ORAL REFERENCE: ICF 1003060R

EFFECTS OF HETEROGENEITY ON THE STRENGTH OF 3D COMPOSITES

Sivasambu Mahesh[1], Irene J. Beyerlein[2], and S. Leigh Phoenix[1]

[1]Theoretical & Applied Mechanics, Cornell University, Ithaca NY 14853. USA.
[2]Materials Science & Technology, Los Alamos National Laboratory, Los Alamos, NM 87545. USA.

ABSTRACT

Monte Carlo simulation interpreted with theoretical modeling is used to study the statistical failure modes in unidirectional composites consisting of a hexagonal array of elastic fibers embedded in an elastic matrix. Composite structure is idealized using the chain-of-bundles model in terms of bundles of length δ arranged along the fiber direction. Fibers element strengths in δ-bundles are taken to be Weibull distributed and Hedgepeth and Van Dyke load sharing is assumed for transverse fiber break arrays.

Simulations of δ-bundle failure reveal two regimes. When fiber strength variability is low, the dominant failure mode is by growing clusters of fiber breaks up to instability. When this variability is high, cluster formation is suppressed by a more dispersed fiber failure mode. Corresponding to these two cases, we construct simple models that predict the strength distribution of a δ-bundle. Their predictions compare very favorably with simulations in the two cases.

KEYWORDS

unidirectional fiber composites, brittle/ductile transition, extreme value problem, chain of bundles model, shear-lag model.

INTRODUCTION

Quasistatic failure of unidirectional composite materials, which consist of long aligned reinforcing fibers embedded in a matrix is a complex stochastic process. While complexity stems from the occurrence of various damage events preceding formation of a catastrophic crack, statistical variation in strength primarily arises due to the variability in fiber strength. Consequently composite tensile strength is itself a statistical quantity and methods to determine its distribution are of considerable significance in assuring composite reliability.

Idealization of composite structure and material properties are found to be inevitable before further analysis can be attempted. In this study we assume stiff linear elastic fibers arranged in a hexagonal array and embedded in a relatively compliant linear elastic non-debonding matrix so that material damage in our idealized composite is limited to fiber breakages alone. The large fiber-matrix stiffness ratio implies that most of the applied load will be borne by the fibers and the role of the matrix is limited to conducting loads from broken fibers to nearby intact fibers. This load transfer occurs through shear deformation that tends to occur over a certain length scale δ. δ is typically only a few fiber diameters and is much less than the composite length L.

As has been common in the literature, we idealize the failure process in terms of a longitudinal partition into $m = L/\delta$ transverse slabs or short bundles of length δ, called δ-bundles. The failure process within a given δ-bundle is treated as mechanically and statistically independent of that in neighboring δ-bundles. The composite is then treated as a weakest-link arrangement of these δ-bundles; that is, the composite fails when the weakest δ-bundle fails. Thus the chain-of-bundles assumption converts the 3D problem of composite failure into the problem of failure of the weakest of several 2D δ-bundles. We also assume that fiber strength X is random and distributed according to the Weibull distribution

$$F(x) = \Pr\{X \leq \sigma\} = 1 - \exp\{-(\sigma/\sigma_\delta)^\rho\} \tag{1}$$

where σ is the stress experienced by the fiber, σ_δ is the scale parameter for a fiber element of length δ and ρ the shape parameter of the distribution.

We use Hedgepeth and Van Dyke's [1] local load sharing model (HVLLS) to determine stress concentrations in the plane of a tranverse array of fiber breaks. While we do not delve into the details of their approach, we note that under HVLLS, the stress concentration around a penny-shaped crack of r fiber breaks is approximately

$$K_r \approx \sqrt{\frac{2\sqrt{r}}{\pi^{3/2}} + 1} = \sqrt{\frac{D}{\pi} + 1} \tag{2}$$

where D is the effective non-dimensional diameter of the penny-shaped crack and $r = \pi D^2/4$. Also, as the crack size becomes large, the stress concentration decays as $1/\sqrt{t}$ in the near-field where t is distance from the crack tip and shares this characteristic of the near-tip stress field with LEFM.

In this work, we take our δ-bundles to be rhombus shaped. In every realization of δ-bundle failure, gradually increasing load is applied to it in the fiber direction until a fiber fails due to the applied load just exceeding its Weibull strength. This fiber breaks causes stress redistribution according to HVLLS which in turn may cause more failures. This back-and-forth process of fiber failures and stress redistribution is continued until either stability (i.e., no further fiber breakage) is reached or the δ-bundle fails. In the case of stability, gradual applied load increment is resumed until the failure of another fiber. Then the above process of stress redistribution and further fiber failures is repeated. Eventually, at some applied load(the δ-bundle strength), a cascade of fiber failures signifies catastrophic δ-bundle failure. We now proceed to describe dominant failure mechanisms of composite failure observed in simulations and to model them in order to analytically predict the statistical strength distribution of composite strength.

FAILURE MECHANISMS AND MODELS IN δ-BUNDLES

1. Small Variability in Fiber Strength (large ρ)

Snapshots of the damage evolution in the $\rho = 10$ median (among 500 simulations) δ-bundle en route to failure are shown in Figure 1. The last stage shown corresponds to the arrangement of breaks immediately after the formation of an unstable system of fiber breaks and before the catastrophic failure of the remaining fibers. Note that the boundary conditions are periodic so that a break cluster appearing at one edge may be continued on the opposite edge. Cluster formation and growth is clearly the dominant failure mode in the specimen shown.

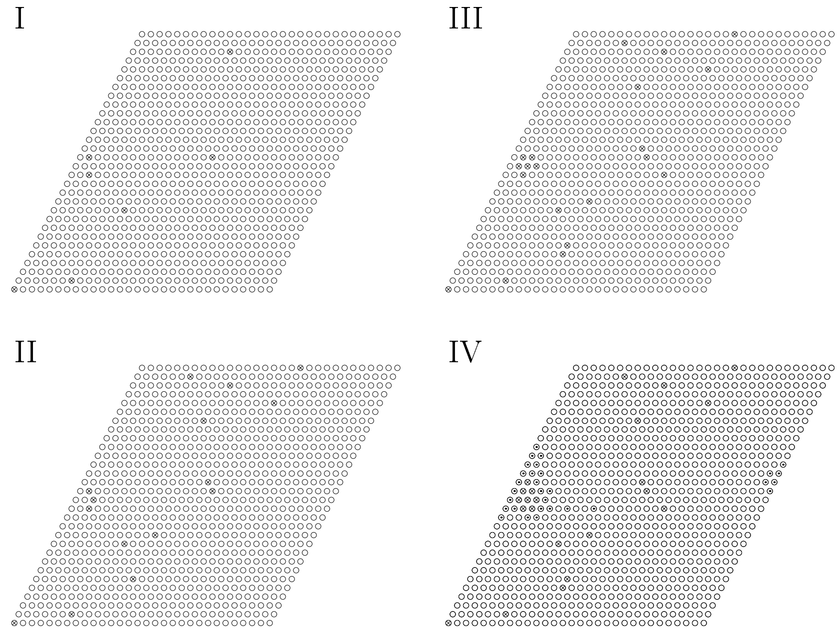

Figure 1: Snapshots of the failure process in the median (among 500 simulations) δ-bundle with 900 Weibull fibers of $\rho = 10$ under periodic boundary conditions. Open circles $\bigcirc$ denote intact fibers, $\otimes$ denote broken fibers.

Following Harlow and Phoenix [2], we have plotted the empirical weakest-link distributions

$$\hat{W}_n(\sigma) = 1 - [1 - \hat{G}_n(\sigma))]^{1/n} \tag{3}$$

on Weibull paper, in Figure 2 obtained from our Monte-Carlo simulations of failure of δ-bundles under HVLLS. Here $\hat{G}_n$ denotes the empirical strength distribution of the n-fiber δ-bundle. For $\rho \geq 2$ the $\hat{W}_n(\sigma)$ curves for $n = 225$, 625 and 900 collapse onto one characteristic curve $\hat{W}(\sigma)$. This n-independent collapse however fails to hold for $\rho = 1$. The collapse of $\hat{W}_n$ into a single curve for $\rho \geq 2$ suggests that the cluster growth failure mode is active for ρ range in the composite sizes that were simulated.

We model the cascade event defining $W(\sigma)$ as the formation of a break cluster at stress σ that goes unstable. The non-dimensional effective diameter D of a tight circular cluster of r breaks was defined earlier as $\pi D^2/4 = r$. The circumference of the circle, $\pi D = \sqrt{4\pi r}$ is approximately the number of intact fibers surrounding this r-cluster. Let N_r be the number of these neighbors that are severely overloaded. The first step is the failure of a given fiber in the δ-bundle under σ, followed by the failure of one of its $N_1 = 6$ equally overloaded neighbors under stress $K_1\sigma$. The resulting pair of fiber breaks has eight intact neighbors of which only $N_2 = 2$ are severely overloaded under stress $K_2\sigma$. The next likely event is the failure of one of these, to form a break triplet with $N_3 = 3$ severely overloaded neighbors, of which one fails, and so on. The critical event is thus the evolution of a growing "tight" r-cluster, with each added break being the failure of one of the N_r severely overloaded fibers surrounding it. We write this as

$$\begin{aligned} W_n(\sigma) \approx\ & F(\sigma)\{1 - [1 - F(K_1\sigma)]^{N_1}\} \\ & \times \{1 - [1 - F(K_2\sigma)]^{N_2}\} \cdots \{1 - [1 - F(K_{n-1}\sigma)]^{N_{n-1}}\}, \end{aligned} \tag{4}$$

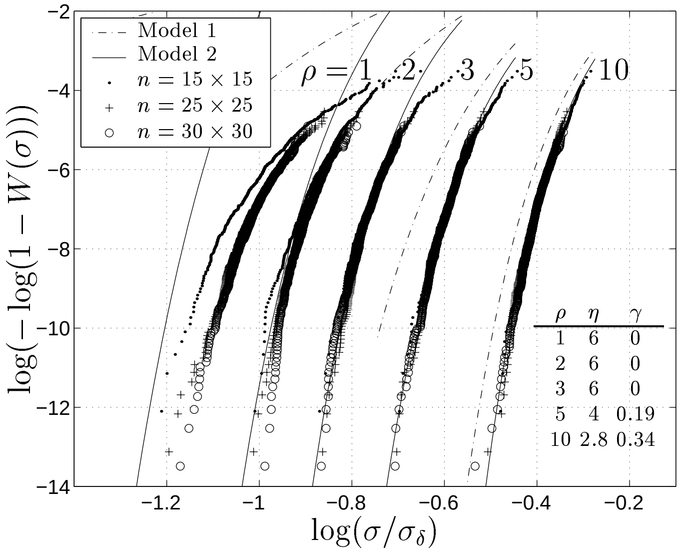

Figure 2: Comparison of the weakest link distribution predicted by the 2D cluster growth model with the empirical weak link distributions $\hat{W}(x)$ obtained from Monte-Carlo simulations. Model 1 assumes $\eta = \sqrt{4\pi}$ and $\gamma = 0.5$ for all ρ. In Model 2, we adjust the parameters η and γ so as to get the best fit with the simulated data. These values of η and γ are listed in the bottom right corner. Weakest link distributions corresponding to $\rho = 0.5$ are not shown because both Model 1 and Model 2 predict distributions that are out of the range of this plot. Also omitted is the Model 1 line for $\rho = 1$ which also lies outside the range of this plot.

where K_r is the stress concentration on the N_r most severely overloaded neighbors of a tight r-cluster and is given by Eqn. 2. We introduce two parameters, η and γ, to account for the actual number of fibers at high risk of failure. Let

$$N_r = \eta r^{\gamma} \tag{5}$$

be the number of severely overloaded neighbors around an r-cluster, where $\eta > 0$ and $0 < \gamma \leq 1/2$. We find this structure for N_r to be essential in order that $W_n(\sigma)$ in Eqn. 4 agree with the form of the simulated $\hat{W}(\sigma)$ distribution especially for small ρ. Taking $\eta = \sqrt{4\pi} \approx 3.55$ and $\gamma = 1/2$, implies counting all the fibers on the cluster periphery to be at risk of failure. Model 1 lines in Figure 2 correspond to this case and do not fit the simulated $\hat{W}(\sigma)$ very well. However model 2 lines in which we vary η and γ as functions of ρ fit the simulated $\hat{W}(\sigma)$ much better. A closed form approximation for $W(\sigma)$ is derived in [5].

2. Large Variability in Fiber Strength (small ρ)

When ρ is small corresponding to large variability in fiber strength, the cluster-driven breakdown mechanism is dominated by a dispersed, strength-driven breakdown mechanism of the δ-bundle. This is clearly seen in the failure snapshots of a δ-bundle for $\rho = 1$, as shown in Figure 3.

In the case of dispersed fiber failure in a δ-bundle, the details of the fiber load-sharing model may not be important provided that the model conserves load. Thus we consider behavior under the equal load-sharing rule or ELS. ELS assumes that the stress concentration factor for each intact fiber in an n-fiber δ-bundle with j broken fibers is $\kappa_{n,j} = n/(n-j)$. Applying a result due to Smith [3] which sharpens one due to Daniels [4] to an ELS bundle of Weibull fibers we find that the bundle strength distribution $G_n(\sigma)$ converges as $n \to \infty$, to the normal form $\Phi((\sigma - \mu_n^*)/s_n^*)$

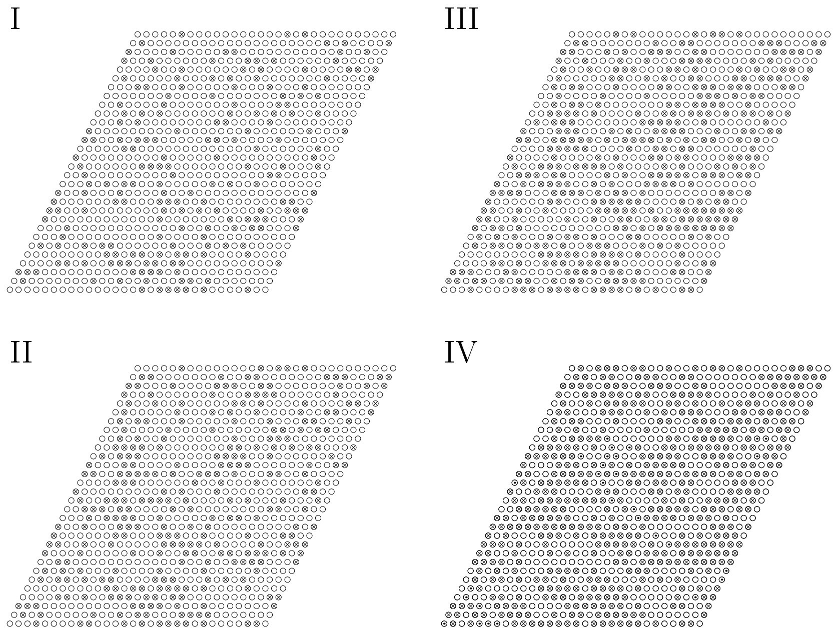

Figure 3: Snapshots of the failure process in the median (among 500 simulations) δ-bundle specimen with 900 Weibull fibers of $\rho = 1$ under periodic boundary conditions. Open circles $\bigcirc$ denote intact fibers, $\bigotimes$ denote broken fibers.

with asymptotic mean

$$\mu_n^* = \sigma_\delta (\rho e)^{-1/\rho} \left\{ 1 + 0.996 n^{-2/3} \left(e^{2/\rho}/\rho \right)^{1/3} \right\} \tag{6}$$

and asymptotic standard deviation

$$s_n^* = \sigma_\delta n^{-1/2} \rho^{-1/\rho} \sqrt{e^{-1/\rho}(1 - e^{-1/\rho})}. \tag{7}$$

2a. Global/Local Model

Unlike in ELS, wherein material damage accrues globally, we speculate that in HVLLS there is a ρ-dependent size scale within which damage accumulates in a dispersed manner and propagates catastrophically from there. That is, failure initiates over $\tilde{m} = n/\tilde{n}$ bundles of $\tilde{n}$ fibers in an ELS-like manner within a localized region and propagates catastrophically from there resulting in composite strength distribution

$$G_n(\sigma) = 1 - \{1 - \Phi[(\sigma - \mu_{\tilde{n}}^*)/s_{\tilde{n}}^*)]\}^{\tilde{m}}, \tag{8}$$

In Figure 4, for highly variable fibers with $\rho = 1$, 2, 3, and 5 we have plotted the strength distribution of the smallest sized δ-bundle $(n_1 \times n_1)$ to which weak-linked distributions of larger bundles collapse. This smallest δ-bundle size approximately corresponds to the critical cluster size defined previously. We also show the distributions of larger bundles of size $(n_2 \times n_2)$ or $(n_3 \times n_3)$ weak-linked to the size $(n_1 \times n_1)$. Note that as ρ decreases, these weak-linked distributions become increasingly Gaussian (indicated by the straightness of the strength distribution on normal coordinates) and are better approximated by the ELS asymptotic distribution. Despite the excellent agreement of the 900-fiber, weak-linked strength distribution with the 625-fiber,

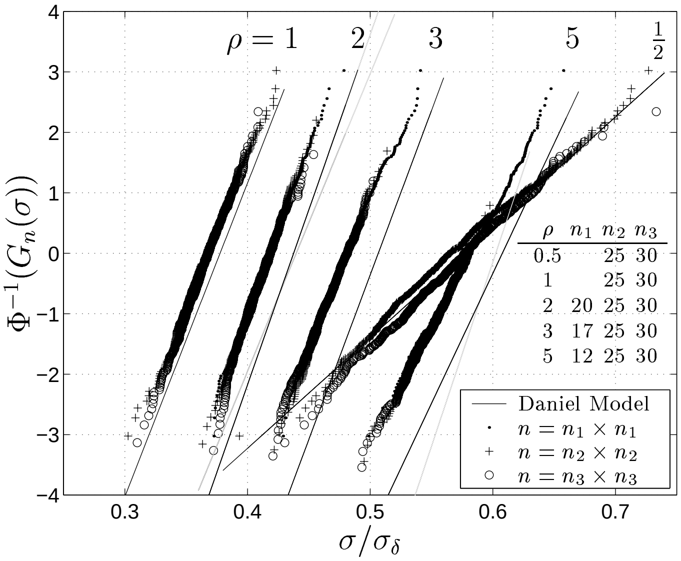

Figure 4: Comparison of $\Phi((\sigma - \mu_n^*)/s_n^*)$ given by Daniel's asymptotic formula for ELS bundles with simulated strength $G_n(\sigma)$ of an $n = n_1 \times n_1$ HVLLS δ-bundle. Also shown are empirical strength distributions of larger δ-bundles weak linked to the size $n_1 \times n_1$ in agreement with those of the $n_1 \times n_1$ δ-bundle.

weak-linked distribution when $\rho = 0.5$, it turns out that they do not agree with a 2500-fiber, weak-linked δ-bundle strength distribution. This suggests that the smallest catastrophic failure event of the bundle occurs over more than 625 or perhaps even 900 fibers.

CONCLUSIONS

In Eqn. 4, we give the weakest-link characteristic distribution function $W(\sigma)$ for δ-bundles. These bundles are links in the chain-of-bundles model for the failure of 3D unidirectional composites. For sufficiently large Weibull modulus ρ, say $\rho > 4$ in 3D composites, the strength distribution of a composite of length $L = m\delta$ and with n fibers is $H_{m,n}(x) \approx 1 - (1 - W(x))^{mn}$. For $\rho \leq 4$, however, we observe that the details of the load-sharing become increasingly unimportant, and the δ-bundle strength distribution for fixed n is not only increasingly Gaussian up to quite large n but also converges to that for ELS whose analytical form is known. For fixed ρ, however, this Gaussian nature is expected to persist only up to a δ-bundle size of the order of the critical cluster size. For composites beyond this critical size the distribution function for δ-bundle strength is that for a chain of Gaussian 'patches' of $\tilde{n}$ fibers in the δ-bundle. Thus the composite can be viewed as a weakest-link arrangement of $m\tilde{n}$ such Gaussian patches.

REFERENCES

[1] Hedgepeth, J.M and Van Dyke, P. (1967), *J. Comp. Mater.*, **1**, 294.

[2] Harlow, D.G. and Phoenix, S.L. (1978), *J. Comp. Mater.*, **12**, 314

[3] Smith, R.L. (1982), *Ann. Probab.*, **10**, 137.

[4] Daniels, H.E. (1945), *Proc. Royal Soc. London A* , **183**, 405.

[5] Mahesh, S., Phoenix, S. L., and Beyerlein I.J. (2001) In preparation.