FRACTURE BEHAVIOUR OF A FIBRE-BRIDGED CRACK

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ABSTRACT

The effects of interfacial debonding and sliding on the fracture characterisation of unidirectional fibre-reinforced composites are studied numerically under small-scale bridging conditions in plane strain. The cohesive zone, more commonly called the fracture process zone (FPZ), is characterised by matrix separation and individual fibre pullout. A bi-linear stress-displacement fibre-bridging law is incorporated into the computational model to account for interfacial debonding and sliding and fibre breakage, instead of the line-spring model in which the fibres remain intact. The matrix fracture process is described by a linear stress-displacement cohesive law. Anisotropic elastic constitutive relations are used for the effective properties of the brittle-matrix composite outside the FPZ. A remote boundary condition is also imposed in terms of the elastic solutions for a mode I crack. Crack advance and fibre breakage are direct consequences of the constitutive modelling without any *ad hoc* crack growth criterion. Numerical implementation of the finite element method with the embedded cohesive zone containing fibre bridging and matrix fracture is elucidated. Fracture resistance (R)-curve is obtained for a range of conditions to highlight the debonding and frictional behaviours at the fibre-matrix interface. It is shown that the fracture toughness can be enhanced by optimisation of the fibre-matrix interfacial properties. Numerical results also show that fibre breakage plays an important role on the reduction of the slope of the R-curve.

KEYWORDS

Cohesive zone, Fibre-bridging, Matrix cracking, Interface behaviour, (R)-curve, Finite element method, Fibre pullout, Fibre breakage

INTRODUCTION

In fibre-reinforced composites, the interaction of a fibre with the matrix is of great interest for failure assessment. Interfacial debonding and frictional sliding associated with the fibre pullout process are regarded as two important mechanisms to increase the toughness of unidirectional fibre-reinforced composites. In general, a cell model for a single fibre pullout test is used to evaluate the relationship between pullout force and pullout displacement. The fibre pullout force-displacement curves obtained provide a good approximation of the crack-face bridging law in the wake region [1-3].

Various approximation methods with regard to the bridged-crack have been developed. Aveston, et al

[1] estimated the applied stress at which matrix cracking takes place with the fibres left completely intact. In Marshall and Cox's work [2], a shear lag analysis, i.e., a constant shear frictional stress, is used to deal with interfacial sliding resistance. This is valid for a composite system with a weak or un-bonded sliding interface. Also, different forms of bridging laws have been presented in the past decade to account for the effects of fibre debonding and fibre sliding [4,5]. Further, the effect of interface roughness on the bridging law was analysed by Liu, Zhou and Mai [6]. Even so, the effect of fibre-matrix interface debonding and sliding on the fracture process has not yet been thoroughly investigated.

In low-density fibre composites a crack is bridged by a few fibres, and the smeared-out model cannot be usefully applied without loss of accuracy. Hence, there is a need to understand the bridging effect by discrete fibres over a finite length. Meda and Steif [4] discussed several reasons for poor agreement between experimental observations and theoretical predictions based on the continuous distribution of tractions along the crack-faces. It is desirable to develop an independent model where discrete fibres are analysed explicitly, rather than implicitly as in earlier crack-bridging models. Fibre breaks play an important role in the softening behaviour of fibre-reinforced composites. However, most solutions available for crack resistance (R)-curves are obtained from approximate shear lag analysis and smeared-out representation of the bridged area and do not include fibre breakage. Also, fracture of composites involves more than one physical process. Thus, rupture in unidirectional fibre-reinforced composites is associated with brittle matrix fracture, fibre pullout and fibre fracture [7]. In principle, distinct failure mechanisms cannot be incorporated in a unified cohesive zone model, although attempts have been made to do so. Hence, matrix cracking and fibre bridging should be modelled separately by a two-part description. That is, the coexistence of a fibre bridging zone and a matrix cohesive zone, whose stresses depend on the crack-face opening displacement.

The aim of this work is to study the toughening mechanisms of aligned fibre composites by finite element analysis. Two failure mechanisms, fibre-bridging and matrix cracking, are considered separate entities, rather than by a single smeared-out bridging law. A bi-linear constitutive relation is employed to describe the fibre-matrix debonding and frictional sliding process. A softening cohesive law is used for matrix separation to avoid the stress singularity in conventional bridged-crack models. Numerical results are calculated for a carbon fibre-epoxy unidirectional composite with a low fibre density. The effects of the interface properties and fibre breakage on the fracture toughness of the composite are presented and discussed in detail.

THEORETICAL ANALYSIS

Bridging Law during Fibre Pullout

Consider an infinite uniformly aligned unidirectional fibre composite containing a semi-infinite crack. The crack-face separation and crack growth are restrained by the bridging fibres. Descriptions of the fibre bridging stress and crack-face displacement relationship can be obtained by a theoretical analysis of a fibre pullout process [8] as typified by the stress-displacement curve in Fig.1, in which σ is applied stress at fibre end and δ is pullout displacement. σ_{1f} and δ_{1f} are the initial debonding stress and displacement, respectively. Two parameters, σ_{1f} and the slope κ (see Fig.1) are used as indicative measures of the interfacial bond strength and friction coefficient at the partially debonded interface, respectively. They define the bilinear law of the stress-displacement relation prior to the full fibre pullout from the matrix (which is not shown). Computer simulation [9] study shows that a strong bond gives a high initial debonding stress and a high interfacial friction. However, a very strong bond will increase the full debonding stress but decrease the pullout-displacement due to fiber breaks.

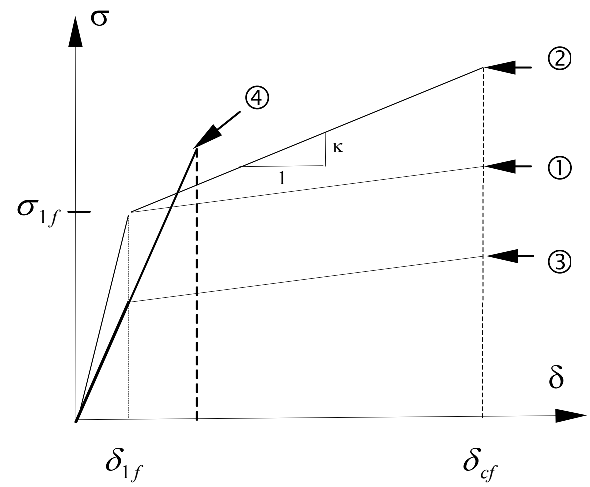

Fig. 1. Schematic representation of fibre-bridging stress with pullout displacement. Four cases are studied in this work are shown here and case 4 is for a very strong bond.

Cohesive Law for Matrix Cracking

The constitutive relationship of the matrix cohesive zone is described by the traction σ and the displacement δ. It is recognised that the specific work of separation, $\Gamma_0 = 2\int_0^{\delta_1} \sigma(\delta) d\delta$, reaches a critical value when crack growth commences. In the numerical results of Xu, Bower and Oritz [10], it was found that fibre debonding occurred as the matrix crack reached a fibre. Hence, we assume that the maximum separation δ_1 across the matrix cohesive zone must equal the fibre pullout displacement δ_{1f} corresponding to σ_{1f} as shown in Fig. 1. The shape of the cohesive law is assumed to be a linear function of the separation displacement and is given by: $\sigma = \sigma_1(1 - \delta / \delta_1)$ so that $\Gamma_0 = \sigma_1 \delta_1$.

Finite Element Method

Finite element simulations are based on the updated co-ordinate Lagrangian formulation for a dynamic case. For quasi-static cases studied here, the loading rate is assumed to be very small to eliminate the kinetic effect. All physical quantities are functions of a set of moving co-ordinates x^i at time t. Fig. 2 shows the schematic diagram of this small-scale bridging problem of a semi-infinite crack in a uniform unidirectional fibre composite. Due to symmetry only a half cracked specimen needs to be considered. The fracture plane is replaced by the cohesive zone with fibre-bridging and matrix separation. The stresses are related to δ by the assumed cohesive laws.

Numerical method is implemented by displacement-based finite element method and uniformly distributed fine meshes are placed along the fracture plane to simulate crack growth. The size of these elements is assumed to be the fibre spacing. A linear interpolation procedure is carried out to obtain values of δ at four Gaussian integration points. Then the stress can be obtained from the cohesive law. A fourth order Gaussian integration scheme is used to obtain contribution to the fracture work from the matrix process zone. The fibre bridging force is a force at each node due to the small fibre radius. Its magnitude is determined by the crack-face displacement. The results are obtained for a semicircular region with initial radius R_0 = 200 mm. It is chosen such that $R_0 = 4000\Delta$ in which Δ is the size of the smallest mesh element ahead of the crack-tip. $\delta_{cf} = 10\delta_1 = 0.1\Delta$ is selected to determine the maximum separation of the fibre and $\delta_1 = 0.01\Delta$ is the maximum matrix separation. A fine mesh with length $L_0 = 48\Delta$ is placed ahead of the crack tip to simulate crack growth.

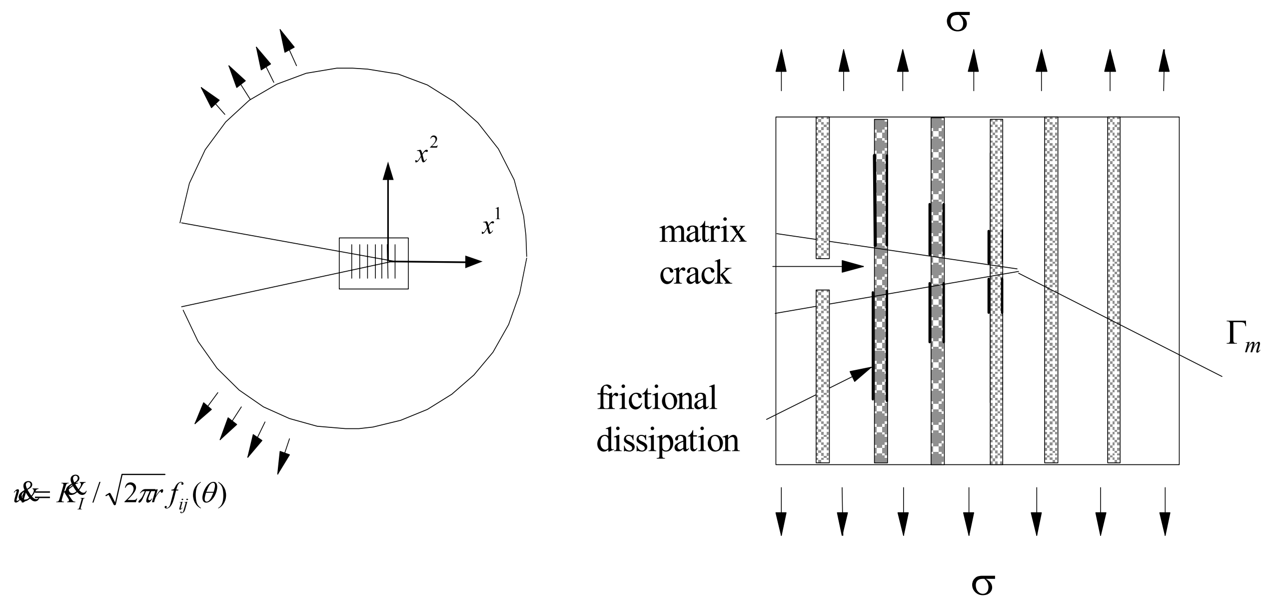

Fig. 2. Schematic diagram of the small-scale bridging problems of a semi-infinite crack in uni-directionally aligned fibre reinforced composites.

Small-scale bridging conditions involve imposing initial and boundary conditions appropriate to the linear transversely isotropic and orthotropic elastic crack-tip fields, respectively. The traction-separation law used to model the fracture process is specified everywhere on the boundary $x^1>0$ and $x^2 = 0$ of the region analysed, while zero traction exists on the boundary $x^1<0$ and $x^2 = 0$. On the outer semi-circular boundary, the displacements, $\dot{u}_1$ and $\dot{u}_2$ are given by the external stress fields, specified by the incremental rate of the mode I stress intensity factor $\dot{K}_I$. Details of the equations, mesh and FEA procedure are given in Ref. [11].

NUMERICAL RESULTS

Numerical calculations are carried out for carbon fibres in an epoxy matrix with material parameters E_f=230 GPa, $E_m = 3$ GPa, $\nu_f = 0.2, \nu_m$=0.4 and V_f=0.1. Thus, we can obtain the effective elastic modulus and Poisson ratio as $\overline{E} = 25.7$ GPa and $\overline{\nu}$=0.38, as well as the mass density 2000 kg/m^3. The computation is stopped when sufficiently long crack growth occurs. We assume the fibre strength to be uniform and the fibres break at the crack plane. (However, non-uniform fibre strength distribution can be included in our computation model as in [12,13].) Hence, there is no fibre pullout due to frictional sliding from the matrix. For numerical calculations, we give the fibre a small extra pullout displacement, $\delta_{1f}/10$, so that the pullout force can be reduced to zero. This also stabilized the computations. For simplicity, the debonding strength σ_{1f} is normalised by the matrix cohesive strength, i.e., $\eta = \sigma_{1f} / \sigma_1$. In the calculations, we assumed fibre debonding begins when the matrix crack-tip reaches this fibre. The non-dimensional cohesive strength of the matrix $\sigma_1 / \overline{E}$ is assumed to be 5×10^{-3}. Thus, the cohesive energy G_{mc} for the matrix is 66.25 J/m^2. Based on the relationship: $K_{mc}^2 = E_m G_{mc}/(1-\nu_m{}^2)$, we obtain the matrix critical stress intensity factor K_{mc}=0.486MPa$\sqrt{\text{m}}$.

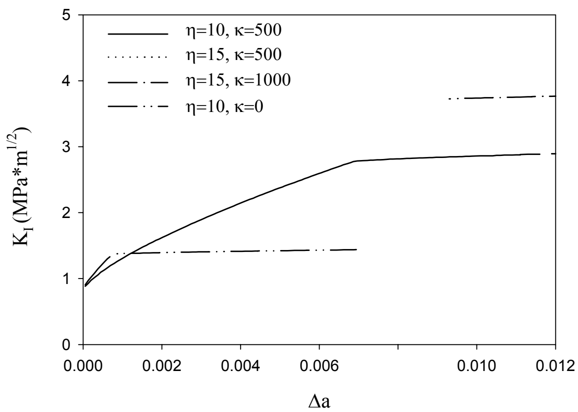

Fig. 3. Fracture resistance curves for several values of parameters η and κ.

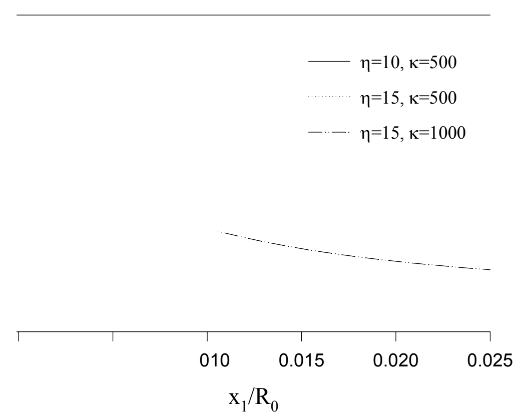

Fig. 4. Fiber stress ($\sigma/\overline{E}$) along the crack-line (x_1/R_0) for several values of parameters η and κ

Figure 3 shows typical R-curves for different debonding energy and interfacial frictional resistance. An increase in σ_{1f} means an improvement in debond toughness and an increase in κ, normalised by $\overline{E}/R_0$, means an increase in frictional sliding resistance. It is clear that crack initiation is delayed by increasing η. Fracture toughness is enhanced as crack advances at a relatively large slope because of the dominant effect of fibre-bridging. An increase in the frictional sliding resistance also increases the fracture toughness. Due to the complexity of the pullout process, the effects of fibre sliding at the debonded interface cannot be separated in the bridging law. However, it is reasonable that an increase in fracture resistance takes place at large κ. There is a transition of the R-curve from an initial rising part to a steady-state in which the length of the bridging zone does not change with subsequent crack extension. This transition occurs when the fibres at the receding edge of the bridging zone begins to break. The very slight rise of crack-resistance in the steady-state region is caused by kinetic effect since extremely large CPU time is required for lower loading rate than that used.

Increasing the interface strength will accelerate fibre breakage due to the stress elevation in the fibres ahead of the crack-tip. The nett effect is a reduction in the fibre pull-out displacement. If we take case 4 in Fig. 1 with η=10 and $\delta_{cf} = 2\delta_1$ for a very strong interface, the steady-state toughness is much lower but slope much larger at the initial stage than that with the same η, Fig. 3. It is unwise hence to increase the interface strength, as the admissible inelastic deformation accompanying fibre sliding is severely curtailed. There are optimal interface properties to obtain maximum fracture toughness.

As fibre stress ahead of the crack-tip is not affected by the amount of crack advance at the stage of steady-state crack growth, the distribution of fibre stress at incipient fibre breaks can be taken as typical for a set of given material parameters. Fig. 4 shows distributions of normalized fibre stresses for three cases studied. With increasing interface strength and frictional resistance, the maximum fibre stress also increases. This corresponds to 0.06, 0.084 and 0.095 for the three cases. Although a large κ can increase the maximum stress in the bridged zone, it decreases rapidly to the same level as low κ's at the crack-tip, as frictional sliding does not come into effect ahead of the crack-tip.

CONCLUSIONS

A finite element analysis was carried out for the failure behaviours of aligned fibre composites in terms of an embedded cohesive zone with matrix cracking and fibre-bridging. These two failure mechanisms, matrix cracking and fibre debond, pullout and fracture, are considered as separate entities, rather than under a single smeared-out bridging law. In describing the properties of fibre debonding and frictional sliding, a bi-linear constitutive law is used and in which, two parameters, η and κ, have been utilised to characterise these two effects on the fracture resistance.

Numerical results show that the composite toughness can be increased by improving the interfacial properties. However, if the interface is too strong, low toughness can be caused by fibre breakage. Hence, high toughness can only be achieved by optimal design of the fibre-matrix interface.

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