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FRACTURE TOUGHNESS ENVELOPE OF A LIMESTONE ROCK AT HIGH CONFINING PRESSURE AND TEMPERATURE

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ABSTRACT

Fracture locus or envelope under a mixed-mode I-II loading can be obtained by plotting the normalized mode-II versus mode-I fracture toughness values. The envelope obtained can be used as a criterion for fracture failure for that material. However, testing conditions have a strong impact on that envelope. The objective of this paper is to present some results of an experimental program that was made to obtain fracture toughness envelope for a limestone rock from Saudi Arabia. Brazilian disks with an inclined central notch were tested under diametral compression, to get variety of mixed mode I-II fracture cases. Tests were conducted using disks with different sizes, and different notch type. Tests were made at different confining pressure from 0 to 28 MPa, and different temperature from 27 to 116°C. Fracture toughness envelopes for the tested rock were obtained for both positive and negative regions (opening and closing of the crack) and at various environmental conditions. A quadratic equation, which fit the experimental results more satisfactory, were proposed as a failure criteria. One of the major contributions of this paper is the effect of high confining pressure and temperature on the fracture toughness envelopes of rocks.

KEY WORDS

Rock fracture, mixed Mode I-II, fracture envelope, high temperature and pressure.

INTRODUCTION

Studying the fracture toughness of rocks at elevated temperatures and confining pressures is valuable for a number of practical situations such as hydraulic fracturing used to enhance oil and gas recovery from a reservoir, and the disposal or safe storage of radioactive waste in underground cavities. Based on the loading type, there are three basic crack propagation modes in a fracture process, namely: Mode I (extension, opening), Mode II (shear, sliding), and Mode III (shear, tearing). Any combination of these modes can occur as a mixed-mode. Most, if not all, studies in the past have focused on fracture toughness determination under confining pressures only for Mode-I failure conditions. Nevertheless, due to randomly oriented cracks in rocks and/or in-situ stress conditions, cracks tend to propagate under the influence of a combined action of the basic failure modes called mixed mode [1,2]. In the case of rocks, the combination of Mode-I and Mode-II (mixed Mode I-II) failure is more common. Therefore, consideration of mixed Mode I-II loading in addition to pure Mode-I becomes important in fracture toughness investigation.

Due to this mixed mode failure pattern, in addition to mode-I, fracture toughness under mode-II becomes important to be considered. When Brazilian disks with an inclined central notch are tested under diametral compression (Figure 1), a variety of mixed mode I-II fracture cases are obtained. For a particular material, a fracture locus or envelope can be obtained by plotting normalized mode-II versus mode-I fracture toughness. The envelope obtained could be used as a failure criterion in fracture toughness study for a particular material and testing condition in a way similar to the use of Mohr-Coulomb failure envelope for strength.

Usually, the fracture toughness of rock is determined at ambient conditions. However, under varying temperatures and confining pressures, the measured fracture toughness has been shown to vary. The fracture toughness behavior of a deep-seated rock formation requires the testing to be conducted in a manner that simulates the *in-situ* conditions such as temperature and confining pressure.

THEORETICAL BACKGROUND

Facture Toughness

When a notched rock specimen is subjected to an externally applied load, stress concentrates in the vicinity of the crack tip. When this concentrated stress reaches a critical value, failure occurs due to propagation of the preexisting crack. The fracture toughness is then calculated in terms of the stress intensity factor (SIF). In this paper, a circular Brazilian disk with a central notch under diametrical compression (Figure 1) was used to investigate fracture toughness. The following mathematical expressions, proposed by Atkinson *et al.* [3], were used for the fracture toughness calculation:

$$K_I = \frac{P\sqrt{a}}{\sqrt{\pi}RB} N_I \tag{1}$$

$$K_{II} = \frac{P\sqrt{a}}{\sqrt{\pi}RB} N_{II} \tag{2}$$

where, K_I is Mode-I stress intensity factor; K_{II} is Mode-II stress intensity factor; R is radius of the Brazilian disk; B is thickness of the disk; P is compressive load at failure; a is half crack length; and N_I *and* N_{II} are non-dimensional coefficients which depend on a/R and the orientation angle (β) of the notch with the direction of loading. For linear elastic fracture mechanics (LEFM), the small crack approximation proposed by Atkinson *et al.* [3] can be used to determine the values of N_I and N_{II} for half crack to radius ratio ($a/R \leq 0.3$), as follows:

$$N_I = 1 - 4\sin^2\beta + 4\sin^2\beta * (1 - 4\cos^2\beta)(a/R)^2 \tag{3}$$

$$N_{II} = \left[2 + \left(8\cos^2\beta - 5\right)\left(a/R\right)^2\right] \tag{4}$$

Fracture toughness for pure Mode-I ($\beta = 0$) is taken as K_{IC}; and that for pure Mode-II ($\beta \approx 29^o$) is taken as K_{IIC}.

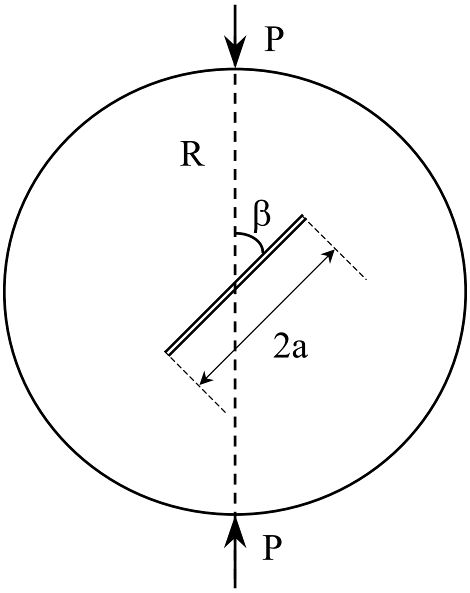


Figure 1: A schematic for Brazilian disk under diametrical compression

Failure Theories

There are numerous failure criteria for crack initiation and propagation under mixed mode I-II loading condition. The most popular ones are: (1) the maximum tangential stress (σ) criterion, (2) the maximum energy release rate (G) criterion, and (3) the minimum strain energy density (S) criterion. The available experimental data shows that no distinct theoretical failure criterion is applicable to all cases. Also, these criteria imply that K_{IC} is larger than K_{IIC}, while experimental data show the opposite. Moreover, due to the fact that the existing failure criteria were developed based on the tensile loading rather than the compressive, they hold good only in the positive region (crack opening) and cannot predict the fracture behavior in the negative zone (crack opening)

Many researchers have recommended using empirical relations for practical applications. Huang and Wang [4] and Sun [5] have used one of three empirical equations of straight line, ellipse, and homogenous quadratic to fit the experimental fracture toughness data in the (K_I/ K_{IC})-(K_{II}/ K_{IIC}) plane. Also, an exponential relationship was used [6,7].

EXPERIMENTAL PROGRAM

Sample Preparation

Rock blocks were collected from a limestone rock formation outcropping in the Central Province of Saudi Arabia. Cores were obtained from these blocks using 98 mm and 84 mm coring tube pits. Cores were sliced into circular disks using a high-speed circular saw. The thickness (B) of the sliced disks was in the range of 20-24 mm. A notch was machined in the center of the disks. Straight notch was made using a 0.25 mm diamond-impregnated wire saw, while chevron notch was made using a slow-speed circular saw. The notch making process is explained in more details elsewhere [8].

Rock properties

The investigated limestone rock was beige in color. Its physical properties included a dry density of 2.586 gm/cm^3, a specific gravity of 2.737, and porosity of 5.4%. The mechanical characteristics included a uniaxial unconfined compressive strength of 105 MPa, a tensile strength of 2.31 MPa, a modulus of elasticity of 54 GPa, and a Poisson's ratio of 0.276.

Testing

A strain-controlled loading frame having a capacity of 100 KN was used for the load application with a strain rate of 0.08 mm/min. Disk specimens were diametrically loaded with different values of the crack inclination angle (β) ranging from 0° to 75° with a 15° increment. The applied load and load-point displacement were acquired using a computerized data logger. Tests were made at ambient conditions, at high confining pressure σ_3) of 28 MPa, and at high temperature of 116°. The details of the experimental program can be found elsewhere [9].

RESULTS AND DISCUSSIONS

The values for mode I and mode II fracture toughness (K_I) and (K_{II}) were determined using equations 1 to 4. Table 1 summarizes the values of K_{IC} and K_{IIC} at different conditions for straight notch. For all conditions, K_{IC} is smaller than K_{IIC}, in contrary to the values provided by the famous failure theories. Figure 2-a shows the variation of K_I and K_{II} with β for D = 98 mm, at different conditions. It can be seen that the high confining pressure has a tremendous impact on the fracture toughness, while the effect of high temperature has a minor effect.

The normalized fracture toughness values of (K_I/K_{IC}) and (K_{II}/K_{IIC}) were determine for various cases. The plot of (K_{II}/K_{IIC}) vs. (K_I/K_{IC}) is named the fracture toughness envelope, which is the fracture locus for the general mixed-mode I-II loading. Crack initiates when a point ((K_I/K_{IC}), (K_{II}/K_{IIC})) falls on the envelope. Figure 2-b gives a comparison between fracture toughness envelopes at different conditions (ambient, σ_3 = 28 MPa, and T = 116 °C). Figure 3-a shows fracture toughness envelopes for D = 98 and 84 mm, at both positive region (crack opening) and negative region (crack closing). Also, Figure 3-b shows similar results for straight and Chevron notches. A second-degree polynomial was used to fit the experimental data at various conditions, and at both positive and negative region, Figures 2 to 3. The general form of the fitting is:

$$\left(K_{II}/K_{IIC}\right)= A + B\left(K_I/K_{IC}\right) + C\left(K_I/K_{IC}\right)^2 \tag{5}$$

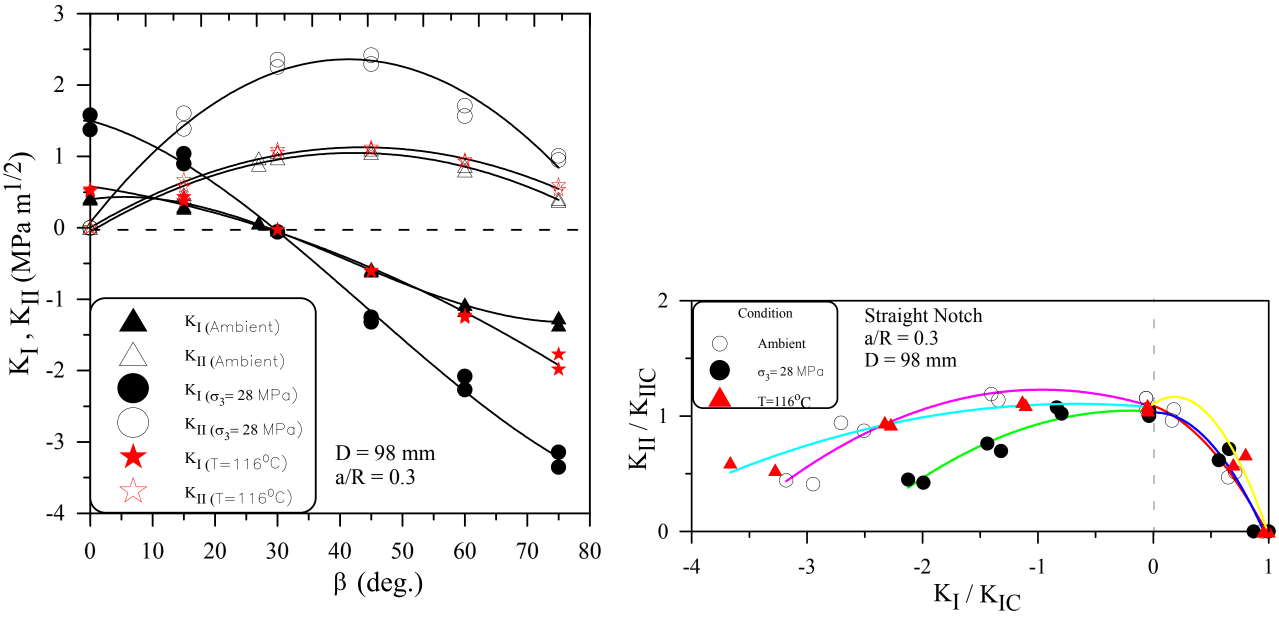
where, *A*, *B*, and *C* are the coefficient for the second order polynomial used for the regression. The values of *A*, *B*, and *C* for various experimental conditions are tabulated in Table 2.

It can be seen from Figure 6 that the results for all three conditions fall into a relatively close bound in the positive zone; however, distinct regions of data exist in the negative side. Note that the results for the specimens tested at high temperature fall close to the data for ambient conditions in the negative zone revealing that the fracture toughness is not very much affected by the temperature used in this study. However, fracture toughness envelope at high confining pressure, in the negative region, is extremely lower than that at ambient condition.

TABLE 1

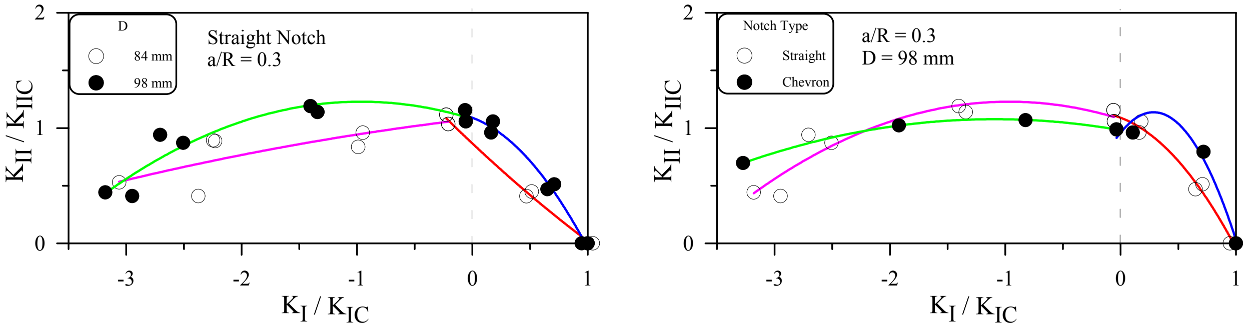
COMPARISON BETWEEN K_{IC}, K_{IIC}, AND THEIR RATIO AT VARIOUS CONDITIONS

Condition	Diameter (mm)	K_{IC} (Mpa $m^{1/2}$)	K_{IIC} (Mpa $m^{1/2}$)	K_{IIC}/K_{IC}
Ambient	98	0.42	0.92	2.19
	84	0.35	0.75	2.14
σ_3 = 28 MPa	84	1.19	1.49	1.25
	98	1.57	2.18	1.39
T = 116°C	98	0.52	1.00	1.92



(a) **(b)**

Figure 2: Compar ison of (a) fracture toughness, (b) fracture envelopes



(a) **(b)**

Figure 3: Fracture envelopes for Brazilian disks with (a) different D, (b) different notches

TABLE 2

REGRESSION PARAMETERS FOR VARIOUS CONDITIONS

Condition	Notch Type	D (mm)	Positive Region			Negative Region		
			A	*B*	*C*	*A*	*B*	*C*
Ambient	Chevron	98	0.9589	1.2436	–2.1709	0.9811	–0.1746	–0.0799
	Straight	84	0.8664	–0.9402	0.0928	1.0797	0.1162	–0.0203
	Straight	98	1.0891	–0.3701	–0.7584	1.0779	–0.3120	–0.1616
σ_3 = 28 MPa	Straight	98	1.0290	0.0461	–1.1663	1.0399	–0.0670	–0.1779
T = 116°C	Straight	98	1.1046	0.6635	–1.7977	1.0759	–0.0882	–0.0661

CONCLUSIONS

For the investigated limestone rock, the effect of confining pressure on K_{IC} and K_{IIC} is much more significant than the effect of temperature. K_{IC} increased by 274% under $\sigma_3 = 28$ MPa, but the corresponding value at T = 116°C was 24%. K_{IIC} increased by 137% under $\sigma_3 = 28$ MPa, but the corresponding value at T = 116°C was only 9%. Also, the effect of confining pressure on K_{IC} is much more significant than its effect on K_{IIC}. The above observations lead to the conclusion that the Mode-II component may be the most critical mode controlling failure at high values of temperature and confining pressure.

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