $\delta(x)$ is the delta-function. The Green function for a single dislocation in a periodically layered composite can be found in a closed form by the representative cell method in the same manner as in [2], [3], [5]. To this end the plane is presented as an assemblage of identical cells denoted by the index $k$ $(k = 0, \pm1, \pm2, ...)$. After application of the discrete Fourier transform

$$u^* = \sum_{k=-\infty}^{\infty} u^{(k)} e^{i\varphi k} \tag{3}$$

the problem for the plane is reduced to a problem for the representative cell consisting of two dissimilar bonded layers. The values of the components of the stress strain state at the lower $(-)$ and upper $(+)$ edges of the cell are found to be related by the Born-Von Karman type boundary conditions

$$\{u^*, v^*, \tau^*_{xy}, \sigma^*_y\}^+ = e^{i\varphi}\{u^*, v^*, \tau^*_{xy}, \sigma^*_y\}^- \tag{4}$$

including the transform parameter $\varphi$. The problem for the representative cell is solved by means of the Fourier integrals. Applying then the inverse discrete Fourier transform one obtains the sought expressions of the Green functions for the stresses in the form of double integrals

$$G^{(k)}_\sigma(x,y,t) = \frac{1}{2\pi}\sum_{j=1}^{2} f_j(t)\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}[A_j e^{-yz} + B_j e^{yz}]e^{i(zx-\varphi k)}dz d\varphi \tag{5}$$

$$G^{(k)}_\tau(x,y,t) = \frac{1}{2\pi}\sum_{j=1}^{2} f_j(t)\int_{-\pi}^{\pi}\int_{-\infty}^{\infty}[C_j e^{-yz} + D_j e^{yz}]e^{i(zx-\varphi k)}dz d\varphi \tag{6}$$

Here $k$ is the number of the cell, $t$ defines the location of the dislocation at the interface within the cell number 0, and $A_j, B_j, C_j, D_j$ are the known functions of the problem parameters and variables $t, z, y, \varphi$. The explicit expressions of these functions are obtained by the use of symbolic computation, they are rather cumbersome but observable.

The procedure employed for the conversion of the interface crack problem to a system of two singular integral equations using the Green function is described by Erdogan and Gupta [8]. The solution is obtained as in that work by representation of the unknown dislocation density $f(x)$ as series of Jacobi polynomials.

## 3 Results

The singular stresses in front of the crack are characterized by the complex stress intensity factor $K = K_1 + iK_2$

$$\sigma_y + i\tau_{xy} = \frac{K}{\sqrt{2\pi r}} r^{i\varepsilon} \; , \tag{7}$$

and the energy release rate

$$G = \frac{(1-\nu_2)(1-\beta^2)}{2\mu_2(1+\alpha)} K\bar{K} , \tag{8}$$

where

$$\varepsilon = \frac{1}{2\pi} \ln \frac{1-\beta}{1+\beta} . \tag{9}$$

and $\alpha$ and $\beta$ are the Dundurs parameters. The case of opening normal tractions ( $\tau_0 = 0$) is

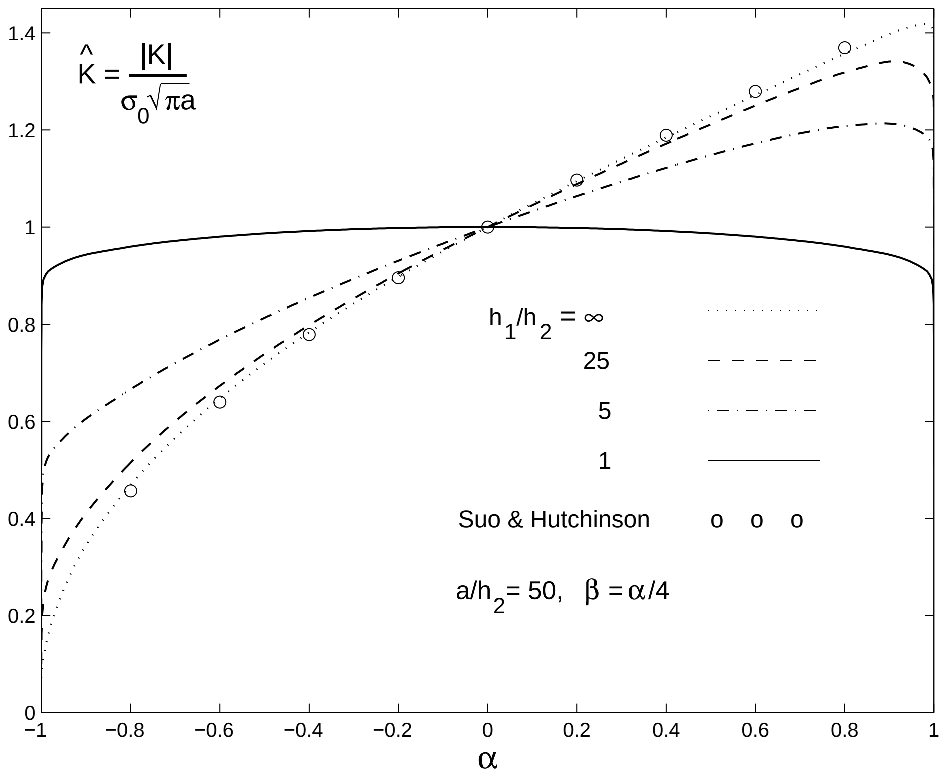


Figure 1: Normalized stress intensity factor vs. elastic mismatch parameter $\alpha$ for $a/h_2 = 50$ and $\beta = \alpha/4$

considered. We will examine first the normalized absolute value of the stress intensity factor

$$|\hat{K}| = \frac{1}{\sigma_0} \left[ \frac{K_1^2 + K_2^2}{\pi a} \right]^{1/2} \tag{10}$$

divided by the stress intensity factor for a corresponding crack in a homogeneous plane $K_{hom} = \sigma_0\sqrt{\pi a}$. The dependence of $|\hat{K}|$ upon the mismatch in the elastic properties of the composite constituents is illustrated in Fig. 1. The mismatch is characterized by the value of $\alpha$ and it is assumed that $\beta = \alpha/4$ which is a good approximation for many engineering materials [10]. Three thickness ratios $h_1/h_2 = 1, 5, 25$ are considered, i.e. the layers of the first type with the thickness $h_1$ are assumed to be not thinner than the layers of the second type. In addition the curve corresponding to the sandwich composite ($h_1/h_2 = \infty$) is depicted for reference. The ratio of the crack length to the thinner layers thickness is taken relatively large $a/h_2 = 50$. Therefore the results for the sandwich composite are very close to the ones obtained from the asymptotic formula for a semi-infinite crack derived in Suo and Hutchinson [11].

Increasing $\alpha$ corresponds to the enhancement of the stiffness of the thinner layers with thickness $h_2$ relative to the $h_1$ layers. In the considered case $\beta = \alpha/4$ the thinner layers are more compliant (i.e. $\mu_2 < \mu_1$) for $-1 < \alpha < 0$. For $0 < \alpha < 1$ the situation is reversed, and when $\alpha = 0$ the elastic properties of the composite constituents coincide. Consequently, in view of the accepted normalization, in the latter case $|K| = 1$. For negative $\alpha$ the general trend observed for periodic composites is the same as for the sandwich. Namely, the behavior of the stress intensity factor is monotonic and its values are less than unity as a result of so called the elastic shielding effect (see, for example, [10]).

For the case of positive $\alpha$ the above similarity does not hold true.

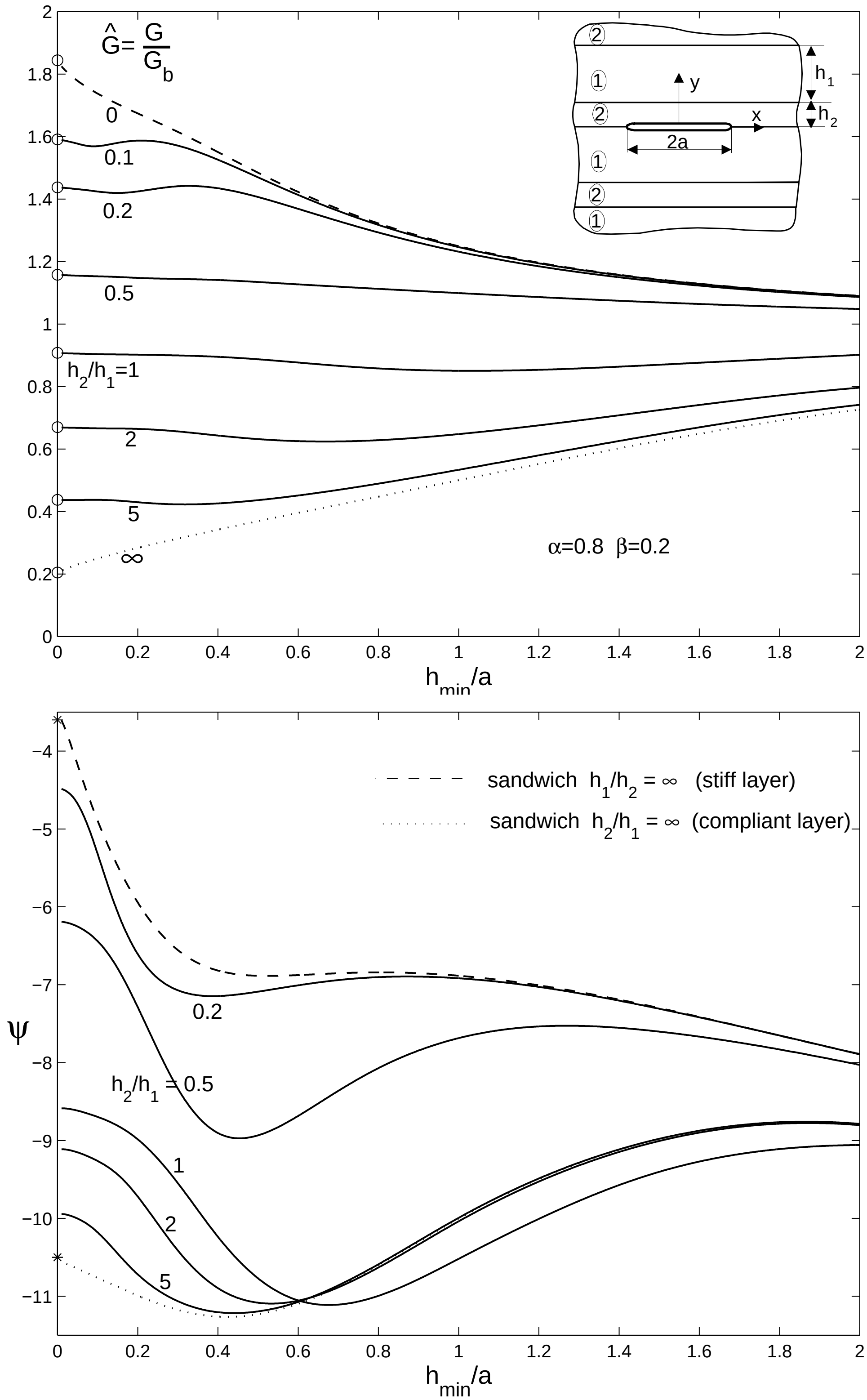


Figure 2: Normalized energy release rate $\hat{G}$ and real phase shift angle $\psi = \mathrm{Arg}[K h_{min}^{i\varepsilon}]$ (in degrees) vs. crack length parameter $h_{min}/a$; $h_{min} = min\{h_1, h_2\}$.

The curve for the case of equal thicknesses $h_2/h_1 = 1$ is symmetric with respect to the line $\alpha = 0$

owing to the symmetry of the problem. Therefore the absolute value of the stress intensity factor for an interface crack will be always less than that for the crack in a homogeneous material. On the other hand, for the sandwich composite a monotonic enlargement of the stress intensity factor with increasing $\alpha$, i.e. with increasing stiffness of the sandwiched layer, is observed which may be referred as the inverse shielding effect. For the bimaterial composites with thin layers $h_1/h_2 = 5,\ 25$ this effect is interfered with an opposite trend induced by the restriction of the displacements of the layered body when the layers of one of the types become stiffer. As a result, the behavior of $|\hat{K}|$ is found to be non-monotonic. This phenomenon is similar to the one observed in the case of a Mode III interface crack [2]. Hence, if we increase stiffness of the thinner layers in a periodic composite it may lead not only to increasing but also to decreasing of the singular stresses in front of the crack tip. Consequently, for some parameter combination the stress intensity factor reaches its maximum value. The magnitude of this value for the given material pair increases with the increasing of the difference in layers thicknesses and can appreciably exceed the value for the crack in homogeneous material.

The influence of the crack length on the energy release rate is presented in the upper part of Fig. 2. It is convenient to consider the normalized quantity

$$\hat{G} = \frac{G}{G_b}\,, \tag{11}$$

where $G_b$ is the energy release rate for the sample problem of a crack at the interface between two half-planes made of the same materials as the layered composite. The crack length is normalized by the value $h_{min} = min\{h_1, h_2\}$ being the thickness of the thinner layers. For the considered elastic materials combination $\alpha = 0.8,\ \beta = 0.2$ the information on all possible thickness ratios is presented. Two reference curves for the sandwich composites with compliant and stiff layers are denoted by the doted and dashed lines respectively.

When the crack is short the layered structure of the composite is not relevant, and, consequently, the asymptote for $h_{min}/a \to \infty$ is unity in view of accepted normalization for all the curves. The increasing of the crack length results in moderate decreasing(increasing) of the energy release rate when the thinner layers are more compliant (stiff). The limiting values of $\hat{G}$ for $h_{min}/a \to 0$ denoted by circles can be evaluated analytically similar to [3]. To this end one has to calculate the energy release rate in terms of the remote stress field. Clearly, for a sufficiently long crack the periodically layered plane composed of isotropic layers can be replaced by an anisotropic one possessing effective elastic properties. For the considered case of normal loading as in the case of mixed mode crack [5]

$$G = \sigma_0^2 \pi a Q\,, \tag{12}$$

the explicit expression of $Q$ in terms of the geometric and elastic parameters of the layers is given in [5].

It may be observed that for the considered elastic mismatch of the materials the analytical limiting values for a semi-infinite crack provide a sufficiently good approximation for the energy release rate in a surprisingly large crack length range. In fact, if the ratio of the thinner layers to the thick ones is more then 0.1, one can use analytic formula in the framework of 5% accuracy for all cracks with $h_{min}/a < 0.4$. The mentioned region, clearly, will extend when the elastic mismatch is diminished i.e. for $\alpha < 0.8$. The case of a large difference in the thickness and the elastic properties of the layers is investigated in [12].

Following [11] we will characterize the argument of the stress intensity factor by a real phase angle $\psi = \mathrm{Arg}[K h_{min}^{i\varepsilon}]$. The dependence of this angle measured in degrees upon the crack length is shown in the lower part of the figure. One can note that the argument of the stress intensity factor is much more sensitive to the crack length than its absolute value related to the energy release. The limiting values for the semi-infinite crack in sandwich composites denoted by asterisks are derived from the asymptotic formula obtained in [11].

In conclusion it should be noted that the behavior of interface cracks in the periodically layered composites is found to be unlike to the one observed for the more simple sandwich models and requiered separate investigation. The employed analysis technique based on the combined use of the representative cell and the dislocation methods provides an exact and convenient mathematical tool for such investigation.

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