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FRACTURE AT COMPRESSION

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ABSTRACT

Fracture processes under compressive loading are considered from two points of view: 1) specific fracture mechanisms at compression and their influence on the material strength and fracture resistance; 2) an influence of the geometric constraints on the stress-deformation state and fracture conditions of bodies (natural objects) with cracks and cracklike faults (in particular, effects of a crack surfaces contact, friction, loading hystory; features of thin bodies fracture).

KEYWORDS

Compression, microstructure, crack, strength, fracture resistance, friction, loading hystory, thin bodies

INTRODUCTION

Fracture at compression is a rather complex process which occurs by combining some features of fracture at tension and shear complicated by an influence of the microstructure and geometric constraints. The observed relations between the real compressive and tensile strengthes can not be described by classical fracture criteria. To overcome this difficulty Griffith suggested to take into account a cracklike defect behaviour under compression. The theory predicted that the ratio of the compressive strength to tensile one equals 8. This result is not confirmed by experimental data. A new stage of studying the fracture at compression started at the early 60^{th} and was associated with a more detailed analysis of micro- and macromechanisms of fracture at compression. Possibilities of microcrack nucleation near the material nonhomogeneities or near existing cracks, crack surface contact accompanied by friction and dilation effects, and, finally, crack surfaces overlapping in thin bodies create diversity of schemes and forms of fracture at compression. The paper is aimed to demonstrate some characteristic features of fracture at compression and models which enable to describe and predict the observed processes. The models are often related to nonstandard using or generalization of fracture mechanics concepts.

FRACTURE AT COMPRESSION

Macrostrength and microfracture

Brittle fracture in bulk under compression is as a rule related to redistribution of general compressive stresses into local tensile ones at the material structural or textural nonhomogeneities (such as pores, inclusions, etc). The fracture mechanism can be illustrated by the classical example: uniaxial compression of an elastic plane with a single circular hole (pore) by the stresses (σ). The tensile stresses occur at a part of the hole contour and attain their maximum value, σ, at the points of the hole contour intersection with its diameter parallel to the compression direction. The tensile stresses act in the transverse direction and lead to nucleation and growth of two cracks emanating from the hole contour. This fracture scheme was, in particular, observed on the prisms of natural quartz single crystals free of cleakage with a small central cylindrical hole drilled normal to the largest face [1]. The crack growth is stable since an influence of local tension decreases as the crack length increases.

In case of a real porous material each pore is a potential source of microcrack nucleation in the direction of sample compression. The distinct difference from the case of a single hole is related to the transition from the initial stage of the stable crack growth to the stage of the unstable coalescence of cracks growing from the adjacent pores. This process leads to formation of a series of macrocracks which separate the sample on the layers parallel to the compression direction if a friction influence was eliminated along the contact surfaces of the compressed sample. The compression level at the attaing the crack instability determines the material compressive strength, σ^{c}_{st}. The process of the microcrack growth is adjusted by the material fracture toughness at the scale of their sizes. In turn, the microcrack size when it becomes unstable depends on the material porosity. As a result the compressive strength of a porous material depends on both the porosity and microfracture toughness. A quantitative model taking into account the aforedescribed mechanism of porous material brittle fracture under uniaxial compression was developed in [2]. The theoretical dependence of the compressive strength on porosity fitted the available and authors own experimental data for a series of brittle effusive rocks. Note, that a close model was suggested to describe the similar fracture mechanism of fracture of ceramics under compression [3].

One of the key points of the aforedescribed fracture mechanism was associated with occuring an instability stage of microcrack growth in the compression direction. However, this process can be arrested by compression in the transverse direction at a certain length of an elongated cracklike defect formed by an array of pores linked by microcracks. In turn, this cracklike defect causes redistribution of the stresses in a material. An analysis shows that a process of nucleation and growth of another adjacent similar cracklike defect can be initiated near the end zones of this cracklike defect, etc. This mechanism leads to formation of an ordered system of cracklike defects (structure of fracture) under bi-axial compression. An approach to modeling formation of ordered crack (or cracklike defect) systems (structures of fracture) was suggested in [4,5] and [6] and summarized in the review paper [7]. Note, that formation of structures of fracture represents an often observed mechanism of fracture transition from one scale to another (see, [7]).

Ordered crack systems are formed not only at brittle fracture under compression. Let us illustrate this possibility for a porous material. Again consider a plane with a circular pore under uniaxial compression. Remind, that the shear stress concentration occurs at the points of the pore contour interaction with the diameter transverse to the compression direction. The maximal shear acts along the planes inclined by the angle $\pm\pi/4$ to this diameter. In case of shear fracture mechanism slip lines can occur along these directions. As a result a net of cracklike elements of rhombic shape will be formed between the adjacent pores. Such structure of fracture was observed on the plates of a material of high plasticity [8] and modeled (see [7]).

Microcracks along with the considered microstructural nonhomogeneities can initiate macrofracture formation in the direction of compression. The basic element of the appropriate well known mechanism (see, e.g. [9-15]) is a microcrack inclined to the compression direction. Slipping effects along a contact zone of the microcrack surfaces lead to formation of the wing cracks.

One of the possibilities of macrofracture occuring is associated with the wing cracks coalescence. Usually the compressive strength is evaluated within the framework of this mechanism for a 2D model. An experimental and theoretical study of the 3D effects was performed in [16]. Experiments on uniaxial compression of transparent samples with inner diskshaped cracks demonstrated that macrocrack formation can only result from the interaction of properly oriented and closely located wing cracks. The critical distance for two coplanar cracks is of order four crack radii. This experimental observation was also explained within the framework of a model of 3D wing crack interaction suggested in [16] using the far field asymptotics of the crack induced stress field given in [6].

Another possible scenario of macrofracture initiated, in particular, by wing cracks can be related to the occuring of compression-shear bands formed by ordered (echelon like) wing crack systems [15]. In turn, the compression-shear bands can initiate inner or surface buckling. Note, that mechanisms and models of global compression fracture were summarized in [15] and used for an analysis of the scale effects at compressive fracture.

Brittle fracture of samples by macrosplitting occurs in conditions of uniform compression in the absence of confining stresses at the end surfaces of a sample. The confining stresses, as a rule, cause a transition of the brittle macrofracture scheme from splitting to inclined fracture. However, similar transition is also observed under compression without confining stresses if material fracture is accompanied by some plastic effects (see, e.g. [17]). Experiments on cementitious materials [17] demonstrated both the splitting mode for pure cement samples and inclined (shear) mode for concrete ones under the same loading conditions. To clarify the mechanism of the fracture mode transition, the tests were performed on samples made from hydrated cement paste with filling of different grade [18]. The results of the experiments and modeling showed that in this case the macrofracture mode under uniaxial compression was adjusted by the microplastic behavior of the material. Splitting was a result of formation and growth of cracklike macrofaults with a long process zone being formed because of the material high microplasticity. On the other hand, in case of a short process zone (in microscopically brittle materials) growth of a macrocrack inclined to the compression direction and shear fracture were caused by an influence of a lateral surface of a sample.

Macrocrack with partial contact of its surfaces

The aforegiven analysis was related to the global fracture at compression based on the mechanisms of multiple microcracking. Now let us consider some features of deformation and fracture of an elastic body with a macrocrack or cracklike cavity at compression.

First, for simplicity we will illustrate the effects for a homogeneous, isotropic, linear elastic space with a plane crack. The influence of a body boundary and the behavior of thin bodies will be discussed later on.

Consider a plane crack-cut under compression transverse to the crack plane. Then the crack surfaces contact occurs along the whole crack domain when an analysis being performed within the framework of the linear elastic fracture mechanics. Indeed, in opposite case the stress intensity factor, K_I, becomes negative and, hence, the crack surfaces overlapping needs to occur. The full crack closure is a consequence of the impossibility of crack surfaces overlapping in bulk [19,20].

More complex behavior is inherent to cracklike cavities. For instance, consider a cracklike cavity of the cross-section Ω in the plane $x_3 = 0$ of the x_1, x_2, x_3 coordinate system. The distance between the cavity surfaces, $w_0\,(x_1,x_2)$ is a single-valued function of (x_1,x_2). The initial cavity opening, $w_0\,(x_1,x_2)$, is small as compared with the characteristic sizes of the domain Ω. A partial contact of the cavity surfaces can occur under the compressive loading at infinity and/or in bulk. The problem under consideration was analyzed in [21] within the assumption on the friction absence in the contact region. The asymptotics of the solution near an arbitrary smoothness point of the boundary separating the contact region and the opening region was obtained for a rather general geometry of the cavity surfaces. As a result the conditions were obtained which enable to determine a priori, without solving the problem, the regions of cavity surfaces where their contact is impossible. We will illustrate these conditions for the cavities with elliptic cross-sections. The initial cavity opening has the following $w_0\,(x_1,x_2) = b\,(1-x_1^2/a_1^2-x_2^2/a_2^2)^{\alpha/2}$. The process of contact region formation starts at the cavity's boundary (in plane $x_3 = 0$) if $\alpha \geq 1$, and inside the cavity if $0 < \alpha < 1$. For this specific geometry of the cavities the contact regions were also studied numerically in [22], where a boundary variational method was suggested for solving the 3D problems of cracks and cracklike cavities with contact regions without friction. Note, that a series of qualitative properties of the problems solution were established in [21,23]. On the basis of these properties another nonvariational numerical method for solving the problems was developed. The method enables to construct a sequence of approximate solutions which monotonically tend to the exact solution from one side.

Note, that in some cases the unilateral constraints of the cavity surfaces displacements related to a possibility of their contact are replaced by the condition implying that these surfaces drawing together w (x_1, x_2) needs to be nonless than the values of a given function $u_0 (x_1, x_2) > 0$ such that $w (x_1, x_2) \geq u_0 (x_1, x_2)$. For instance, this condition is inherent to the statement of the problem on evaluation of the drawing together of the surfaces of a mine (see, [22]). The regions where the strict inequality or equality in the above condition are fulfilled need to be determined.

Having the solution of the appropriate elasticity problem with unknown boundaries separating the fulfillment the conditions in the form of equalities and inequalities one can calculate the stress state near the cavity (crack) surfaces and analyze possible fracture mechanisms and conditions.

We would like to pay attention to a model of fracture and stability near a mine suggested in [24]. A model is based on the consepts of the LEFM. A mine is modeled as an elongated striplike narrow cavity in an elastic space such that really a 2D elasticity plane strain problem on a cavity of the characteristic height h and length ℓ ($h << \ell$) located transverse to the compression direction is considered. At the scale ℓ the mine can be modeled as an effective crack under compression with additional condition which admits the crack surfaces overlapping less than the half of the mine height (the outer problem). Evidently, the stress intensity factor, K_I, near the tip of this crack will be negative. Further, at the scale h one has an inner problem on a half-infinite mine of height h. The conditions of biaxial loading near its end section (the mine working face) are determined by the stress intensity factor of the outer problem. Solving the inner problem one can search for the characteristic regimes of fracture, rockburst and loss of stability of a mine. Examples of the analysis were given in [24]. An energetic approach based on a concept of a release of the energy stored in the rock massiff was also suggested for an analysis of the similar problems of mining engineering (see, e.g. [15]).

Macrocrack. Friction and loading hystory effects

We considered a partial contact of the cavity (crack) surfaces without friction under the compressive load normal to the plane $x_3 = 0$ along which the cavity is elongated. Friction between the cavity (or crack) surfaces within the region of their contact leads to some new effects in the deformation and fracture processes. In particular, the loading hystory and related stick-slip phenomena become to be essential. The general statement of the 3D problems was given in [25] and their analysis was performed in [25-29]. Some aspects of the crack problems with friction were considered in [30,31] for 3D case and in [32-35] for 2D case. For the sake of definiteness and simplicity consider a crack in the plane $x_3 = 0$ of an elastic space. Assume that the loads acting at infinity and (or) in bulk are slowly varying in dependence on a loading parameter θ (complex loading) such that the inertia effects are negligible. A crack surfaces interaction within the contact region obeys the Coulomb friction law: $F_{3i} = \rho\sigma V_i/V$, where ρ is the friction coefficient, $\sigma (x_1,x_2,\theta)$ is the normal stress (pressure), $V_i = \partial U_i/\partial\theta$ are the components of the slipping velocity, $U_i = U_i^+ - U_i^-$ are the components of the displacement jump $(i = 1,2)$, $V = (V_1^2 + V_2^2)^{1/2}$. Note, that the sign in the right hand side of the friction law was chosen taking into account that the displacement jump is counted off from the upper surface of the crack-cut.

The problem is subdivided on symmetric and antisymmetric relative to the plane $x_3 = 0$ ones related to the normal displacement jump, U_3, and shear displacement jumps, U_1, U_2, respectively. It is possible to separate the initial problem in spite of its nonlinearity since the normal and shear components of the displacement jumps do not cause shear and normal stresses at the crack plane, respectively. Then the initial problem is solved sequentially. The contact region and distribution of the normal stresses $\sigma_{33}^{\pm} (x_1,x_2,\theta)$ in this region are determined within the framework of the symmetric problem (the normal problem). The antisymmetric problem (the shear problem) implies searching for the regions of slipping, $G_{s\ell}$, and sticking, G_{st}, where

$V_i \neq 0$ and $V_i = 0$, respectively, and the distribution of shear stresses in these regions at the known contact region and distribution of the normal stresses.

Note the essential feature of the crack problem with friction at the complex loading [25]. The sticking region is determined by the condition of vanishing the slipping velocity. Hence, two tipes of the sticking regions can occur: the sticking regions with zero and non-zero shear displacement jumps, $U_i = 0$ $(i = 1,2)$ and $U_i \neq 0$ $(i = 1$ and/or $2)$, respectively. Occuring the sticking regions with the fixed shear displacement jump is related to a memory of the loading process. The loading process leads to the evolution of contact, slipping and sticking regions in dependence on the loading hystory. In turn, this evolution determines (through the stress intensity factors) occuring the crack limit equilibrium and its growth.

To solve the appropriate boundary value problem taking into account the loading hystory one needs to know some properties of the normal and shear problem solutions and additional conditions proved in [25, 27, 28]. First of all, the shear displacement jumps, U_1, U_2, are continuously differentiable functions of the loading parameter. The distribution of the shear displacement jumps and the slipping rates are uniguely determined at the end point of the given loading trajectory. Further, denote by $\sigma_{31}^0\,(x_1,x_2,\theta)$, $\sigma_{32}^0\,(x_1,x_2,\theta)$, $\sigma_{33}^0 = -\,\sigma\,(x_1,x_2,\theta)$ the loading trajectory. Then an increment of the shear displacement jumps δU_1, δU_2 can occur if it exists a point (x_1,x_2) within the crack region Ω such that $\delta\tau\,(x_1,x_2,\theta)\cos\gamma \geq \rho\delta\sigma\,(x_1,x_2,\theta)$ where $\tau = [(\sigma_{31}^0)^2 + (\sigma_{32}^0)^2]^{1/2}$, γ is the angle between the vectors $(\delta\sigma_{31}, \delta\sigma_{32})$ and $(\delta U_1, \delta U_2)$, ρ is the friction coefficient. An essential property of the solution is related to a possibility of the slipping continuation at the normal load decreasing, $(\partial\sigma/\partial\theta) < 0$ [27, Part II]. Indeed, it means that an increment of the displacement jumps and stress intensity factors can occur at the unloading stage if the boundary of the slipping region and the crack contour have common points. Hence, a crack can attain a limit equilibrium state at decreasing the normal load. This effect is associated with a possibility of a more fast decreasing of the friction force as compared to the shear load. Note, that this effect is inherent to the problem with friction. It was demonstrated in an explicit form for the problem on an annular crack in an elastic space under the action of a single compressive concentrated load acting at some distance from the crack plane [27, Part II). The properties of the slipping angle were studied in [28]. In particular, it was shown that the slipping angle near the zones of the positive pressure is a continuous function of the loading parameter both in the slipping region and during its extension into the sticking one. This property enables to write the initial nonlinear problem in the increments and to obtain a linear problem relative to the slipping angle increment. Then the slipping angle increment can be calculated by a step-wise procedure at the given slip line field and increments of the external loads.

The slipping process near the regions of the crack opening was also analyzed. It was shown that the slipping angle is not a continuous function near the boundary of the opening region. Hence, generally, two types of the regions were separated relative to the slipping regime: regions of the stable slipping (the slipping angle and slipping rate are continuous functions of the loading parameters) when the problem admits linearization and regions of the unstable slipping (infinitesimal variations of the external loads lead to finite increments of the slipping angle) where the problem does not admit linearization. Numerical algorithms for solving the normal and shear problems were suggested in [27,28] taking into account the aforedescribed properties and conditions determined by the asymptotic behavior of the solutions at the boundaries of the regions of the different types within the crack [25]. These conditions enable to search for the location of the unknown boundaries (in particular, the boundary of the slipping and sticking regions are determined by the condition that the solution needs to be nonsingular at this boundary [25]).

Several analytical and numerical examples demonstrating the efficiency of the suggested methods for solving the crack problems with friction at complex loading were given in [26-28]. In particular, the elliptic crack with interacting surfaces was analyzed in [26] under the uniform loading depending on the loading parameter. In this case the component of the shear displacement jump along each of the main axes of the ellipse only depends on the appropriare component of the shear stress. The slipping angle is independent on the coordinates x_1, x_2 and is determined by an ordinary differential equation of the first order as a function of the loading parameter. A geometric interpretation of the slipping regimes was suggested such that a slipping cone can be constructed at each point of the loading trajectory. The cone determines the continuity of the slipping process and transition to the sticking state. The geometric interpretation enables before the problem solving to separate a possible slipping regime, a friction force direction and initial conditions at the transition from the sticking state to the slipping one. It is essential that the evolution of the far field is closely related to the processes of continuing loading or unloading at the crack such that the observations of the displacement and stresses far fields can be used for an analysis of a limit equilibrium of a crack. This effect can be used in geophysics.

Let us pay attention to an effect in a simple axisymmetric crack problem. A singular point (the crack center) exists in this problem. Neither displacement nor stress direction are not determined at this point. This circumstance leads to a qualitative effect in the problem with friction. Indeed, a finite sticking zone occurs near the crack center, e.g., at the classical loading trajectory: compression to the value σ and then axisymmetric shear τ (r).

Remarks. I. We considered the behavior of the plane cracks. The compression and friction effects essentially influence the conditions of the curvilinear cracks limit equilibrium and propagation.

Remind, that the contact zones can occur along a curvilinear crack even under uniform tension (e.g. for an arc crack of sufficiently large angle [36]).

Further, the stability of the trajectory of a slightly curved crack is influenced by the compressive loads acting along the initial nonperturbated crack direction. It is essential that the conditions of the stability are determined by a nonsingular term of stresses near the crack tip caused by this compressive load [37].

An analysis of the limit equilibrium of a curvilinear crack taking into account friction in the contact zones at complex loading was recently performed in [38].

II. Fracture at compression of bodies (media) with many cracks is complicated by the crack influence on the effective deformation characteristics of the object under consideration. The crack interaction leads to an induced deformation anisotropy, nonhomogeneity and irreversibility of deformations in the loading-unloading regime, as well as to essential dilatation effects both at compression and shear. These effects were observed experimentally and described theoretically (the references are given, e.g., in [39-41]). Note, that an induced nonhomogeneity causes redistribution of a stress state of a body [41]. For instance, longitudinal compressive deformations occur in a cracked plate at bending; the Euler critical load of a plate loss of stability is also changed for a cracked plate.

Geometric constraints influence the mechanisms of fracture at compression. Thin bodies

First, let us consider a specific mechanism of fracture at compression along a body surface or interface associated with formation and growth of a crack parallel to the surface (delamination in case of an interface). This fracture mechanism is, in particular, inherent to composites, e.g., layered systems and systems with coatings. An energetic approach was suggested in [42,43] (see also [44-46]) to search for the conditions of a dilamination limit equilibrium if its characteristic size is much larger than the thickness of a delaminated layer. According to this approach the energy balance is evaluated by modeling the delamination as a beam (or plate) rigidly clamped at its ends (contour). An asymptotic approach to the problem based on the method of the matched asymptotic expansions was suggested in [47,48]. This approach enables to make more precise the estimates given by the energetic approach by replacing the condition of rigid clamping on a condition of elastic clamping obtained in [47,48]. A similar condition for the 2D case was given in [49].

Effects of loss of stability and breakdown were also observed in systems with coatings (see, e.g., [50,51]). The systems based on polymer (polyethylene terephthalate) with a thin stiff coating (platinum or SiO_2) with a thickness of several nanometers were studied. The samples were loaded by uniaxial tension in the direction parallel to the interface. The fracture process of two stages was observed. First, a system of parallel cracks occured in the coating separating it on a series of strips. Then loss a stability and regular fracture of separate strips appeared under the action of a transverse compression of the deformed polymeric substrate. A model of the process was also suggested in [50,51].

Possible mechanisms of loss of stability of surface layer on an elastic halfspace or a surface layer of an elastic halfspace with elasticity parameters variable through its thickness under the action of bi-axial compression caused by steady termal loading were studied in [52]. In particular, it was shown that if the substrate (half-space) is more compliant as compared with the surface coating then loss of stability will be accompanied by formation of concavities with a characteristic length essentially exceeding the coating thickness.

In thin bodies with elongated through crack (or cracklike defects) of characteristic length much larger than the body thickness one can observe a specific mechanism of fracture at compression associated with a possibility of crack surfaces overlapping [53-55]. For instance, consider a thin plate with a stright through crack under uniaxial compression transverse to the crack plane. The zone near the crack is more compliant than the initial plate. As a result the crack surfaces can be shifted in the direction transverse to the plate plane. Then an overlapping of the crack surfaces occur at least within the central part of the crack, while the zones of the compressive stress concentration will be localized near the crack edges. The fracture process in these end zones is adjusted by the value of the negative stress intensity factor. Different fracture schemes in the end zones are related to different ranges of the negative stress intensity factor. The aforedescribed fracture mechanism is inherent to the ice cover compression with ridge formation. It was also observed in special experiments performed on thin paraffin plates placed on a rubber substrate when a constraint of the plate displacements in the direction transverse to the plate plane was provided by a special glass plate fixed on a small distance from the paraffin one [53,54]. Models of the observed local fracture schemes were also suggested. Note, considering a crack with overlapping surfaces as a crack of compression one can generalize the LEFM concept for this case. In particular, one can introduce a critical value of the negative stress intensity factor which characterizes the ice cover resistance to the ridge growth, K_r, [53]. The estimates and experiments on thin ice plates showed that the modulus of K_r exceeds the ice fracture toughness K_{Ic} on an order of value. The concept of cracks of compression was also used for description and classification of the regimes of ice cover fracture by icebreaker [56,57].

The aforediscussed features of fracture are inherent to both static and dynamic compression. Additional effects of dynamic compression were considered in the recent review [58].

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