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Evaluation of Stress Intensity Factors of Spot Welded Joints Using Meshless Method

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ABSTRACT

It has been required strongly to evaluate fatigue strength of spot welded joint accurately in the stage of designing automobile body structure. There is a evaluation method of fatigue strength based on stress intensity factors K in linear fracture mechanics. We applied Element Free Galerkin Method (EFGM) developed by T. Belytschko et al. to calculating stress intensity factors K of spot welded double cup specimen (is called DC for short) under multiaxial loads, and investigated the accuracy of the solutions. This paper shows that K obtained by using EFGM are appropriate results.

KEYWORDS

Meshless Method, Fractures, Stress Intensity Factors, Spot Weld

INTRODUCTION

It has been required strongly to evaluate fatigue strength of spot welded joint accurately in the stage of designing automobile body structure. There is a evaluation method based on stress intensity factors K in linear fracture mechanics. In order to obtain value of K accurately using finite element method (FEM), the detail division of elements on spot welded joint model is essential. In crack growth problem, the rearrangement of nodes and elements around a crack tip is needed through progress of crack. Especially, this rearrangement is the most difficult work in three-dimensional problems. Meshless method, which needs no elements on analytical model, is of great advantage to solving crack growth problem.

In this paper, we applied EFGM developed by T. Belytschko et al. [1] to calculating stress intensity factors K of double cup specimen (DC) under multiaxial loads, and investigated the accuracy of the solutions.

DC SPOT WELDED JOINT AND ANALYTICAL MODEL

The test specimen composed of two cups, called DC shown in Figure 1, is spot-welded at center in the bottom of the cups. Dieter Radaj et al. [2] suggested DC for fatigue strength under multiaxial load. We consider the part around nugget as a ringshaped crack and apply EFGM to linear fracture mechanics and evaluate K of DC.

We consider analytical model as only bottom of the cups as shown in Figure 2. When we analyze DC acted on multiaxial loads, the DC is analyzed as superposition of the load which is divided into cross tension (β =90deg) and tensile shear(β =0deg). When cross tension(β =90deg) is acted on DC, the elastic stress analysis comes to be a axisymmetric problem. When tensile shear(β =0deg) is acted on DC, the elastic stress analysis comes to be a axisymmetric body problem.

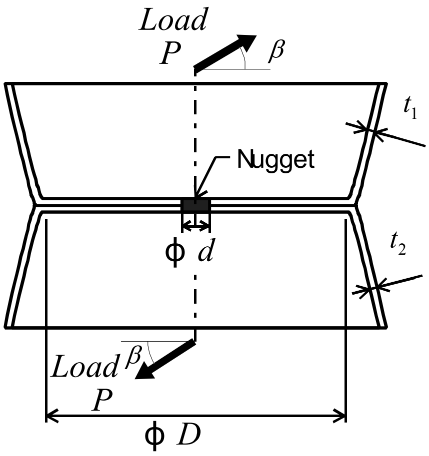

Figure 1: Double cup joint under out-of-plane

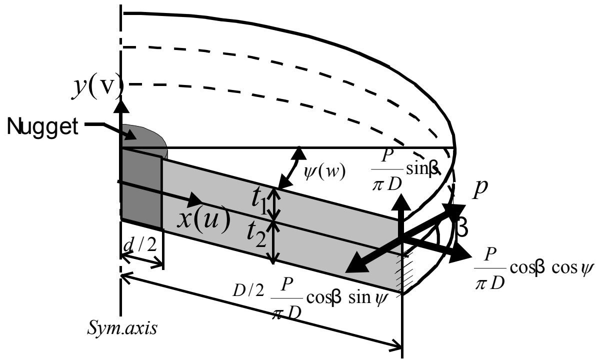

Figure 2: Analytical model

GOVERNING EQUATION AND BOUNDARY CONDITIONS

Analyzing stresses and strains of DC by theory of elasticity are boundary-value problems of partial differential equations expressed as following equations:

$$\sigma_{ij,j} + b_i = 0 \qquad \text{in}\,\Omega \tag{1-a}$$

$$u_i = \bar{u}_i \qquad \text{on}\,\Gamma_u \tag{1-b}$$

$$n_j \sigma_{ij} = \bar{t}_i \qquad \text{on}\,\Gamma_t \tag{1-c}$$

Where Eqn. 1-a is a governing equation of stress field, Eqn. 1-b is a fundamental boundary condition, and Eqn. 1-c is a natural boundary condition. The notation $,j$ represents partial different with respect to space coordinate $x_j\left(x_1 = x, x_2 = y\right)$. Eqn. 1-a is expressed by using Einstein's summation convention. σ_{ij} is stresses, b_i is a body forces, Ω is an analytical domain bounded by Γ , Γ_t is a boundary acted by given surface forces $\bar{t}_i$, and Γ_u is a boundary prescribed displacements $\bar{u}_i$. n_j can be considered to give the direction cosines of the unit normal of the interface on which the traction force is desired.

A equilibrium conditions for a linear elastic body are identical with minimizing total potential energy expressed in Eqn. 2:

$$\pi(u_i) = \int_\Omega \left(\frac{1}{2}\sigma_{ij}\varepsilon_{ij} - u_i b_i \right) d\Omega - \int_{\Gamma_t} u_i \bar{t}_i d\Gamma + \int_{\Gamma_u} \frac{\alpha}{2}(u_i - \bar{u}_i)^2 d\Gamma \tag{2}$$

Where ε_{ij} is component of strain. α is a penalty number which is a large positive number for reliving a fundamental boundary condition with a penalty method.

Interpolation function in FEM is set by each element but the one of EFGM is locally set by using moving least square method (is called MLSM for short) . In the MLSM of two-dimensional problem, the function u_0 of arbitrary evaluation point $\mathrm{x}^T = \lfloor x\ y \rfloor$ in the domain is approximately expressed as:

$$u_0(x, y) = p_j(x, y) c_j(x, y) \equiv \mathrm{p}^T(x, y)\mathrm{c}(x, y) \quad i, j = 1 \sim 3 \tag{3}$$

Where p_j are multinomial expression including space coordinates (x, y), c_j are coefficients undefined.

$$\mathrm{p}^T(x,y) = \lfloor p_1\ p_2\ p_3 \rfloor = \lfloor 1\ x\ y \rfloor \tag{4-a}$$
$$\mathrm{c}^T(x,y) = \lfloor c_1\ c_2\ c_3 \rfloor \tag{4-b}$$

In the MLSM, the coefficients c_j are obtained by minimizing the following weighted square expression J.

$$J = \sum_i^n w(\|\mathrm{x} - \mathrm{x}_i\|)[u_0(\mathrm{x}_i) - u_i(\mathrm{x}_i)]^2 = w_i \Delta u_i^{\ 2} \tag{5}$$
$$w_i = w(\|\mathrm{x} - \mathrm{x}_i\|) \tag{6-a}$$
$$\Delta u_i = u_0(\mathrm{x}_i) - u_i(\mathrm{x}_i) = p_{ji}(x_i, y_i) c_j(x, y) - u_i(x_i, y_i) \tag{6-b}$$

Where $\mathrm{x}_i = \lfloor x_i\ y_i \rfloor$ is the coordinate of a node i in the neighborhood of evaluation point x and u_i is the nodal value at $\mathrm{x} = \mathrm{x}_i$. n is the number of nodes in the neighborhood of x. $w(\|\mathrm{x} - \mathrm{x}_i\|)$ is a weight function defined in the neighborhood of x. We used four-dimensional spline function in Eqn. 7 as $w(\|\mathrm{x} - \mathrm{x}_i\|)$. $\|\mathrm{x} - \mathrm{x}_i\|$ is norm of vector $\mathrm{x} - \mathrm{x}_i$ and stands for the distance from the evaluation point x to node x_i.

$$w(r_w) = 1.0 - 6.0\left(\frac{r_w}{\rho}\right)^2 + 8.0\left(\frac{r_w}{\rho}\right)^3 - 3.0\left(\frac{r_w}{\rho}\right)^4 (0 \le r_w \le \rho) \tag{7}$$

Where $r_w = \|\mathrm{x} - \mathrm{x}_i\|$ and ρ is a radius of circular domain of influence in two dimensions. The stationary of J in Eqn. 5 with respect to $\mathrm{c}(\mathrm{x})$ leads to the following equation.

$$\mathrm{A}(x,y)\mathrm{c}(x,y) = \mathrm{B}(x,y)\mathrm{u} \tag{8-a}$$

or

$$\mathrm{c}(x,y) = \mathrm{A}^{-1}(x,y)\mathrm{B}(x,y)\mathrm{u} \tag{8-b}$$

Where A , B and u in Eqn. 8 are defined by following equations.

$$\mathrm{A} = \lfloor A_{ij} \rfloor, \quad A_{ij} = w_k p_{ik} p_{jk} \qquad i,j = 1\sim 3 \quad k = 1\sim n \tag{9-a}$$
$$\mathrm{B} = \lfloor B_{ij} \rfloor, \quad B_{ij} = w_i P_{ji} \qquad i,j = 1\sim 3 \tag{9-b}$$
$$\mathrm{u}^T = \lfloor u_1\ u_2 \cdots u_n \rfloor \tag{9-c}$$

Eqn. 8-a must be solved accurately to retain the accuracy of the MLSM interpolant. When the matrix A is not well conditioned, Eqn. 8-b cannot be solved with desired accuracy. However the necessity for solving Eqn. 8-a can be eliminated by diagonalizing the matrix A . To diagonalize the matrix A , we obtained orthogonal basis function $q_i(\mathrm{x}, \mathrm{x})$ by using Schmidt orthogonalization procedure. The coefficients c_j can be expressed by using Eqn. 8-a with $q_i(\mathrm{x}, \mathrm{x})$. Thus, by substituting the coefficients c_j for Eqn. 3, the MLSM interpolant approximation can be expressed as follows:

$$u_0(x,y) = w(\|\mathrm{x} - \mathrm{x}_i\|) \frac{q_k(\mathrm{x}, \mathrm{x}) q_k(\mathrm{x}_i, \mathrm{x})}{w(\|\mathrm{x} - \mathrm{x}_j\|) q_k^{\ 2}(\mathrm{x}_j, \mathrm{x})} u_i \tag{10}$$

By using a method developed by T. Belytschko et al, we divide the analytical domain into regular and latticed domain named background cell. The cell has a unit which is integrated variation $\delta\Pi$ of integral equation 2. We apply Gauss integral to integrating each cell. We use MLSM for evaluating an integrand of Gauss integral point. We obtained stiffness equation by discreting variation $\delta\Pi$ on the value of node.

STRESS INTENSITY FACTORS

Stress intensity factors K_I, K_{II}, K_{III} can be obtained directly by using stresses or displacements around crack tip. We set newly a coordinate system which has a origin around crack tip as shown in Figure 3.The relation between K, Δu which is a relative displacement in x_c direction and Δv which is relative displacement in y_c direction and Δw which is relative displacement of circumferential direction are expressed as:

$$\Delta u = \frac{8K_{II}(1-\nu^2)}{E}\sqrt{\frac{r}{2\pi}} \quad \text{(11-a)}$$

$$\Delta v = \frac{8K_{I}(1-\nu^2)}{E}\sqrt{\frac{r}{2\pi}} \quad \text{(11-b)}$$

$$\Delta w = \frac{8K_{III}(1+\nu)}{E}\sqrt{\frac{r}{2\pi}} \quad \text{(11-c)}$$

Where E is a young's modulus, ν is a Poisson's ratio, r is a distance from crack tip. K_I, K_{II}, K_{III} are obtained by substituting relative displacements Δu , Δv , Δw calculated by EFGM for Eqn.11. We can obtain K by extrapolating K_I, K_{II}, K_{III} to $r \to 0$.

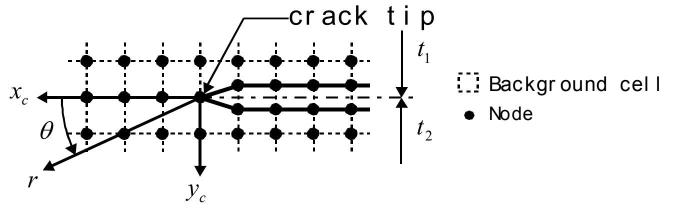

Figure 3: Distribution of nodes and background cells around crack tip

By adopting the maximum tangential stress criterion (F. Erdogan and G. C. Sih`s criterion), initial crack under K_I, K_{II} mixed modes grows in the direction of angle $\theta = \theta_0$ inclining to x_c axis in Eqn. 12. Maximum principal stress intensity factor $K_{\theta\max}$ is obtained by Eqn. 13.

$$K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1) = 0 \quad (12)$$

$$K_{\theta\max} = \cos\frac{\theta_0}{2}\left(K_I \cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{II}\sin\theta_0\right) \quad (13)$$

ANALYTICAL RESULTS

Before we calculate K of DC using EFGM, we apply EFGM to calculating K of finite flat plate named single edge crack of which exact solution has been known (Figure 4), and examine the accuracy of the solution. Table 2 shows the results of K of single edge crack using EFGM under the analytical condition shown in Table 1. We get analytical domain to be half of the total length. The K values of crack obtained by using EFGM are in agreement with the exact solutions.

TABLE 1
ANALYTICAL CONDITION

Height b(mm)	Width w(mm)	Load σ (N/mm^2)
30	20	1000
The number of nodes	The number of cells	Node distribution
29×43	20×30	Uniform distribution

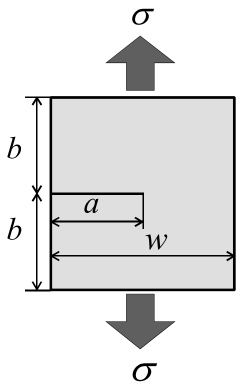

Figure 4: Finite flat plate with single crack subjected to uniform tension

TABLE 2

STRESS INTENSITY FACTORS OF SINGRE EDGE CRACK

Crack length		Stress intensity factors K(×10^{4}N/mm$^{3/2}$)		Error
a(mm)	a/w(-)	Analytical solution	Exact solution	e (%)
5	0.25	0.594	0.595	0.10
10	0.5	1.611	1.584	-1.68
15	0.75	5.800	5.799	-0.02

On the basis of the analytical results shown in Table 2, we will arrange nodes and background cells uniformly and calculate stress intensity factor K of DC by EFGM. Figure 3 shows the distribution of nodes and background cells around crack tip.

Because DC shown in Figire 1 has a crack in analytical domain, there is a possibility of including the boundary between nodes and evaluation point. Therefore giving equal weight to the nodes across and beyond a boundary causes to decrease accuracy of the solution and therefore we define the domain of influence around a boundary (crack) as shown in Figure 5.

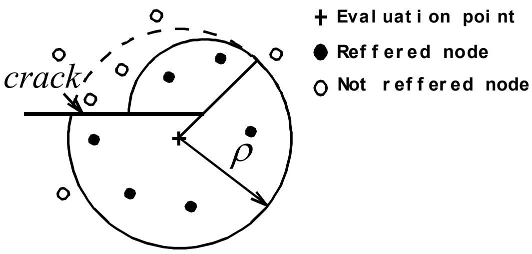

Figure 5: Domain of influence

Table 3 shows the analytical results of DC which is changed load direction β (from 0deg to 90deg) by using EFGM. We get diameter of nugget to be $5\sqrt{t}$.

TABLE 3

STRESS INTENSITY FACTRORS OF DC

Load direction β (deg)	Diameter D(mm)	Plate thickness t_1=t_2(mm)	Load P(N)	ナゲット径 d (mm)	Stress in K ($\times 10^3$ N/mm$^{3/2}$)				
					$K_I(\Psi=0)$	$K_{II}(\Psi=0)$	K_I/K_{II}	$K_{\theta\,max}$	θ_0(deg)
0	34.2	0.8	1000	4.48	0.000	0.152	0.000	0.176	-70.5
15	34.2	0.8	1000	4.48	0.222	0.147	0.664	0.319	-46.0
30	34.2	0.8	1000	4.48	0.428	0.132	0.308	0.481	-29.7
45	34.2	0.8	1000	4.48	0.605	0.108	0.178	0.632	-19.1
60	34.2	0.8	1000	4.48	0.741	0.076	0.103	0.753	-11.5
90	34.2	0.8	1000	4.48	0.856	0.000	0.000	0.856	0.00

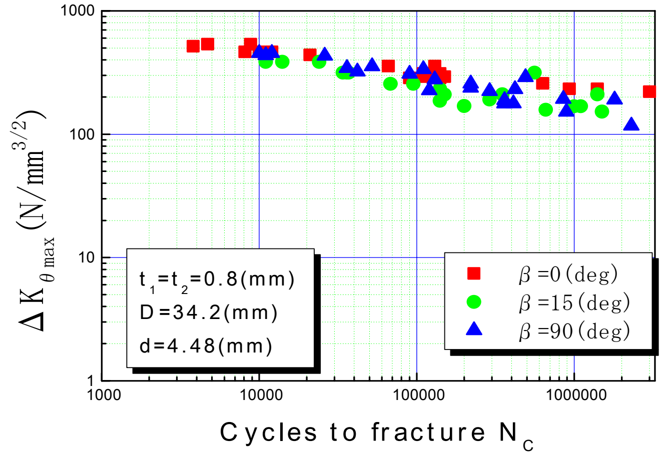

Figure 6: $\Delta K_{\theta\max} - Nc$ curve

Figure 6 shows $\Delta K_{\theta\max} - Nc$ which can be obtained by the data of fatigue test ($\Delta p - Nc$ curve) and $K_{\theta\max}$ obtained by EFGM. Figure 6 show the data are gathered in narrow range. So we can say K obtained by using EFGM are appropriate results.

CONCLUSIONS

We applied EFGM to calculating stress intensity factors K of DC under multiaxial loads, and investigated the accuracy of the solutions obtained.
Main results were as follows.

(1) In order to examine the accuracy of solutions of K obtained by using EFGM, finite flat plates named single crack under uniform tension were analyzed by applying EFGM as two dimensional elastic problem.
(1) The K values of cracks obtained by EFGM were in agreement with the exact solutions.
(2) When we analyzed DC acted on multiaxial loads, the DC was analyzed as superposition of the load which was divided into cross tension and tensile shear. From the value of K obtained by EFGM, the data of fatigue could be arrange systematically. So we could say K obtained by EFGM were appropriate results.

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