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ERROR ESTIMATION OF SHAPE CHANGES DURING FATIGUE CRACK GROWTH

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ABSTRACT

Accurate determination of the shape of a crack is important during fatigue. On the other hand, simulation may require a considerable number of fatigue evaluations that correspond to a wide range of crack shapes. Therefore, the numerical methods employed to carry out this task need to address the issues of accuracy, robustness and efficiency. In this paper, a fourth order Runge-Kutta scheme is used for crack shape evaluation and its accuracy is first assessed by comparison to closed form solutions for a surface cracked plate under tension and by means of a convergence study for the surface-cracked butt welded plate in tension. Finally, a convergence study is carried out on the fatigue lives of Monte Carlo samples of the latter geometry. This last study is carried out by assuming that crack growth follows a bi-linear law.

KEYWORDS

Semi-elliptical surface crack, fatigue, shape changes, Monte Carlo simulation

INTRODUCTION

Under fatigue, cracks grow while retaining a semi-elliptical shape. Accurate determination of the crack shape is important at all times during the crack evolution since accurate stress intensity factor determination and hence rate of crack growth is heavily reliant on the dimensions of the crack. For a two degree-of-freedom surface crack, and assuming that the Paris C parameters are the same in both directions [1], the pertinent equations are:

$$\frac{da}{dN} = C(\Delta K_A)^m \tag{1}$$

$$\frac{dc}{dN} = C(\Delta K_C)^m \tag{2}$$

Here, the point A refers to the deepest point of the crack border and the point C to its surface counterpart. The differential Eqns. 1 and 2 may be solved for a plate with thickness t by solving one of the two as a differential equation and integrating the other one thus,

$$\left(\frac{a}{c}\right)_f = f\left(\left(\frac{a}{c}\right)_{in}, \left(\frac{a}{t}\right)_{in}, \left(\frac{a}{t}\right)_f, m\right) \tag{3}$$

$$N = \frac{1}{C}\int_{a_{in}}^{a_f} \frac{da}{\left(S_r\ Y_A \sqrt{\frac{\pi\ a}{Q}}\right)^m} \tag{4}$$

where S_r is the stress range, Y_A is the stress intensity magnification factor for A, Q is the elliptical shape factor [1], and the subscripts *in* and *f* denote the initial and final values, respectively. Note that Y_A is a function of both a/t and a/c, the latter being calculated from Eqn. 3. Eqn. 4 may be solved using any numerical integration scheme and here the integral is calculated using 24-point Gauss integration. Eqn. 3 may be obtained by using any high order method. Here, the fourth order Runge-Kutta scheme is investigated using different number of steps. In order to demonstrate the robustness of the Runge-Kutta scheme, two geometries are investigated in this paper, both under tension. First, the case of the semi-elliptical surface crack in a plate under tension is investigated since analytical solutions of Eqn. 3 exist for different integer m [2] and these are directly compared with the results of the proposed numerical scheme. Then the case of the semi-elliptical surface crack in a butt-welded plate under tension is examined by means of a convergence study on the statistical characteristics of the crack shape and the fatigue lives while the latter are calculated based on a bi-linear crack growth law.

ERROR ESTIMATION

Semi-elliptical surface crack in a plate under tension

The factor Y_A used here is the Newman-Raju solution [3]. An analytical expression of Eqn. 3 in this case was supplied by Wu [2] for m=2 and m=3. The results for m=5 and m=6 are straightforward to derive and are given by Eqns. 5 and 6 for m=5 and 7 and 8 for m=6.

$$\frac{a}{c}=\left\{F\left(\frac{a}{t}\right)-\left[\frac{(a/t)_{in}}{(a/t)}\right]^{7/2}\left[F\left(\left(\frac{a}{t}\right)_{in}\right)-\left(\left(\frac{a}{c}\right)_{in}\right)^{-7/2}\right]\right\}^{-2/7} \tag{5}$$

where

$$F\left(\frac{a}{t}\right)=1.6105+1.6305\left(\frac{a}{t}\right)^{2}+0.7609\left(\frac{a}{t}\right)^{4}+0.1911\left(\frac{a}{t}\right)^{6}+0.0251\left(\frac{a}{t}\right)^{8}+ 0.0013\left(\frac{a}{t}\right)^{10} \tag{6}$$

$$\frac{a}{c}=\left\{F\left(\frac{a}{t}\right)-\left[\frac{(a/t)_{in}}{(a/t)}\right]^{4}\left[F\left(\left(\frac{a}{t}\right)_{in}\right)-\left(\left(\frac{a}{c}\right)_{in}\right)^{-4}\right]\right\}^{-1/4} \tag{7}$$

where

$$F\left(\frac{a}{t}\right)=1.7716+2.2547\left(\frac{a}{t}\right)^{2}+1.3451\left(\frac{a}{t}\right)^{4}+0.4565\left(\frac{a}{t}\right)^{6}+0.0908\left(\frac{a}{t}\right)^{8}+ 0.0099\left(\frac{a}{t}\right)^{10}+0.0005\left(\frac{a}{t}\right)^{12} \tag{8}$$

The robustness of the Runge-Kutta (R-K) scheme employed here was investigated by assuming different combinations of $(a/t)_{in}$, $(a/c)_{in}$ taken from the interval (0,1), calculating $(a/c)_f$ for $(a/t)_f = 1$ by using Eqn. 3 and comparing with their analytically derived counterparts. In total, 400000 different combinations of these variables were used. The statistical characteristics of the differences between the numerical approximation of Eqn. 3 and the analytical solution are shown in Figure 1 and are presented in the form of mean and mean + standard deviation curves of the percentage difference between the proposed numerical scheme and the exact solution. The mean error lines demonstrate that, larger errors irrespective of the number of R-K steps used are observed for lower *m* values. However, the scatter is significantly smaller for low *m* values and decreases as the number of R-K steps increases.

Semi-elliptical surface crack in a butt-welded plate under tension

The factor Y_A used for this geometry is given in the Appendix. In this case, a closed form solution cannot be obtained as was done for the previous geometry. Consequently, differences are reported here with respect to the forty-step R-K scheme. Errors in $(a/c)_f$ were calculated again for the same values of $(a/t)_{in}$, $(a/t)_f$ (=1) and $(a/c)_{in}$ as in the previous section. The results are shown in Figure 2. The scatter, which is depicted by the solid lines, diminishes with decreasing *m*. The highest mean errors are here recorded for *m*=6, which drop below 5% for R-K steps greater than 10. The highest scatter is again recorded for m=6 and this drops below 5% for R-K steps greater than 18. These results indicate that use of lower order schemes at the same small number of steps could result in unacceptably high errors. The convergence characteristics may also be investigated in a more comprehensive manner by looking at the fatigue lives calculated via Eqn. 4 in a probabilistic way. Here, the bi-linear crack growth law proposed in BS 7910 [1] is used.

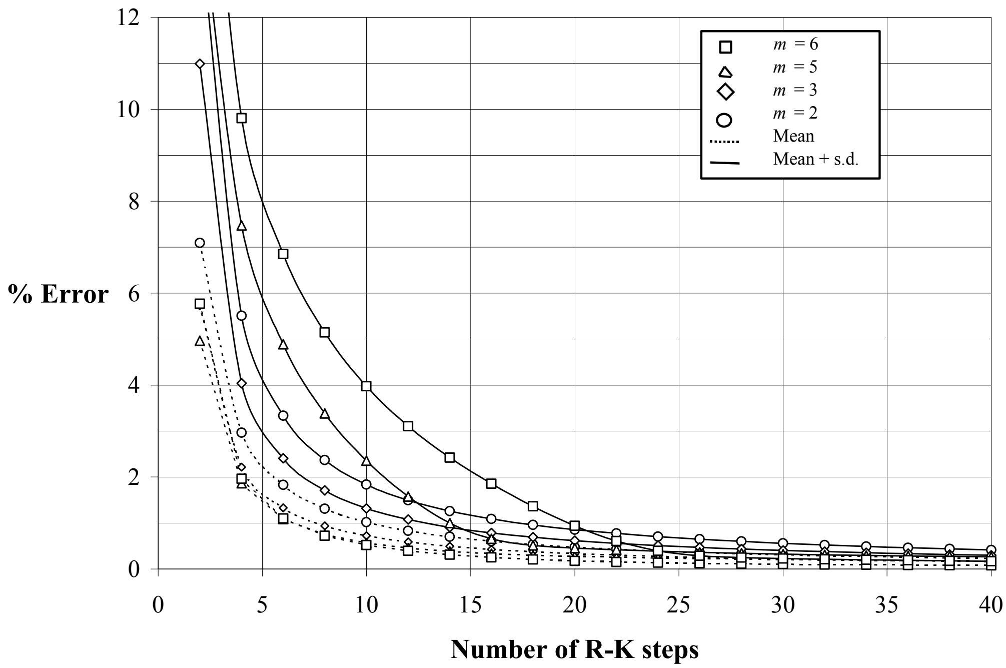

Figure 1: Mean and mean + s.d. curves of the errors in $(a/c)_f$ as functions of the R-K steps for a cracked plate

The crack growth model comprises two lines, each being described by its own Paris parameters. The crack growth law is described by the following equation

$$\frac{da}{dN} = \begin{cases} 0 & \Delta K \le \Delta K_{thr} \\ C_1(\Delta K)^{m_1} & \Delta K_{thr} < \Delta K \le \Delta K_{tr} \\ C_2(\Delta K)^{m_2} & \Delta K_{tr} \le \Delta K \end{cases} \tag{9}$$

where ΔK_{thr} is the threshold stress intensity range and ΔK_{tr} is the stress intensity range corresponding to the intersection of the two crack growth lines. The deterministic and probabilistic variables and their characteristics used here are the same as in Reference [4] with the added random variable of $(a/c)_{in}$, which was assumed to be lognormally distributed with mean 0.01 and coefficient of variation 0.5 . The Paris parameters m_1 and m_2 were taken to be deterministic and equal to 5.10 and 2.88, respectively, in accordance with BS 7910 [1]. Errors here were calculated with respect to the estimate arising from the application of the highest number of R-K steps (n_I, n_{II} = 10) in both crack growth regions. Results are shown in Figure 3. Additional information is given by the inset of this figure. The figure shows that in this case, the influence of the number of R-K steps in the near threshold crack growth region (I) is far more important than in the Paris region.

Consequently, it is quite clear that acceptable accuracy (error < 5%) may be obtained by using a $(3,n_{II})$ R-K with $n_{II}>1$.

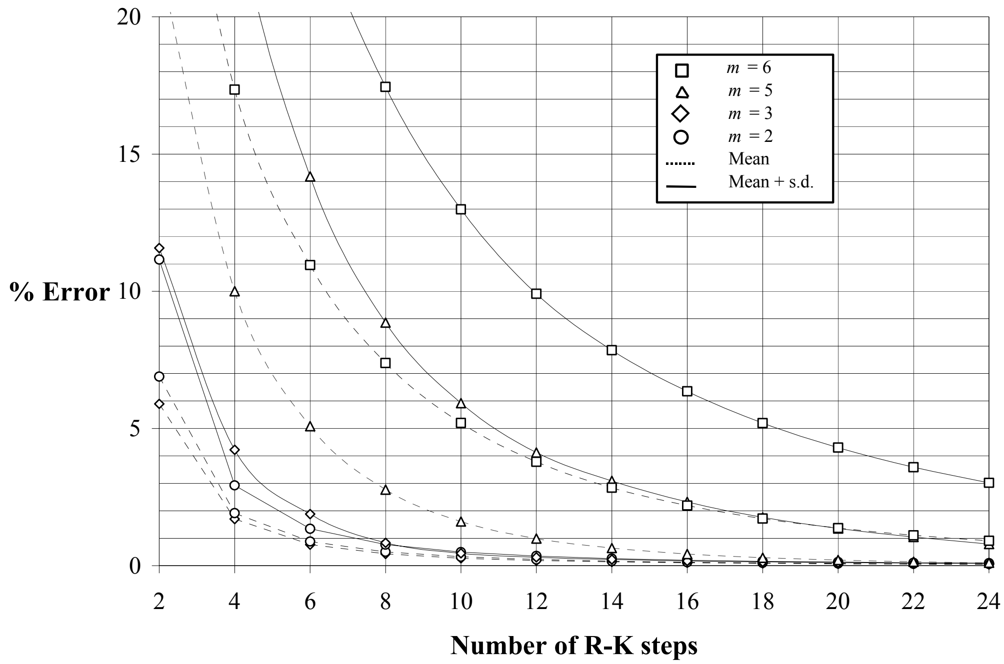

Figure 2: Mean and mean+s.d. curves of the errors in $(a/c)_f$ as functions of the R-K steps for a cracked butt-welded plate

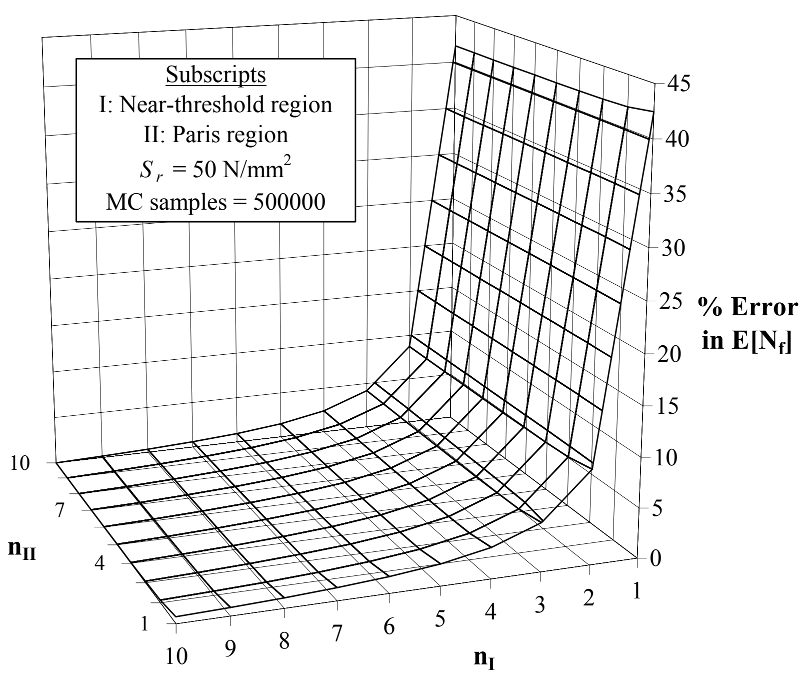

Figure 3: Variation of error in mean fatigue life with number of R-K steps in the two crack growth regions

CONCLUSIONS

The numerical characteristics of a fourth order Runge-Kutta scheme used to evaluate the crack shape during fatigue crack growth were presented in this paper. These were first compared against available analytical solutions for the case of a surface cracked plate under tension. Errors were then calculated in the form of a convergence study for a cracked butt-welded plate under tension. Both studies were carried out for a wide range of initial crack depths and shapes. The robustness of the scheme was also investigated with reference to fatigue lives as these were calculated using Gauss integration of a bi-linear crack growth law. Motivation for this work was supplied by the need to determine accurately fatigue lives within Monte Carlo simulation of the recently proposed bi-linear crack growth model of BS 7910 [1].

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APPENDIX

For a welded plate, the factor Y_A is given by

$$Y_A = M_{km}\, M_m$$

where M_{km} is the magnification factor associated with the weld [1] and M_m is the Newman-Raju plate solution [3].