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FORMATION OF HIERARCHICAL ORDERED CRACK SYSTEMS AT FRACTURE

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ABSTRACT

Fracture processes in different materials, structural components and natural objects are often occured by formation and transformation of ordered systems of cracks or cracklike faults (structures of fracture). Scale interaction can lead to formation of a hierarchy of structures of fracture of different length scales. Ordered systems of cracks or cracklike faults are observed, e.g., at fracture of ceramics and rocks under biaxial compression, pipeline steels and titanium under the hydrogen absorption, ice cover under the wind action.

In the paper an approach is developed for modeling of processes of formation of hierarchical systems of structures of fracture. The approach implies, first, an analysis of a sequence of events leading to formation of separate elements of a structure of fracture of a certain scale (rank) taking into account local perturbations of the stress-deformation field caused by an influence of the initial structure and/or texture of the object under consideration. Then a set of such elements forms a basic element of a structure of fracture. In turn, a structure of fracture of a given scale can serve as a basic element for formation of a structure of fracture of another scale. The approach enables to take into account specific mechanisms of fracture and analyze formation of hierarchy of structures of fracture. Structures of fracture of long-range action and short-range action are considered. Coupled structures of fracture forming at smaller and larger scales relative to the scale of a basic structure of fracture are analyzed. The results of modeling different types of structures of fracture correlate with the available experimental data.

KEYWORDS

Odered systems, crack, cracklike faults, structure of fracture

INTRODUCTION

Fracture of materials, structures and nature objects is often accompanied by occuring ordered systems of faults-structures of fracture [1]. These structures can form hierarchical systems. The structures of fracture were observed at quasibrittle fracture of polymers and rocks, localization of plastic deformations [2,3,4], slip planes formation in quick media [5], energy dissipation on the deformation wave front [6], splitting off fracture [7], etc. Structure of fracture formation occurs at small variations of the parameters near the limit equilibrium conditions for the media under consideration.

Several approaches were developed to analyze formation of structures of fracture within the framework of fracture mechanics. In particular, these processes are treated as an instability of deformation at a scale of a whole body [3,8]. Another approach, based on an analysis of a sequence of events associated with the fracture mechanism development and accompanied by formation of separate elements of the structure and the structure on the whole, was suggested by the authors [1,9,10,11]. In the paper we performed further development of this approach and considered some problems related to formation of a hierarchy of structures of fracture.

STRUCTURES OF FRACTURE

Conditions for increasing the rank of the structure

Remind necessary and sufficient conditions for increasing the rank of the structure of fracture, i.e. formation of a basic scale of a new structure having the elements larger than those of the "old" structure [1,9].

It is assumed that the medium has a structure (of rank i) which is active relative to the external loading. The response of the medium can be described by a distribution (field) of characteristics at the level of a structural element (SE) of rank i or a combination of such elements. The SE can attain a limit state at certain combinations of the values of these characteristics. Generation of a SE of rank (i+1) will be associated with the localization of response processes in the medium at the level of rank i, i.e. with the instability of a SE of rank i.

The formation of a structure of rank (i + 1) requires, on the one hand, the local instability of an SE of rank i to initiate the process of successive local changes in the structure of rank i and, on the other hand, the inhibition of this instability to complete the formation of the element of rank (i + 1). These two processes are necessary for the formation of a structure of rank (i + 1). In some cases, the formation of one element of rank (i + 1) does not lead to the formation of a system of such elements and, hence, does not lead to an increase in the rank of the entire structure. For these reason, we need also sufficient conditions for the change in the rank of the structure. To initiate the formation of the structure of rank (i + 1), it suffices that the perturbation caused by the appearance of the previous SE of rank (i -+- l)be able to lead to an instability necessary for the formation of the next SE of rank (i + 1).

The spatial configuration of the element of the new structure depends on the form of the initial structure and on the position of the site of generation of the local instability of a typical element of the initial structure. Since this position depends on the combination of the external field and the perturbation of this field by the element exhibiting instability, it is important that specific response mechanisms determining the spatial nonuniformity of the corresponding fields be considered in the model of the process. The basic mechanisms defining the change in the structure are the mechanism of excitation of the local instability of an element of rank i and that of instability restraint for the corresponding SE of rank (i + 1). Note that in some mechanisms restraining the instability, the spatial nonuniformity of the stress fields leads to the fracture of separate SE's of rank i. This provides the possibility of decreasing the rank of the structure combined with the increase of this rank (the combined formation of SE's of ranks (i + 1) and (i - 1)).

Fracture of a porous body

We will consider typical examples of structures of fracture. In [I], the increase in the rank of the structure of an elastic porous body is illustrated by the formation of the structure of rank i + 1 of crack elements interacting in short range in the tensile crack echelon. Similar constructions can be carried out also for other sorts of stress concentrators, for example, for inclusions or macrocracks. We will briefly consider the plane model of the process [1,9]. Let the body contains circular pores of radius R which are uniformly rarely arranged. Let the distance between the centers of the neighboring pores have an order of magnitude of 2L

(Fig. 1a*)*. The cracks formed by the external compression (i) came against the pore boundary in the

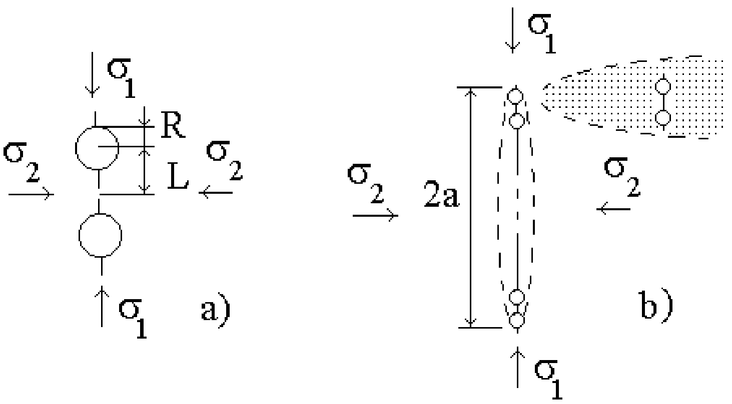

Figure 1: Fracture of a porous medium under uni-axial (a) and bi-axial (b) compression

diametrical direction, (ii) have identical lengths ℓ*,* and (iii) are oriented along the principal compressive stresses. The initiation of the main crack occurs by means of coalescence ofmicrocracks growing near typical pores, i.e., with the condition $\ell \sim$ L being satisfied. The propagation of the main crack under the uniaxial loading of the body is unstable. Hence, the stress intensity factor at the tips of the main crack exceeds that of an isolated microcrack of length $\ell \sim L$.

The effective value of the stress intensity factor for a main crack comprising several pores in the case of the uniaxial compression can be represented in the form

$$K_{Ia} = \sigma_1 \sqrt{R}\, f\left(\frac{R}{a}, n\right), \qquad f\left(\frac{R_0}{a}, n\right) \approx \frac{0.24 n^{1/3}}{1 - n^{1/3}\left(3\frac{R}{a} - 1\right)}, \quad \frac{a}{R} > 3. \tag{1}$$

where a is the half-length of the main crack and n is the porosity, R is a constant radius of curvature at the vertex of a narrow elongated ellipse equivalent to the main crack.

It was assumed that ℓ = L and (L/R) ≈ $n^{1/3}$.

In a similar way, one can obtain the value of the stress intensity factors of isolated cracks having the lengths at which these cracks join together. Thus we obtain

$$K_{IR} = \sigma_1 \sqrt{R}\, f_1(n), \quad f_1(n) = \frac{8n^{1/3} - 1}{80(1 - n^{1/3})}. \tag{2}$$

If an additional lateral compression σ_2 acts, then the stress intensity factor can be expressed by the sum

$$K_I = K_I(\sigma_1) + K_I(\sigma_2). \tag{3}$$

For the case of an elongated ellipse mentioned above we have

$$K_I(\sigma_2) = \sigma_2 \sqrt{a}\, f_2\left(\frac{R}{a}, n\right), \quad f_2\left(\frac{R}{a}, n\right) = \left[\frac{1}{\sqrt{2}} + \left(0.32\sqrt{\frac{R}{a}} - 0.05\right)\frac{0.95n^{2/3} - 2n^{1/3} + 1}{n^{1/3}(1 - n^{1/3})}\right] \tag{4}$$

The functions f, f_1, f_2 were constructed on the basis of the results of the numerical solving the appropriate elasticity problems [12].

The size of the limit equilibrium crack is defined by the condition

$$K_I = K_{Ic} \tag{5}$$

It is apparent from Eqs. (3.2) and (3.3) that a porous material subjected to the uniaxial compression is fractured by main cracks intersecting the volume being loaded along the principal compressive stresses, since the inequality $K_{Ia} > K_{IR}$ is valid for any $a < 3R$. This statement is valid also for mechanisms of brittle fracture of heterogeneous materials with different stress concentrators [9-11]. It is in good agreement with the experimental data concerning the brittle fracture of rocks [12]. Main compression discontinuities represent one of the basic forms of brittle fracture of heterogeneous materials.

For a porous body subjected to the biaxial compression, main cracks are bounded in length. The occurrence of the unstable phase of the development of the main discontinuity formed by a chain of pores implies that the necessary condition for the generation of a higher-rank structure is satisfied. To satisfy the sufficient conditions, it is necessary that the perturbation of the stress field in the neighborhood of the main crack would initiate the unstable development of another main crack at a distance from the former crack. Depending on the parameters of the initial structure (for example, on the porosity) and loading condition, the stress field perturbation can initiate a fracture site in a short-range or long-range neighborhood of the main crack. The conditions for the generation of the fracture site near the tip of an arrested crack are most favorable in the neighborhood of a tensile crack at a distance of an order of magnitude of 2L from the crack tip [13]. At this distance, the shear perturbation zone covers two interacting pores (Fig. 1b*).* The condition for these pores to join together provides the sufficient conditions for an increase in the structural rank to be satisfied. The latter conditions imply the relations

$$\begin{aligned}
&K_{Ic} = K_{IR}(\sigma_1 + \Delta\sigma_1) + K_{IR}(\sigma_2 - \Delta\sigma_2), \quad \Delta\sigma_{1,2} \approx \pm\frac{K_{Ia}}{\sqrt{2L}}, \\
&K_{IR}(\sigma_1 + \Delta\sigma_1) = (\sigma_1 + \Delta\sigma_1)\sqrt{R}\, f_2(n), \\
&K_{IR}(\sigma_2 - \Delta\sigma_2) = (\sigma_2 - \Delta\sigma_2)\sqrt{R}\, f_3(n), \quad f_3(n) = \frac{5 + 8.4n^{1/3} - 13.4n^{2/3}}{n^{1/3}(22 - 21n^{1/3})}
\end{aligned} \tag{6}$$

If the boundary conditions specified in terms of stresses (σ_1, σ_2) are fixed, then the relations (6) define the basic parameters of the crack echelon in the volume being loaded. These parameters are the lengths of isolated main cracks in the echelon and the distance between parallel main cracks. The configuration of the echelon (the relative arrangement of main cracks) depends on the conditions of interaction of neighboring cracks in the echelon.

Note that the location of the fracture site in the neighborhood of the main fracture and the direction of the development of the next main fracture can be different and determined by stress fields in the long-range zone.

Echelon structures in ice cover

Necessary and sufficient conditions of structure of fracture formation can be provided by different mechanical processes.

As an example let us consider a type of largescale structures in the ice cover-formation of an echelonlike system of faults in thin level ice under uniform tension [14]. The velocity of growth of a single fault in a viscoelastic thin plate on a hydraulic foundation is changed with nonmonotone manner at increasing the fault length [8]

$$\frac{d\ell}{dt} \sim \frac{\vartheta^*}{f_1}\ell^{3/4}\left(1-\frac{bf_1}{\vartheta^*}\ell^{1/4}\right)^2 \quad b=\frac{\eta}{E};\ f_1=f(E,K_{Ic},\overline{\sigma}) \tag{7}$$

where η is the ice cover viscosity, ϑ^* is the threshold rate of the fault surfaces displacement characterized by compensation of a mean stress $\overline{\sigma}$ by hydrodynamic resistance. Local instability is inherent for a small size of the fault, while fault growth arrest is related to its large size. Hence a necessary condition of a structural element formation is fulfilled. A sufficient condition occurs when the resulting stress in the vicinity of the fault tip becomes equal to the stress of new fault nucleation such that the size of a nucleous fault provides its growth with a larger velocity according to (7) than for the initial fault. One can show that the energy dissipation rate for an echelon of large size faults is larger than for a single fault of the same total length. As a result structure formation becomes the leading process at preparing large scale fracture in the conditions close to the limit equilibrium.

A possible role of structure formation in rocks was analyzed in [11] in case of preparing when the local instability is provided by the gas pressure in pores, while the arrest mechanism by the rock pressure.

A similar scheme of fracture characterized by formation of a system of microcracks under the action of the hydrogen diffusion is observed at the cracking of pipeline steels modeled in [15].

Basic and conjugate structures

Basic structures forming mechanisms of fracture at the initial stage of the medium deformation are of particular importance in the scheme of multiscale fracture. It seems that in the fragment of the hierarchy of structures which can be observed [8-10], the fracture structures play a leading role only within a limited range of scales, within the limits of the influence of certain basic structures. Note, that small perturbations of physical fields are important for occuring a basic structure in a quasihomogeneous body. One can show [11] that the interaction between the elements of this structure is implemented by means of long-range perturbations of stress fields.

The transformation of the basic structure into a higher-rank structure is not the only possible response of the medium to an input influence. Experiments with rocks and model media subjected to a quasistatic deformation reveal the process of combined change of the rank with respect to the initial block structure. This process involves the agglomeration of some blocks (formation of a structure of rank i + 1) combined with the refinement of other blocks (formation of a structure of rank i - 1).

This combined initiation of structures of rank i ± 1 can be activated by the mechanism of deceleration of the SE of rank i + 1 consisting of several elements of rank i in the effective medium determined by the basic structure of rank *i*. Additional stresses (stress concentration) in the vicinity of the growing element of rank i +1 can transfer some elements of rank i to the critical state, which leads to the local fracture of these elements and, hence, to a reduction in the rank of the structure. This, in turn, changes the boundary conditions on the scale of the growing element of rank i + 1, leading, in particular, to a stop of the element of rank i and initiation of the development of the neighboring structural element of rank i + 1. Note, that in this scenario of the development of the structures, the structure of rank i - 1 do not influence all elements of rank i and, hence, is not global for the medium subjected to a loading. The scheme of development of the structures just described cover three hierarchical levels. The further extension of the hierarchical systems of structures of fracture to cover all ranks can be implemented as a repetition of the three-stage procedure of formation of the hierarchical blocks. In each of these blocks, the basic structures for the formation of structures of higher ranks (i.e., the structures of rank i + 2, i + 3, etc.) have a rank which is lower by unity then the rank of the structure to be formed. The lower the rank, the less the volume covered by the local fracture process. The development of the process in accordance with such a scheme implies an increasing differentiation of the stress fields and geometrical parameters in the structures of the hierarchical system. This is due to the selective nature of the rank decreasing processes in space and also due to the fact that if the rank increases beyond level i + 1, the role of the lower rank structures in the three-stage hierarchical blocks, is again played by separate portions of the hierarchical structure which has already been formed. However, these portions are subjected to additional local stress fields.

The relations between the sizes of similar elements of ranks i, (i - 1) and (i + 1) for a plane model of a three linked hierarchy of shear block structures were obtained in [11,16]

$$L_i / L_{(i-1)} \sim 1 + \frac{n^2}{4}, \quad n \sim L_{(i-1)}/L_i \tag{8}$$

The used model of shear fracture is similar to the aforedescribed model of fracture of a porous body at tension. The observed fragmentation effects [17,18] lead to $(L_i/L_{(i-1)}) \sim 3$ at $n \sim 3$ which correlates with (8).

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