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EFFECT OF BRIDGING LIGAMENTS UPON CRACK KINKING IN GRADED INTERFACES

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ABSTRACT

A graded interface involves a spatially changing composition gradient at an interface which removes the stress singularity which normally occurs at bimaterial interfaces. A number of analytical models have shown that, when a crack initiates perpendicular to the direction of a composition gradient, the cracks will kink as it propagates. This is a significant issue in the design of functionally graded materials. These existing analytical models show that the nature of kinking is controlled by the size and profile of the graded region and the elastic moduli of the two constituents. In this present work a model ceramic/polymer interface is constructed and the nature of fracture within this interface investigated experimentally. The sample developed ensures mixed-mode loading of the crack while ensuring no material composition gradient in the crack-tip field. Additionally, the choice of polyester and alumina means that there are negligible thermal residual stresses in the sample. It is found that the extensive crack bridging by the polymer phase occurs and that crack kinking is significantly less than that predicted by using existing analytical models which are based upon the elastic moduli of the ceramic and polymer. A numerical model is developed and demonstrates that it is the observed crack bridging which hinders crack kinking. An analytical model is then presented which confirms this hypothesis and also elucidates the influence of interface microstructure upon crack kinking in graded interfaces.

KEYWORDS

composite, graded, interface, kinking, fracture, crack, bridging

INTRODUCTION

Functionally graded materials often consist of a gradual spatial change from a ductile to a brittle material, e.g. a metal to ceramic. Often this results in the same type of structure as occurs in a ductile phase reinforced brittle material except, that in this case, there is a spatial change in the volume fraction of reinforcing phase. Because the 'matrix' and 'reinforcing' phases have different failure strains, the ductile phase leads to crack bridging and subsequent crack growth resistance or R-curve behaviour results.

A significant number of modelling studies have shown the effects of stiffness gradients upon strain energy release rate, G, in bodies containing a stiffness gradient for cracks which initiate both perpendicular and parallel to the gradient direction [1]. Crack deflection has also been considered where cracks initiate perpendicular to the gradient direction [2] and also observed experimentally, even in shallow stiffness gradients [3]. The majority of these studies however ignore two important aspects of fracture: (i) the effect of spatially changing fracture resistance, R, and (ii) the effect of an R-curve, which is an inevitable feature of many of the composite materials used in FGMs.

It has been shown that crack growth resistance behaviour may have a significant effect upon G for a crack which propagates parallel to the gradient direction from a brittle to a ductile composition or reverse [4, 5]. In this case no crack deflection occurs.

When a crack initiates perpendicular to the gradient direction both mode I and II stress intensity factors, K_I and K_{II}, result in a material containing a stiffness gradient. This leads to crack deflection. In many cases, it would appear advantageous for the crack to propagate towards the tougher material where, depending upon component shape, it may halt prior to ultimate failure. Crack bridging, however may cause significant reductions in mode I crack-tip stress intensity factors, and presumably has an effect upon mode II ones also. It is the purpose of this work to assist in elucidating the effects of R-curve behaviour, resulting from ductile phase bridges, upon crack deflection of a crack which initiates perpendicular to the gradient.

EXPERIMENTAL WORK

An experiment was devised which modelled the mixed-mode crack-tip loading in a gradient material. A piece of polyurethane foam was placed at one end of a mould into which an alumina slip was poured, penetrating the foam. This was then dried and sintered, during which time the foam pyrolised leaving a section at one end of the ceramic with interconnecting porosity. This was then placed into another mould and polyester resin poured in, penetrating the porous section of the ceramic. The sample was then machined into a single-edge tensile sample with a sharp-tipped notch placed perpendicular to the loading direction as shown in Fig. 1. The sample was ~40 mm wide by ~10 mm thick with the composite 'interface' region being ~10 mm high. Notches were placed at varying positions across the gradient with a notation of 0 closest the polymer and 1closest the ceramic, as shown in Figure 2. The composite region was measured to contain ~45vol% ceramic and was ~11% porous due to incomplete penetration of the polyester resin.

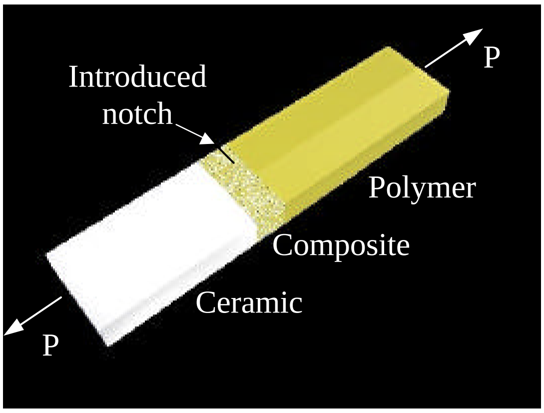

Figure 1: Schematic representation of single-edged tensile sample

The sample was then loaded to fracture. Figure 2 shows the nature of crack propagation and deflection, observed after failure. It was found that the fracture energy/area uncracked region was in the order of 650-900 J/m^2, depending upon initial crack position, compared to 55 J/m^2 for a similar bimaterial interface. Crack deflection occurred due to mixed mode loading resulting from the stiffness gradient along the sample and across the crack plane. The crack deflection angle, θ, was invariably positive and in the direction of the more compliant polymer. As intrinsic toughness was constant across the interface, this deflection could not be due to changing intrinsic fracture resistance. The deflection angle as a function of crack propagation in the initial crack plane was determined from a digitised post-failure image and results are shown in Figure 3.

It can be seen that as the crack propagates the deflection angle decreases. The fracture surface revealed evidence of crack bridging by the polymer phase with ligaments appearing to debond from the ceramic.

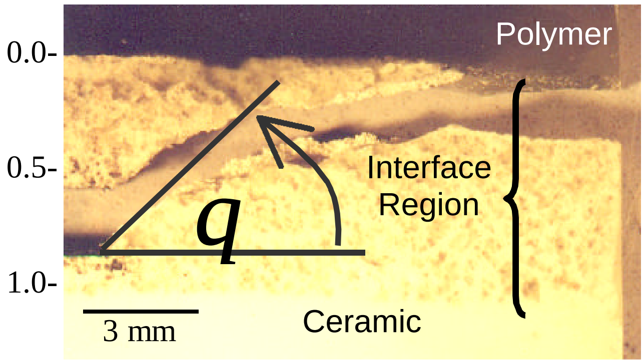

Figure 2: Fractured sample showing crack deflection, θ, with 'interface' and nomenclature used to define position of initial crack within the interface (0.0, 0.5 and 1.0 positions).

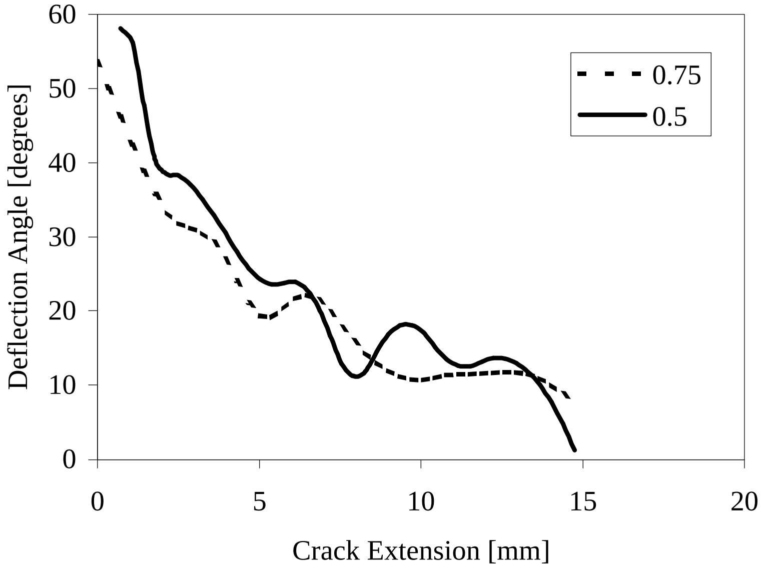

Figure 3: Experimentally determined crack deflection angle as a function of crack extension for initial cracks at the 0.75 and 0.5 position

FINITE ELEMENT MODELLING

A two-dimensional finite element model of the test piece was made using the Ansys FEM package. Plan2 elements were used in plane strain. These elements can handle the singularities associated with a crack tip. Cracks were introduced at 0.3, 0.5 and 0.7 positions with the Young's moduli of the Al_2O_3 and polyester taken to be 440 and 4.5 GPa, respectively, with Poisson's ratios of 0.4 and 0.22. An effective mean approximation was used to calculate the Young's modulus of the composite region, E_c, as 87.05 GPa and Poisson's ratio taken to be 0.3 with volume fraction of ceramic taken as 0.6. Crack bridges were introduced in the 0.5 crack position model as Beam3 elements with a diameter of 0.2 mm and initial length of 0.5 mm and with elastic properties of the polymer.

Using the co-ordinate system shown in Fig. 4, K_I and K_{II} for the crack were calculated from displacements of the crack face nodes in the directions perpendicular and in-plane to the crack direction, v and u, by solving:

$$v = \frac{K_I\left(1-\nu^2\right)}{E_c}\sqrt{\frac{8x}{\pi}} \tag{1}$$

$$u = \frac{K_{II}\left(1-\nu^2\right)}{E_c}\sqrt{\frac{8x}{\pi}} \tag{2}$$

where x is the distance from the crack tip. These were then used to determine the hoop stress around the crack tip:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2r}}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right]\cos\frac{\theta}{2} \tag{3}$$

The angle of crack deflection, θ, was then determined by calculating the value of θ for which $\sigma_{\theta\theta}$ was maximum. Following this another model was constructed in which the crack was incremented ~1 mm in the direction of the calculated deflection and the process repeated.

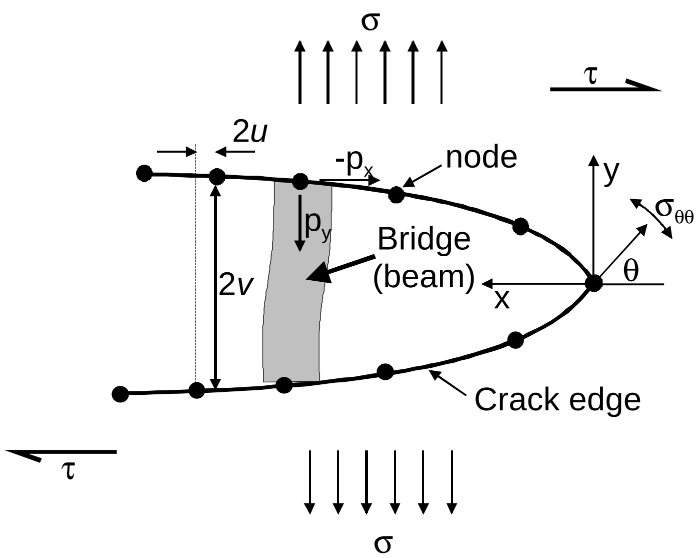

Figure 4: Model of crack showing bridging fibre and nomenclature used in calculations

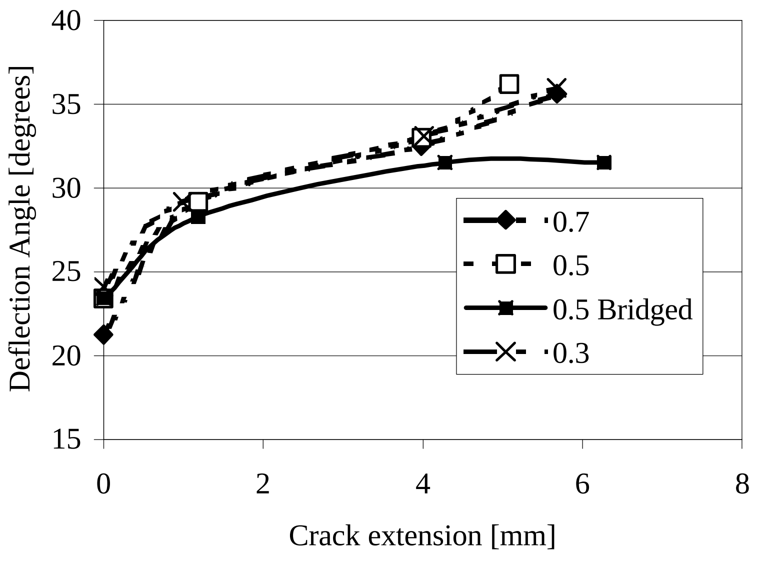

Figure 5: Crack deflection angles as a function of crack extension for bridged and unbridged cracks as calculated in FE model.

Figure 5 shows the calculated deflection angles as a function of crack extension for an unbridged crack with initial positions of 0.3, 0.5 and 0.7 and for a bridged crack with initial position of 0.5. It can be seen that the crack deflects towards the polymer as observed in the experiments. It can also be seen that for the unbridged crack models that the deflection angle *increases* as the crack propagates. This is in contrast to the experiment where the deflection angle *decreased* with crack extension. However, when crack bridges are incorporated into the model then the crack deflection is less than an unbridged crack and, as crack extension becomes large, begins to decrease. This analysis presents only a qualitative comparison with the experimental work as elastic constants and bridge geometry are dissimilar.

ANALYTICAL MODEL

In the FE model the effect of ligament diameter and stiffness and applied stress intensity were not considered. An analytical model is now developed to elucidate these effects upon crack defection for a bridged versus an unbridged crack.

A phase angle is used to describe the extent of mode mixity:

$$\psi = \tan^{-1}\left(\frac{K_{II}}{K_I}\right) \tag{4}$$

Considering Fig. 4, when a bridged crack is loaded in mixed mode then the bridge causes crack closure stresses both perpendicular, p_y, and parallel, p_x, to the crack plane. These lead to a resistance energy in both mode I and II directions according to:

$$R_{II} = 2\int_0^{u^*} p_x du \tag{5}$$

and

$$R_I = 2\int_0^{v^*} p_y dv \tag{6}$$

where u* and v* are crack face displacements at the position of the crack bridge furthest from the crack tip which was taken to be 20 times the fibre diameter in this example. This leads to crack-tip shielding, K_s, and Eq(3) becomes, for a bridged crack:

$$\sigma_{\theta\theta}^{br} = \frac{1}{\sqrt{2r}}\left[\left(K_I - K_s^I\right)\cos^2\frac{\theta}{2} - \frac{3}{2}\left(K_{II} - K_s^{II}\right)\sin\theta\right]\cos\frac{\theta}{2} \tag{7}$$

where $K_s = \sqrt{E_c R}$ separately for modes I and II.

To obtain the crack closure function, p, it is assumed that the bridging fibre acts as an elastic beam. There is assumed to be fibre delamination of length, L, equivalent to twice the fibre diameter, d, prior to loading. In Mode I the fibre acts as a beam loaded in tension while in mode II it is assumed to be fixed at the crack faces leading to a bending moment. Bending moments due to p_y in mode II are assumed to be negligible for simplification of calculations. The beam is assumed to be rectangular with width d and unit depth.

One then obtains:

$$p_y = \frac{E_f d}{L} \cdot v \tag{8}$$

and

$$p_x = \frac{E_f}{10} \cdot \left(\frac{d}{v}\right)^3 u \tag{9}$$

where v and u are defined in equations (1) & (2).

Figure 6 shows the calculated extent to which crack bridging reduces the deflection angle of a crack under mixed-mode loading. It can be seen that increasing the fibre diameter, and consequently the bridging length, and fibre stiffness reduces the deflection angle. It was also found (not shown) that, as applied stress intensity factor ($K^* = \sqrt{K_I^2 + K_{II}^2}$) is increased, the deflection angle decreases. It should be noted that this does not represent an equilibrium solution because for a bridged crack (1) and (2) are functions of $(K\text{-}K_s)$ and not just K as taken here. Nevertheless, the relative effects of bridging upon $\sigma_{\theta\theta}^{br}$ should remain correct.

DISCUSSION

Crack deflection is a result of mixed-mode loading which is inherent to a crack propagating in a material containing a stiffness gradient. Numerical and analytical calculations show that decreasing crack deflection with crack extension, which was observed in the experiment, can be explained by the effects of the observed fibre bridging.

While the introduction of a graded interface may enhance the structural integrity of a material joint, the benefits that may be obtained by a crack deflecting into a more compliant and often tougher material, may be not fully realised. This is because crack bridges, inherent to a crack propagating in a composite of two materials with different failure strains, may reduce the extent of crack deflection such that the crack propagates through a more brittle region of the gradient structure.

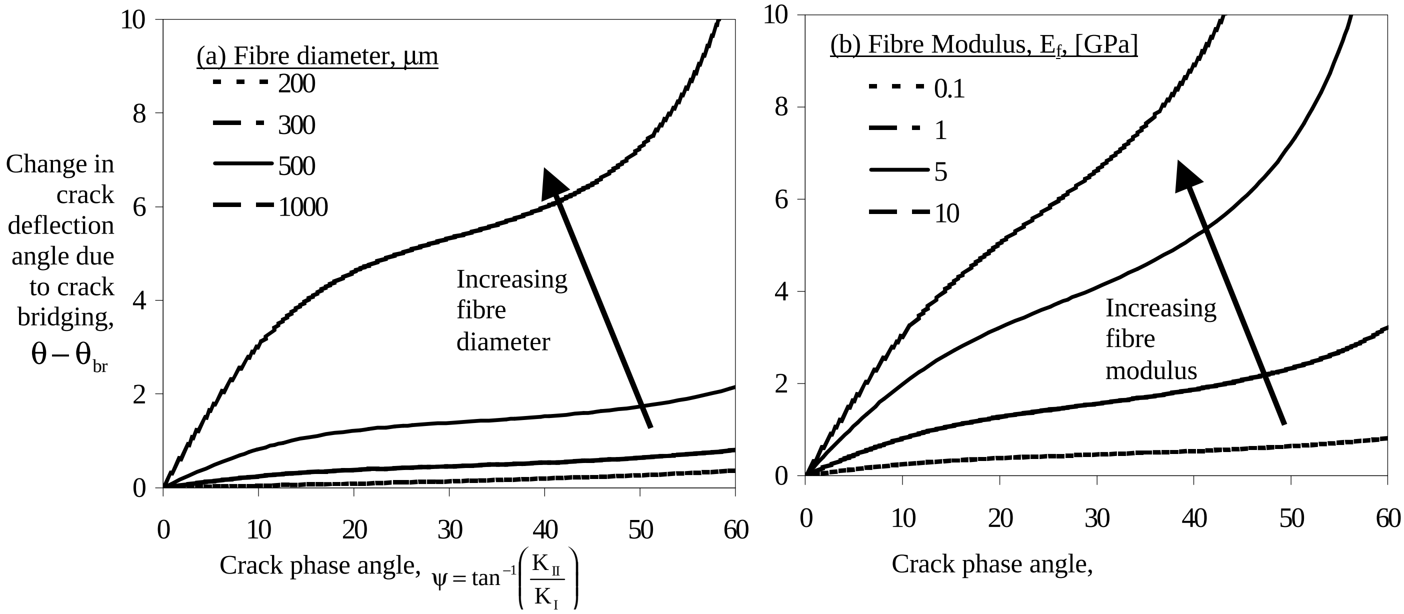

Figure 6 Effect of fibre (a) diameter and (b) modulus upon crack deflection (in degrees) as a function of phase angle (in degrees) of applied stress intensity factors.

CONCLUSIONS

From this work a number of conclusions can be made in regards to crack deflection of a crack propagating perpendicular to the direction of spatial material gradient which incorporates a stiffness gradient:

1. Crack bridging leads to a reduction in the extent of crack deflection and that
2. This is dependent upon the (a) diameter, (b) stiffness, (c) bridging length of the bridging fibres and also upon the (d) applied stress intensity factor.

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