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ESTIMATION OF DYNAMIC STRESS INTENSITY FACTORS FOR BEAM AND CYLINDRICAL SPECIMENS

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ABSTRACT

A simple numeric-experimental method for estimation of dynamic stress intensity factors (DSIF) at impact tests of cylindrical and beam specimens is proposed. For the case of solid cylindrical specimen the formulas are derived for estimation of DSIF from tensile impact tests (K_{I}) and impact torsion (K_{III}), while for a hollow cylindrical specimen only tensile impact tests are used (K_{I}). For the beam specimens symmetrical and eccentric three-point bending loading schemes are considered. The formulas derived contain parameters, estimated from modal analysis. They were evaluated using commercial Finite Element Analysis (FEA) package MARC. Based on the obtained numerical data and using the least square method, simple relations are derived for these parameters. These relations account also for some geometric parameters of specimens considered.

For use of these formulas the experimental loading diagram “applied force – time” is necessary. Therefore, special testing equipment was developed for every type of loading and type of specimen (named above). In order to verify the proposed formulas, the values of DSIF are estimated numerically using FEA package MARC and Newmark incremental integration method for all the cases considered. Good correlation between the results obtained numerically and based on proposed formulas is observed.

KEYWORDS

Impact test, modal analysis, experimental loading diagram, finite element analysis.

INTRODUCTION

Until recently the dynamic problems of fracture mechanics were studied only for idealized cases (for instance for infinite solid). Nowadays, using advanced numerical methods (namely, FE method [1, 2]) it became possible to find a solution to the dynamic problems for solid with more real shape.

For evaluation of dynamic stress intensity factors (SIF) one can use also experimental approach. One way is to use a small-spacing strain gage. However, the drawback of this approach is a necessity to glue every strain gage to the specimen with following calibration. Another way consists in use of the photoelasticity method. This method requires high-speed photography and, hence, is very cumbersome and is used only in for calibration.

Even very complex mathematical models describe only part of dynamic effects and still require introduction of experimentally estimated parameters. In the other words, currently combined experimental-numerical methods are widely used to solve dynamic problems. As an example the methodology [3-8] of data handling can be mentioned, which uses the "force – time" diagram, obtained from impact tests of beam specimens at one- and tree-point bending. In [9, 10] simple formulae for calculation of dynamic SIF for cylindrical specimens was proposed.

PROBLEM FORMULATION AND SOLVING

Tension of cylindrical specimen, weakened by surface circular crack. Let us consider cylindrical specimen with length $2h$ and radii r, which is contains an external circular crack of depth a at the middle. Two equal forces $P(t)$ are applied to the ends of the specimens. Approximating the function $P(t)$ by piece-wise linear function:

$$P(t) = \sum_{i=1}^{N} (k_i - k_{i-1})(t - t_{i-1}) H(t - t_{i-1}), \tag{1}$$

where k_i are the slope coefficients ($k_0 = 0$); t_i denote the breaking points ($t_0 = 0$); $H(t)$ is the Heavyside function,
after simple transformations [9] one can write:

$$K_{\mathrm{I}}(t) = K_{\mathrm{IS}}^{(1)} P(t) - K_{\mathrm{IS}}^{(1)} \sum_{i=1}^{N} (k_i - k_{i-1}) H(t - t_{i-1}) \sum_{j=1}^{n} (\eta_i / \omega_i) \sin \omega_j (t - t_{i-1}). \tag{2}$$

Here ω_i are the circular frequencies of natural oscillations of half of cylindrical specimen; n is a number of modes of natural oscillations, which are accounted for in the analysis;

$\eta_i = K_{\mathrm{I}i} u_{0i} \Big/ \omega_i^2 K_{\mathrm{IS}}^{(1)} \left(\sum\limits_{i=1}^{n} \eta_i = 1 \right)$;

u_{0i} is the i^{th} component of the displacement vector of the point, where the force is applied; $K_{\mathrm{IS}}^{(1)} -$ is static SIF, when the specimen is loaded (in tension) by single force; $K_{\mathrm{I}i}$ denotes SIF in the specimen deformed by i^{th} normed mode of autonomous oscillations.

The frequencies of natural oscillations ω_i and the corresponding to them displacements (eigenvectors) are estimated from the generalized problems on eigenvalues [1, 11]

$$[K]\vec{\mathbf{u}} = \omega^2 [M] \vec{\mathbf{u}} \tag{3}$$

and norming conditions

$$\mathrm{u}_i^{\mathrm{T}} [M] \mathrm{u}_j = \delta_{ij}, \tag{4}$$

where $[K]$ i $[M]$ are the stiffness and mass matrixes [11]; δ_{ij} is the Kroneker matrix; index "T" means transposition [11].
The formulae (2) represents the dynamic SIF as a sum of two additives: a quasistatic part and an inertial correction.

Dimensionless coefficients η_i and eigenfrequencies $\omega_i^* = \omega_i a \big/ \sqrt{E/\rho}$ (here E is the Young modulus and ρ - material density) do not depend on the specimen dimensions, but they do depend on the ratio of specimen length to its radii, relative depth of crack and Poisson ratio ν [1].

In [9] simple relations are given for estimation of ω_i^* та η_i for a range $0.27 \le \nu \le 0.33$, and in [10] – for the range $0.2 \le \nu \le 0.4$ under the condition $4 \le 2h/a \le 20$.

Tension of the hollow cylindrical specimen, weakened by external circular crack. We assume that a hollow cylindrical $2h$ - long specimen (with internal and external radii are correspondingly r_1 i r_2) is weakened by external circular crack, which radii is "a", and tensed by equally distributed forces, applied to the ends (along the ruling line).

If to represent the main vector of loading, applied to cylinder, as the piece-wise linear function (1), the dynamic SIF can be written in the form (2).

The dimensionless frequencies of natural oscillations $\omega_i^* = \omega_i r_2 \big/ \sqrt{E/\rho}$ and the coefficients η_i were calculated numerically using commercial FEA package MARC for following values of geometric parameters: $9 \le 2h/r_2 \le 14$; $0.35 \le r_1/r_2 \le 0.80$; $0.2 \le (r_2 - a)/(r_2 - r_1) \le 0.5$. After the relation $\sum_{i=1}^{n} \eta_i = 1$ was analyzed we concluded that in this range of cylinder' relative length it is enough to restrict oneself by three modes of autonomous oscillations, in order to reach sufficient accuracy.

Making use of the least square method, following relations for ω_i, та η_i were obtained:

$$\begin{aligned}
\omega_1^* &= 0.2348 + 7.495\lambda - 10.4915\lambda^2 + \left(-0.8447\lambda + 1.6701\lambda^2 - 0.7952\lambda^3\right)\left(4.8875\sqrt{\gamma} - 0.5728\gamma\right) + \\
&\quad + \left[5.1393\lambda - 26.346\lambda^2 + 31.5317\lambda^3 + \gamma\left(-0.0109\lambda + 0.0403\lambda^2 - \right.\right. \\
&\quad \left.\left. - 0.0437\lambda^3\right)\left(0.418 + 0.8312\gamma\right)\right]\left(1.0089\sqrt{t} - 0.726t\right), \\
\omega_2^* &= 0.6313 + 8.4907\lambda - 6.5496\lambda^2 + \left(-1.7829\lambda + 1.8045\lambda^2 - 0.4464\lambda^3\right)\left(2.004\sqrt{\gamma} - 0.1745\gamma\right) + \\
&\quad + \left[-11.139\lambda + 47.53\lambda^2 - 4.8197\lambda^3 + \gamma\left(-0.292\lambda + 1.2431\lambda^2 - \right.\right. \\
&\quad \left.\left. - 1.2791\lambda^3\right)\left(0.0916\gamma - 3.7866\right)\right]\left(2.3233t - 2.9088\sqrt{t}\right), \\
\omega_3^* &= -0.4108 - 0.2956t + 0.7003t^2 + \left(4.3683 - 0.5161\lambda\right)\left(1/\gamma + 0{,}364/\sqrt{\gamma}\right)\left(2.1493 - 2.5253t\right) + \\
&\quad \left(1/\gamma - 5.1156/\gamma^2\right)\left(44.945t - 18.849t^2\right).
\end{aligned} \tag{5}$$

$$\begin{aligned}
\eta_1 &= 1.2597 + \left(1.4352\lambda - 2.001\lambda^2\right)\left(0.7997\gamma^{-1} - 7.807\gamma^{-2}\right)\left(3.1391t - 0.0882t^2\right), \\
\eta_2 &= 0.2175 + 7.1996\lambda - 12.381\lambda^2 + 0.6053\lambda^3 + \left(1.7254 + 19.5782\lambda - 32.173\lambda^2\right) \times \\
&\quad \left(26.804/\gamma^2 - 1.837/\sqrt{\gamma}\right)\left[1.4574\lambda - 2.337\lambda^2 + \left(18.09\lambda - 27.859\lambda^2\right) \times \right. \\
&\quad \left. \left(17.058/\gamma^2 - 2.2894/\gamma\right)\right]\left(9.3297t - 10.541\sqrt{t}\right), \\
\eta_3 &= -4.3725 + 16.7274t - 11.967t^2 + \left(-8.3858 + 33.344t - 24.169t^2\right) \times \\
&\quad \times \left(19.164/\gamma^2 - 2.2466/\sqrt{\gamma}\right) + \left[0.2475 + 0.136\lambda - 0.089\lambda^2 + \right. \\
&\quad \left. \left(-4.9837\sqrt{\lambda} + 5.506\lambda - 1.5625\lambda^2\right)\left(5.388/\gamma - 1.5171/\sqrt{\gamma}\right)\right]\left(4.1364\sqrt{t} - 3.892t\right).
\end{aligned} \tag{6}$$

At the same time the static SIF under the loading with the main vector equal to unity was calculated. Processing the obtained data with a help of the least square method the following approximation for $K_{\mathrm{IS}}^{(1)}$ derived:

$$\begin{aligned}
K_{\mathrm{IS}}^{(1)} &= \left(-0.246 + 12.791\sqrt{\lambda} + 6.1363\lambda + 27.55\lambda^2 + \left(6.5745\sqrt{\lambda} + 4.4422\lambda + \right.\right. \\
&\quad \left.\left. + 15.962\lambda^2 - 0.562\lambda^3\right)\left(-9.7118\sqrt{t} + 21.019t - 20.277t\sqrt{t} + 7.3759t^2\right)\right) \Big/ r_2\sqrt{r_2}
\end{aligned} \tag{7}$$

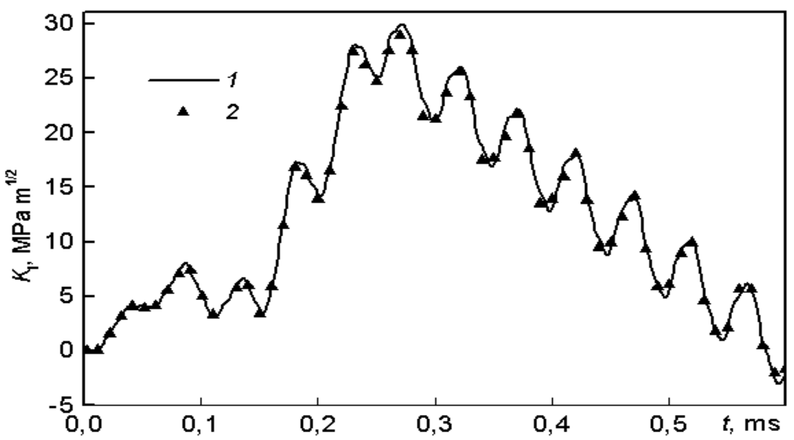

Figure 1: Dependence of the dynamic SIF vs. time for tension of the hollow cylindrical specimen (1 – dynamic SIF calculated using equations (1, 2); 2 – dynamic SIF calculated using FEA).

Using FEA (namely Newmark method) we estimated dynamic SIF for cylindrical specimen with following dimensions: $r_1 = 3$ mm; $r_2 = 8$ mm; $h = 60$ mm; $a = 5{,}5$ mm; $E = 200$ GPa, $\rho = 7870\ \mathrm{Mg/m^3}$,

$\nu = 0.3$. The loading diagram was obtained experimentally according to [12]. Based on these results, following parameters for approximation of the "force-time" diagram were chosen:
$k_1 = 11760$ Pa; $k_2 = 8330$ Pa; $k_3 = 53950$ Pa; $k_4 = 0$ Pa; $t_1 = 75\,\mu$s; $t_2 = 140\,\mu$s; $t_3 = 250\,\mu$s; $t_4 = 600\,\mu$s. Comparison of the diagrams, obtained numerically (FEA) and one calculated using simplified formulae (2), (5) – (7) validates the approach developed.

Torsion of cylindrical specimen, weakened by surface circular crack. Let us consider cylindrical specimen with following dimensions: length is $2h$ and radii is a. At the middle the specimen has a circular crack of depth c. The load, which main vector is equal to T, is applied along the ruling line of the ends of specimens.

After the force T is represented by the piece-wise function (1), the dynamic SIF can be written as follows:

$$K_{\mathrm{III}}(t) = K_{\mathrm{IIIS}}^{(1)} P(t) - K_{\mathrm{IIIS}}^{(1)} \sum_{i=1}^{N} (k_i - k_{i-1}) H(t - t_{i-1}) \sum_{j=1}^{n} (\eta_i / \omega_i) \sin \omega_j (t - t_{i-1}) \tag{8}$$

where ω_i are the circular frequencies of torsional natural oscillations of half of cylindrical specimen; n denotes number of modes of natural oscillations; $\eta_i = K_{\mathrm{III}i} u_{0i} \Big/ \omega_i^2 K_{\mathrm{IIIS}}^{(1)} \left(\sum_{i=1}^{n} \eta_i = 1 \right)$; u_{0i} is the i^{th} component of the displacement vector in the point, where load is applied; $K_{\mathrm{IIIS}}^{(1)}$ means the static SIF for the specimen, loaded by the momentum, equal to a; K_{IIIi} is the SIF in the specimen, deformed with the i^{th} normed mode of autonomous oscillations.

For the range of specimen' dimensions $0{,}3 \le \lambda \le 0{,}7$ and $8{,}0 \le \gamma \le 13{,}0$ ($\lambda = c/r$; $\gamma = 2h/r$) for ν=0,3 making use of the Lancosh' blocks method we evaluated the circular frequencies of torsional natural oscillations and corresponding to them eigenvectors. The data were analyzed using the least square method with following dependencies as the result:

$$\begin{aligned}
\omega_1^* &= 0.158 - 1.113\lambda + 2.706\lambda^2 - 1.959\lambda^3 + \lambda\left(13.79 - 34.1\lambda + 21.94\lambda^2\right)/\gamma, \\
\omega_2^* &= 0.5295 - 3.376\lambda + 7.484\lambda^2 - 5.312\lambda^3 + \lambda\left(39 - 95.3\lambda + 68.1\lambda^2\right)/\gamma, \\
\omega_3^* &= 1.052 - 6.456\lambda + 13.317\lambda^2 - 8.916\lambda^3 + \lambda\left(60.1 - 135.9\lambda + 94.5\lambda^2\right)/\gamma, \\
\omega_4^* &= 1.5047 - 9.202\lambda + 18.602\lambda^2 - 12.198\lambda^3 + \lambda\left(81.68 - 176.16\lambda + 119.4\lambda^2\right)/\gamma.
\end{aligned} \tag{9}$$

$$\begin{aligned}
\eta_1 &= 1.058 + 1.9975\lambda - 5.5285\lambda^2 + 5.5249\lambda^3 + \left(-0.044 + 0.2144\lambda - 0.1977\lambda^2 + 3.5249\lambda^2\right)\lambda\gamma, \\
\eta_2 &= -0.0978 - 3.181\lambda + 9.64\lambda^2 - 7.033\lambda^3 + \left(0.0596 - 0.3317\lambda + 0.3315\lambda^2\right)\lambda\gamma, \\
\eta_3 &= 0.1894 + 0.9271\lambda - 4.3216\lambda^2 + 3.8868\lambda^3 + \left(0.954 - 0.131\lambda\right)\lambda^2\gamma, \\
\eta_4 &= -0.7987 + 3.9245\lambda - 6.412\lambda^2 + 3.481\lambda^3 + \left(0.046 - 0.3236\lambda + 0.6356\lambda^2 - 0.385\lambda^3\right)\gamma.
\end{aligned} \tag{10}$$

The accuracy of approximations (9, 10) does not exceed 2%. For calculations of the dynamic SIF using equations (2), (9) and (10) one must have the static SIF' value for the cylindrical specimen, loaded by torsion. There are several solutions to this problem, known from literature. For a range of λ parameter, based on the integral equations method, the SIF values have been obtained in [12]. Based on these values and using the least square method we got following dependence for $K_{IIIS}^{(1)}$

$$K_{\mathrm{IIIS}}^{(1)} = \left(3{,}403 - 29{,}637\lambda + 104.613\lambda^2 - 156{,}12\lambda^3 + 89{,}14\lambda^4\right)/r\sqrt{r}\,. \tag{11}$$

Eccentric bending of a beam specimen. At the initial stage of the loading history the interaction between the specimen and beam seats does not influence the dynamic SIF value. In the other words, for the brittle materials and high-speed loading a crack propagation begins before the interaction between the specimen and the beam seats start and, as a result, the subcritical loading is not influenced. These conditions are called "one-point-bending" or "bending without support". First case is considered in [3]. Let us consider a case, when the crack is located not in the middle of the beam specimen (fig. 2).

As in the previous cases we represent the "force-time" diagram by piece-wise function. It is possible to show, that for such representation of the "force-time" diagram the dynamic SIF can be written as follows:

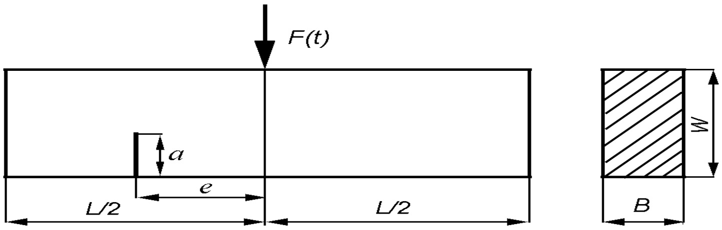

Figure 2: The loading scheme of the beam specimen with eccentric crack.

$$\begin{aligned} K_{\mathrm{I}}(t) &= K_{\mathrm{IS}}^{(1)}F(t) - K_{\mathrm{IS}}^{(1)}\sum_{i=1}^{N}(k_i - k_{i-1})H(t - t_{i-1})\sum_{j=1}^{n_1}(\eta_{1i}/\omega_i)\sin\omega_j(t - t_{i-1}) \\ K_{\mathrm{II}}(t) &= K_{\mathrm{IIS}}^{(1)}F(t) - K_{\mathrm{IIS}}^{(1)}\sum_{i=1}^{N}(k_i - k_{i-1})H(t - t_{i-1})\sum_{j=1}^{n_2}(\eta_{2i}/\omega_i)\sin\omega_j(t - t_{i-1}) \end{aligned} \tag{12}$$

where ω_i denotes the circular frequencies of natural oscillations of the beam without support; n_1, n_2 mean the number of modes of natural oscillations, which are taken into account in analysis; $\eta_{1i} = K_{\mathrm{I}i}u_{0i} \Big/ \omega_i^2 K_{\mathrm{IS}}^{(1)} \left(\sum_{i=1}^{n}\eta_{1i} = 1\right)$, $\eta_{2i} = K_{\mathrm{II}i}u_{0i} \Big/ \omega_i^2 K_{\mathrm{IIS}}^{(1)} \left(\sum_{i=1}^{n}\eta_{2i} = 1\right)$; u_{0i} is the i^{th} component of displacement vector for the point of load' application; $K_{\mathrm{IS}}^{(1)}$, $K_{\mathrm{IIS}}^{(1)}$ are the static SIF for the tension of the specimen by force, uniformly distributed in the volume; $K_{\mathrm{Ii}}, K_{\mathrm{IIi}}$ are the SIF in the specimen, deformed by i^{th} normed mode of autonomous oscillations.

While calculating $K_{\mathrm{IS}}^{(1)}$, $K_{\mathrm{IIS}}^{(1)}$, the volume loading was substituted by equally distributed loading applied to the top and bottom part of specimen. For this the FEA approach was used. The mesh quality was verified by comparison with the results, obtained in [14]. The results in the range $4 \le \gamma \le 6$, $0.3 \le \lambda \le 0.7$; $\varepsilon = e/W = 1$ can be described by equations (with error less than 1.5%):

$$\begin{aligned} K_{\mathrm{IS}}^{(1)} &= L\sqrt{l}\,\Big[-1.3461 + 4.1651\lambda - 6.2731\lambda^2 + \gamma\Big(1.5083\sqrt{\lambda} - 1.9329\lambda + 0.0333\lambda^2 + \\ &\quad + 2.3095\lambda^3\Big)(1.0861 - 0.0606\gamma)\Big]\Big/BW^2, \\ K_{\mathrm{IIS}}^{(1)} &= \lambda L\sqrt{l}\,\Big[0.262 - 0.0861\lambda + \lambda\gamma\Big(0.2024 - 0.4365\lambda + 0.3122\lambda^2\Big)(0.7705 - 0.1051\gamma)\Big]\Big/BW^2. \end{aligned} \tag{13}$$

The dimensionless coefficients η_{1i}, η_{2i} and eigenfrequencies $\omega_i^* = \omega_i a\big/\sqrt{E/\rho}$ do not depend on the specimen dimensions. For $\varepsilon = e/W = 1$, $0.3 \le \lambda \le 0.7$, $4.0 \le \gamma \le 6.0$ (here, $\lambda = a/W$; $\gamma = L/W$) and ν=0,3 using the Lancosh blocks method we found the circular frequencies of natural oscillations and corresponding to them eigenvectors. Analyzing the relations $\sum_{i=1}^{n}\eta_{ji} = 1 (j = 1,2)$, we concluded that it is enough to chose $n_1 = 5$ and $n_2 = 6$ for evaluation of $K_{\mathrm{I}}(t)$ and $K_{\mathrm{II}}(t)$ correspondingly. Processing the numerical data using the least square method, following polynomial approximations were proposed for ω_i^*:

$$\begin{aligned} \omega_1^* &= 0.2935 + 4.617\lambda - 6.2351\lambda^2 - \lambda\gamma\Big(1.1088 - 1.4685\lambda + 0.0763\lambda^2\Big)(1.4143 - 0.105\gamma), \\ \omega_2^* &= 0.8996 + 2.8588\lambda - 4.0988\lambda^2 - \lambda\gamma\Big(0.7018 - 1.0621\lambda + 0.3022\lambda^2\Big)\Big(1.7487 - 0.0096\gamma - 0.0096\gamma^2\Big), \\ \omega_3^* &= 0.6612 + 6.0468\lambda - 5.6163\lambda^2 - \lambda\gamma(1.2356 - 1.1247\lambda)(1.509 - 0.104\gamma), \\ \omega_4^* &= 1.2866 + 5.6316\lambda - 6.4583\lambda^2 - \lambda\gamma\Big(0.5026 - 0.7079\lambda + 0.2123\lambda^2\Big)(4.2913 - 0.2151\gamma), \\ \omega_5^* &= 1.5634 + 8.5572\lambda - 10.092\lambda^2 - \lambda\gamma\Big(0.7007 - 0.8957\lambda + 0.1549\lambda^2\Big)(4.1461 - 0.2309\gamma), \\ \omega_6^* &= 1.6014 + 10.416\lambda - 13.535\lambda^2 - \lambda\gamma\Big(0.8715 - 1.2239\lambda + 0.1916\lambda^2\Big)(3.8153 - 0.2075\gamma). \end{aligned} \tag{14}$$

The approximations (14) describe the numerical data with the deviation less, than 2%. For coefficients η_{1i} following dependencies are proposed:

$$
\begin{aligned}
&\eta_{11} = 1.1814 + 14.703\lambda - 19.443\lambda^2 + \lambda\gamma(1.4708\lambda - 1.122)(4.0276 - 0.3145\lambda),\\
&\eta_{22} = 0.0175 - 6.5325\lambda + 25.07\lambda^2 - 15.82\lambda^3 + \lambda\gamma\left(0.6726 - 2.5323\lambda + 1.629\lambda^2\right)(2.9651 - 0.229\gamma),\\
&\eta_{13} = 0.0471 + 0.6677\lambda - 0.7738 + \lambda\gamma(0.1183\lambda - 0.1073)(2.5695 - 0.2013\gamma),\\
&\eta_{24} = 0.4548 - 2.0633\lambda + 2.1258\lambda^2 + \lambda\gamma(0.8861 - 0.816\lambda)(0.8801 - 0.0806\gamma),\\
&\eta_{25} = 0.2915 - 8.2444\lambda + 36.28\lambda^2 - 36.92\lambda^3 + \lambda\gamma\left(0.5803 - 2.9915\lambda + 3.17\lambda^2\right)(3.0027 - 0.193\gamma).
\end{aligned}
\tag{15}
$$

For η_1, η_2 and η_4 the relative error does not excide 3%, but for approximations of η_3, η_5 the absolute error does not excide 0.003. Similar relations were obtained for η_{2i}:

$$
\begin{aligned}
&\eta_{21} = 0.531 + 8.5372\lambda - 24\lambda^2 + 16.017\lambda^3 - \lambda\gamma\left(0.1678 - 3.227\lambda + 2.184\lambda^2\right)(1.4914 - 0.0439\gamma),\\
&\eta_{22} = 0.0175 - 6.5325\lambda + 25.07\lambda^2 - 15.82\lambda^3 + \lambda\gamma\left(0.6726 - 2.5323\lambda + 1.629\lambda^2\right)(2.9651 - 0.229\gamma),\\
&\eta_{23} = -0.2065 + 0.0792\lambda - 0.0076\lambda^2,\\
&\eta_{24} = 0.4548 - 2.0633\lambda + 2.1258\lambda^2 + \lambda\gamma(0.8861 - 0.816\lambda)(0.8801 - 0.0806\gamma),\\
&\eta_{25} = 0.2915 - 8.2444\lambda + 36.28\lambda^2 - 36.92\lambda^3 + \lambda\gamma\left(0.5803 - 2.9915\lambda + 3.17\lambda^2\right)(3.0027 - 0.193\gamma),\\
&\eta_{26} = -0.026 + 0.2252\lambda - 0.4055\lambda^2 - \lambda\gamma\left(0.3281 - 2.358\lambda + 5.0185\lambda^2 + 0.4439\lambda^3 - \right.\\
&\qquad \left. - 8.9764\lambda^4 + 5.3806\lambda^5\right)\left(8.3904 - 0.8796\gamma - 0.4054\gamma^2 + 0.0524\gamma^3\right).
\end{aligned}
\tag{16}
$$

For first five coefficients the deviation of proposed approximation is the same as in previous case, but for η_{26} parameter the absolute error does not exceed 0.004.

Using the dependencies (13) – (16) one can estimate the dynamic SIF for testing of beam specimen with eccentric crack.

REFERENCES

1. Parton V. Z., Boriskovski V. G. (1985). *Dynamic Fracture Mechanics*. Mir, Moscow (in Russian).
2. Chirino F., Dominguez J. (1989). *Eng. Fract. Mech.* **34**, 1051.
3. Andreykiv A. E., Rokach I. V. (1989) *Physicochemical mechanics of materials.* **25**, 42(in Russian).
4. Rokach I. V. (1990) *Physicochemical mechanics of materials,* **26**, 79(in Russian).
5. Kishimoto K., Fujino K., Aoki S., Sakata M. (1990) *JSME Int. J.*, Ser. 1. **33**, 51.
6. Andreykiv O. Ye., Kovchyk S. Ye., Khodan I. V., Bojko V. M. (1997) *The problems of machinebuilding and automatization*. 22 (in Russian).
7. Rokach I. V. (1998) *Fatig. Fract. Eng. Mat. Struct*. **21,** 1007.
8. Rokach I. V. (1998) *Fatig. Fract. Eng. Mat. Struct*. **21,** 1015.
9. Andreykiv O. Ye., Boyko V. M., Kovchyk S. Ye., Khodan I. V. (2000) *Physicochemical mechanics of materials*. 42(in Ukrainian).
10. Andreykiv O. Ye., Boyko V. M., Kovchyk S. Ye., Khodan I. V. (2000). In: *Abstract volume the 13th ECF: Fracture mechanics: Applications and Challenges*, p. 132, Fuentes M., Elices M., Martin-Meizoso A., Martinez-Esnaola J. M. (Eds). Elsevier, Oxford.
11. Zienkiewicz O. C. (1971). *The Finite Element Method in Engineering Science.* McGraw-Hill, London.
12. Kovchyk S. Ye., Khodan I. V., Zamora T. S. Zaydel B. M. (1994). *Physicochemical mechanics of materials.* **30**, 133(in Ukrainian).
13. Shibuya T., Koizumi T., Okuya T. (1979) *Bull. JSME.* **22**. 1049.
14. Jen Wang Ke, Lin Hsu Chi, Hua Kao (1977). In: *Adv. Res. Strength and Fract. Mater. 4th Int. Conf Fract.* . McGraw-Hill, New York.
15. MARC, Volume A: Theory and User information, Rev. K.7, 1998, MARC Analysis Research Corp.