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FATIGUE FRACTURE PLANE DIRECTION ASSESSMENT THROUGH THE WEIGHT FUNCTION METHOD

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ABSTRACT

In the present paper, the expected principal stress directions under proportional and nonproportional multiaxial high-cycle fatigue loading are determined by averaging the instantaneous values of the "principal" Euler angles. Such angles are employed to describe the position of the principal stress axes at the generic time instant, and the averaging procedure is performed through suitable weight functions. Three possible types of weight functions based on stress parameters are adopted and verified using available experimental results related to some metallic materials. It is shown that the fatigue fracture plane position under multiaxial loading may be established on the basis of the averaged direction of the maximum principal stress, by employing proper weight functions.

KEYWORDS

Multiaxial fatigue, weight function method, fatigue fracture plane

INTRODUCTION

According to several well-known stress-, strain- and energy-based models for multiaxial fatigue (for instance, [1-9]), the failure assessment is usually performed by referring to the "critical plane". The concept of the critical plane, which is related to the crack initiation phenomenon, was firstly proposed by Stanfield [1] in 1935, and has been developed since then.

On the other hand, from a review of fatigue test results under in-phase and out-of-phase cyclic multiaxial loading, it appears that the final fracture plane very often corresponds to:

(a) the plane on which the normal stress, or strain or strain energy density attain their maximum;
(b) one of the two maximum shear planes;
(c) an intermediate position between (a) and (b).

Case (a), case (b) and case (c) are typical for brittle materials, ductile materials and semiductile materials, respectively. The fracture plane for case (b) is one of the two maximum shear planes in which the highest normal stress, or strain or strain energy density occurs [4,6,8]. Case (a) is preferred when the ratio of the shear stress amplitude τ_a to the normal one σ_a is less than 0.63 [10], or when the ratio of the shear strain range $\Delta\gamma$ to the normal one $\Delta\varepsilon$ is less than 1.5 [3]. Such ratios depend on the kind of material, stress or strain level and temperature. Some theoretical predictions of the fatigue fracture plane position have been presented by Kanazawa et al. [11], Findley [12] and Stulen and Cummings [13].

Numerous models of fatigue crack initiation and propagation for multiaxial cyclic loading do not take into account the change of the principal strain and stress axes. That is a possible reason why they can only be applied to some particular cases. Consequently, averaged principal stress directions should be determined, and the averaging procedure could be carried out, for instance, by employing suitable weight functions [14-17]. In the following, the position of the principal stress axes 123 at the generic time instant is described by the three "principal" Euler angles φ, θ, ψ.

The aim of the present paper is to verify whether the weight function method can allow us to determine the expected fatigue fracture plane position. The algorithm presented in [15–17] is applied to experimental data related to one steel and two cast irons [18,19].

AVERAGING PROCEDURE

The principal axes 123 system can be considered as a transformation of the XYZ system, fulfilling a condition of orthogonal conversion or specific rotation [20]. A transformation between two Cartesian systems can be described in different ways, but the direction cosines matrix is usually employed. On the other hand, such a matrix includes nine dependent components, and we cannot average all the matrix elements since the averaged matrix would not satisfy all the conditions of transformation. Moreover, we are not able to select which three independent elements of the direction cosines matrix should be averaged. Thus, the three independent Euler angles φ, θ, ψ are selected for description of the transformation from the XYZ system to the 123 system (Fig.1). Let us assume that the 1-axis and the 3-axis represent the directions of maximum and minimum principal stresses, respectively.

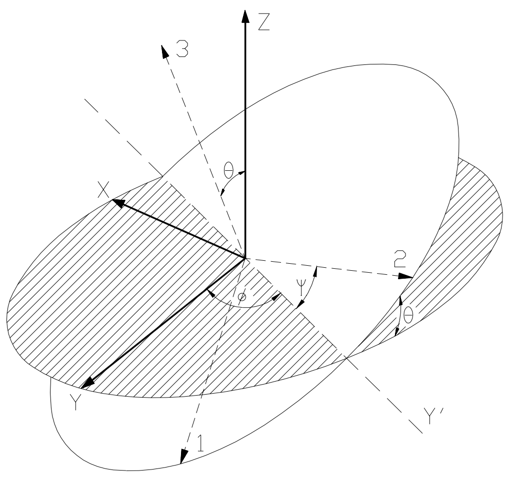

Figure 1: Principal stress axes 123 described by the Euler angles φ, θ, ψ

Under variable loading, the Euler angles are time-varying functions as well as the stress tensor components. Different sets of the Euler angles could describe the same position of the principal axes 123. Consequently, in order to average correctly the results determined for different time instants, the ranges of the Euler angles have to be reduced through a two-stage procedure, bringing them to $0 \leq \varphi, \theta \leq \pi/2$ and $-\pi/2 \leq \psi \leq \pi/2$. A detailed description of an algorithm to perform such a reduction can be found in Refs [15,16].

From a physical point of view, it seems logical to carry out the Euler angles averaging through suitable weight functions [14-17] in order to include various factors influencing the fatigue fracture behaviour (stress amplitudes, exponents of the Wöhler curve and so on). Therefore, the mean directions of the principal stress axes may be described by the weighted mean Euler angles:

$$\hat{\varphi} = \frac{1}{W}\sum_{t_1}^{t_N}\varphi(t_k)W(t_k), \qquad \hat{\theta} = \frac{1}{W}\sum_{t_1}^{t_N}\theta(t_k)W(t_k), \qquad \hat{\psi} = \frac{1}{W}\sum_{t_1}^{t_N}\psi(t_k)W(t_k) \tag{1}$$

where $W = \sum_{t_1}^{t_N} W(t_k)$ is the summation of the weights, and N is the number of time instants considered.

Three kinds of weight functions are examined in the following:

Weight I

$$W_1(t_k) = 1 \tag{2}$$

According to such a weight function, each positions of the principal axes influences the mean position of the principal axes to the same degree, irrespective of the stress values. Application of this weight function gives us W = N, and leads to the arithmetic means.

Weight II

$$W_2(t_k) = \begin{cases} 0 & for \quad \sigma_1(t_k) < c\sigma_{af} \\ \left(\dfrac{\sigma_1(t_k)}{c\sigma_{af}}\right)^{m_\sigma} & for \quad \sigma_1(t_k) \geq c\sigma_{af} \end{cases} \tag{3}$$

where:

m_σ – exponent of the fatigue curve ($\sigma_a - N_f$)

σ_{af} – fatigue limit

c – constant

This weight function includes only those principal axes positions for which the maximum principal stress $\sigma_1(t_k)$ is greater than the product of c and the limit stress σ_{af}; the participation of such positions in averaging exponentially depends on the parameter m_σ of the Wöhler curve.

Weight III

$$W_3(t_k) = \begin{cases} \left(\dfrac{\sigma_1(t_k)}{\sigma_{af}}\right)^{\frac{m_\sigma}{2}} & for \quad \sigma_1(t_k) < \sigma_{af} \\ \left(\dfrac{\sigma_1(t_k)}{\sigma_{af}}\right)^{m_\sigma} & for \quad \sigma_1(t_k) \geq \sigma_{af} \end{cases} \tag{4}$$

This weight function is similar to the weight function II, but it includes all the principal axes positions and, when $\sigma_1(t_k) < \sigma_{af}$, the weight function exponent is divided in half.

RESULTS OF THE SIMULATION CALCULATIONS

The theoretical procedure above is applied to three metallic materials (Table 1). Fatigue test results including the actual position of the fatigue fracture plane are presented in Refs [18,19]. Thus, the position of the vector $\overline{\eta}_{exp}$ normal to the actual fracture plane can be compared with the calculated position of the vector $\overline{\eta}_{cal}$, assumed to be coincident with the averaged position of the maximum principal stress axis. In the following, the solid angle α between the X-axis and the vector $\overline{\eta}$ ($\overline{\eta}_{cal}$ or $\overline{\eta}_{exp}$) is considered. The parameter c for the weight function II is assumed to be equal to 0.5. The generated cyclic courses are similar to those of fatigue tests (N = 100 discrete values at wave period T of the sinusoid are applied).

TABLE 1
MECHANICAL PROPERTIES OF THE TESTED MATERIALS

Material	R_e[MPa]	R_m [MPa]	E[GPa]	ν	m_σ	σ_{af}[MPa]
GGG40	334.0	447.0	165	0.28	11.0	244.0
GTS45	305.0	449.0	160	0.27	18.5	250.0
18G2A	394.0	611.0	213	0.31	8.2	204.0

where R_e – yield point; R_m – tensile strength; ν – Poisson's ratio

Cast irons GGG40 and GTS45

Neugebauer [18] tested round specimens made of two cast irons: GGG40 and GTS45. The specimens were subjected to combined bending and torsion, with different amplitude ratio λ and phase difference δ (Tables 2 and 3).

TABLE 2
CALCULATION RESULTS FOR CAST IRON GGG40 [18]

No.	σ_a [M.Pa]	τ_a [MPa]	λ	δ	α_{exp} [rad]	$\alpha_{cal}(w_1)$ [rad]	$\alpha_{cal}(w_2)$ [rad]	$\alpha_{cal}(w_3)$ [rad]
1	200	200.00	1.000	0	0.144π	0.493π	0.176π	0.176π
2	200	200.00	1.000	$\pi/4$	0.217π	0.419π	0.177π	0.177π
3	220	220.00	1.000	$\pi/2$	0.183π	0.339π	0.138π	0.156π
4	215	123.63	0.575	0	0.113π	0.493π	0.136π	0.136π
5	220	126.50	0.575	$\pi/4$	0.129π	0.417π	0.125π	0.123π
6	265	152.38	0.575	$\pi/2$	0.010π	0.337π	0.080π	0.087π

TABLE 3
CALCULATION RESULTS FOR CAST IRON GTS45 [18]

No.	σ_a [M.pa]	τ_a [MPa]	λ	δ	α_{exp} [rad]	$\alpha_{cal}(w_1)$ [rad]	$\alpha_{cal}(w_2)$ [rad]	$\alpha_{cal}(w_3)$ [rad]
1	160	160.0	1.000	0	0.139π	0.493π	0.176π	0.176π
2	170	170.0	1.000	$\pi/2$	0.133π	0.339π	0.133π	0.142π
3	248	142.6	0.575	0	0.136π	0.493π	0.136π	0.136π
4	248	142.6	0.575	$\pi/2$	0.117π	0.334π	0.038π	0.052π

Steel 18G2A

Pawliczek [19] tested round specimens made of 18G2A steel under combined bending and torsion, with different values of amplitude ratio λ (Table 4).

TABLE 4
CALCULATION RESULTS FOR 18G2A STEEL [19]

No.	σ_a [Mpa]	τ_a [MPa]	λ	α_{exp} [rad]	$\alpha_{cal}(w_1)$ [rad]	$\alpha_{cal}(w_2)$ [rad]	$\alpha_{cal}(w_3)$ [rad]
1	464	0.0	0.0	0.005π	0.245π	0.000π	0.000π
2	374	0.0	0.0	0.004π	0.245π	0.000π	0.000π
3	447	224	0.5	0.137π	0.493π	0.125π	0.125π
4	364	182	0.5	0.142π	0.493π	0.125π	0.125π
5	315	157	0.5	0.129π	0.493π	0.125π	0.125π
6	298	149	0.5	0.110π	0.493π	0.125π	0.125π
7	447	447	1.0	0.162π	0.493π	0.176π	0.176π
8	403	403	1.0	0.160π	0.493π	0.176π	0.176π
9	368	368	1.0	0.150π	0.493π	0.176π	0.176π
10	344	344	1.0	0.167π	0.493π	0.176π	0.176π
11	338	338	1.0	0.170π	0.493π	0.176π	0.176π

ANALYSIS OF THE CALCULATION RESULTS

Table 5 presents the absolute values of the mean errors in determination of fatigue fracture planes for the examined materials. The results obtained by using of the weight functions II and III are almost identical since their mathematical forms are very similar (Fig.2). The agreement between calculation and experimental results is quite good for all the materials considered: the best results are found out for steel 18G2A, while the greatest error occurs for cast iron GGG40, i.e. 0.036 π (about 6^{o}). For the analysed materials, the fracture plane seems to be perpendicular to the weighted mean direction of the maximum principal stress.

TABLE 5.
ABSOLUTE VALUES OF MEAN ERRORS (IN [RAD]) IN DETERMINATION OF FATIGUE FRACTURE PLANE POSITIONS ACCORDING TO THREE WEIGHT FUNCTIONS

Material	Loading	W_1	W_2	W_3
GGG40	bending with torsion	0.284π	0.036π	0.034π
GTS45	bending with torsion	0.284π	0.029π	0.028π
18G2A	bending with torsion	0.326π	0.012π	0.012π

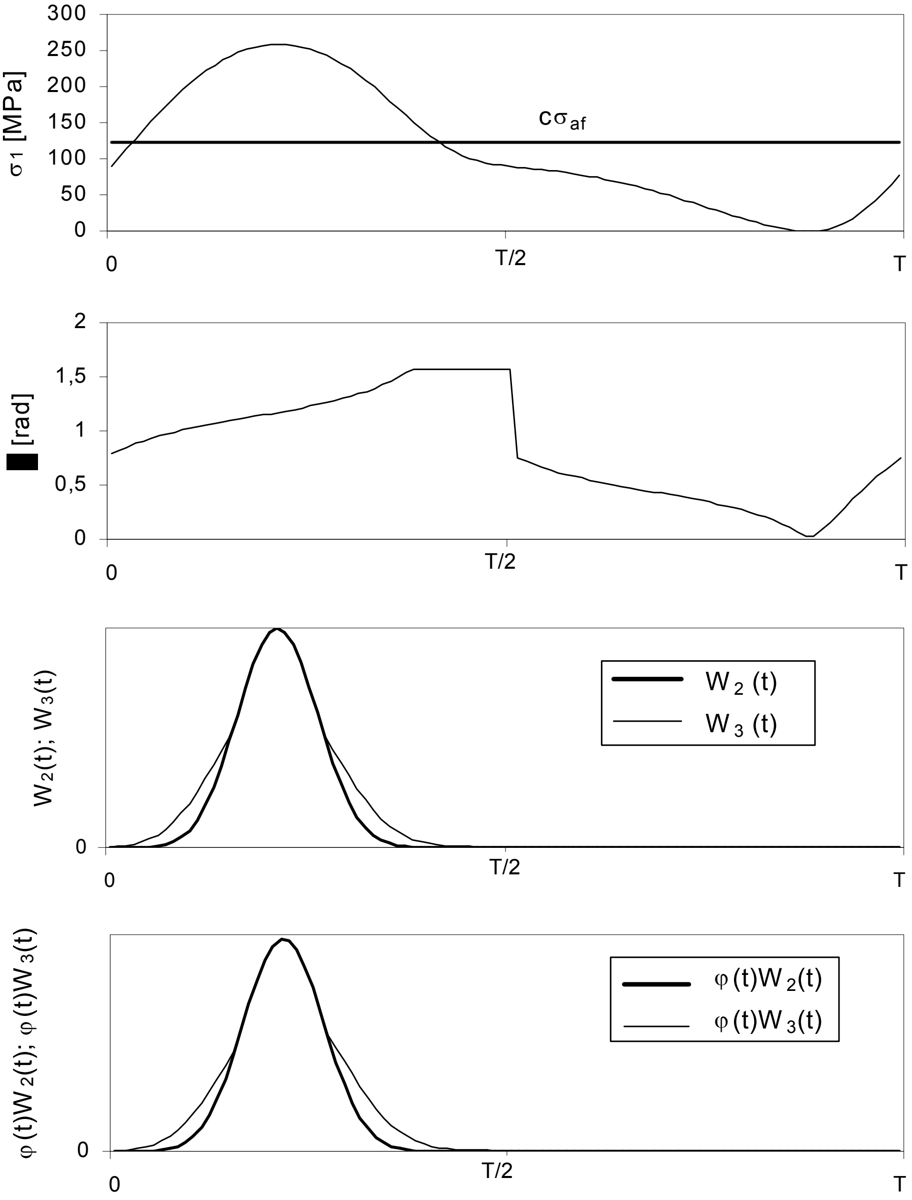

Figure 2: Modification of the Euler angle φ by the weight functions II and III (test No. 5 in Table 2)

CONCLUSIONS

Some experimental data related to one steel, 18G2A, and two cast irons, GGG40 and GTS45, under multiaxial high-cycle fatigue have been compared with the theoretical results of the expected principal stress directions. For the three materials examined, the fatigue fracture plane position is about perpendicular to the weighted mean direction of the maximum principal stress. Such a direction is determined by averaging the instantaneous positions of the principal stress axes through suitable weight functions. For each time instant, the most appropriate weight functions take into account whether or not the maximum principal stress is greater than a certain level dependent on the fatigue limit. Furthermore, such weight functions are influenced by the value of the Wöhler curve exponent.

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