DEVELOPMENT AND VALIDATION OF A ROBUST FRACTURE MECHANICS METHODOLOGY FOR DAMAGE TOLERANCE OF ROTORCRAFT

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ABSTRACT

The research presented in this paper is aimed at overcoming a major obstacle that currently slows effective damage tolerance assessments of rotorcraft fuselage and drive system components. Because cyclic loads are accumulated at a very high rate in rotorcraft, a substantial portion of the fatigue crack growth lifetime can be associated with cracks that do not lend themselves well to conventional fracture mechanics analyses. Accordingly, this effort was undertaken to address the small, arbitrarily shaped and warped, intrinsic, fabrication and service-induced cracks that can initiate fatigue crack growth in rotorcraft components. The resulting methodology is based on an innovative approach in which a symmetric Galerkin boundary element method (SGBEM) alternates with a finite element method (FEM). This technology is uniquely able to provide stress intensity factors (and, when appropriate, elastic-plastic crack tip criteria), to enable accurate and efficient fatigue crack growth predictions to be obtained for conditions pertinent to the full range of rotorcraft applications. This paper outlines the computational fracture mechanics analysis procedure that was developed. It also reviews the validations of the resulting methodology that were made in terms of critical comparisons with existing literature solutions for complex crack shapes. In addition, to illustrate the potential for practical applications of the methodology for rotorcraft, and for other applications where the safe operating lifetime is dictated by load cycles that are amassed at an ultra high rate, example computational results are presented for progressive fatigue crack growth from an elliptical crack initially inclined to the loading direction.

KEYWORDS

Damage tolerance, rotorcraft, fatigue crack growth, non-planar cracks, SGBEM, alternating FEM

INTRODUCTION

The damage tolerance (DT) methodology, largely spearheaded in work initiated over three decades ago by the U.S. Air Force, is now firmly based and widely applied for life determinations and for setting inspection intervals in both military and commercial aircraft. Applications of the DT methodology to a given aircraft structural component require comparable technological capabilities in four distinct areas: (1) performing nondestructive inspections to detect, or postulate, the existence of a crack-like defect, (2) anticipating the applied cyclic loads, (3) measuring the characteristic fatigue crack growth and fracture properties of the material, and (4) devising fracture mechanics analysis techniques for quantifying fatigue crack growth and the ultimate failure state. As concluded in a recent international workshop focused on rotorcraft damage tolerance (RCDT), the currently existing capabilities in all of these four areas are insufficient for RCDT applications [1]. The main reason is the combination of complex structural configurations with very rapid accumulations of cyclic loads. For example, there would be one, four and eight load cycles per blade revolution in the main rotor system that is shown in Figure 1 for which the rotational speed would be about 300 rpm.

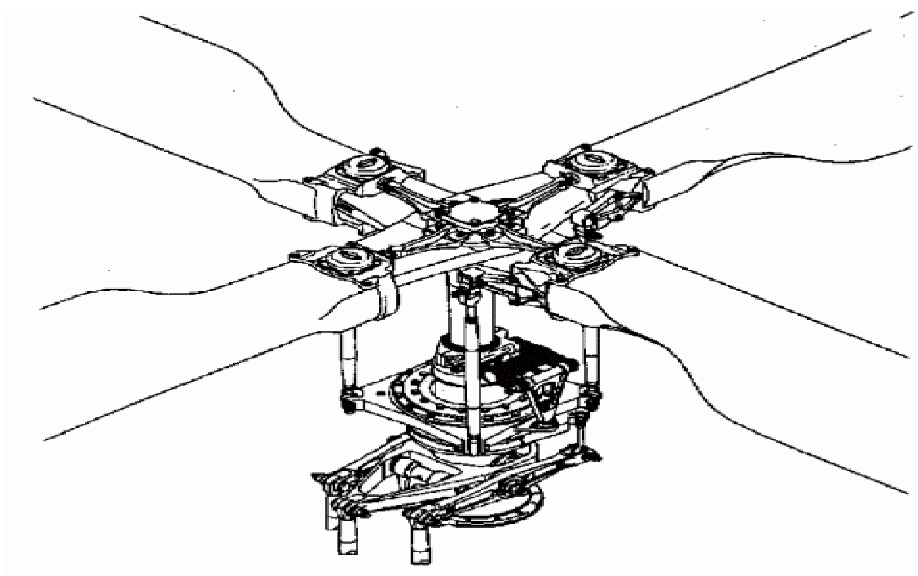

Figure 1: A typical rotorcraft drive and blade system

Because rotorcraft structural components accumulate cyclic loads at very high rates, very small arbitrarily shaped and warped cracks will often be the initiators of fatigue crack growth. Other barriers to practical RCDT applications also exist; e.g., rotorcraft structures make extensive use of surface treatments to retard fatigue crack growth, and they also are generally operated in ways that require highly variable and complex flight load spectra to be used for fatigue life predictions. Notwithstanding, with limited resources available for rotorcraft research, a simultaneous attack on all of the outstanding issues is not possible. To accelerate the practical implementation of RCDT, the Federal Aviation Administration (FAA) has focused its research on advancing the accuracy and efficiency of fracture mechanics calculations. While fuselage structure is certainly a concern for RCDT, the more challenging problem is involved with the dynamic components in the drive system; c.f., Figure 1. This paper describes an innovative fracture mechanics approach aimed at providing the basis for RCDT applications for this class of components.

ANALYSIS METHODOLOGY

An efficient and highly accurate technique utilizing a combination of the symmetric Galerkin boundary element method (SGBEM) and the finite element method (FEM) was developed for the analysis of three-dimensional non-planar cracks. This methodology addresses not only the initiation of growth, but also the subsequent unconstrained growth (i.e., as exclusively dictated by the deformation state existing at and near to the current crack tip) in structural components of non simple geometries. In this approach the crack is modeled by the SGBEM as a distribution of displacement discontinuities, as if in an infinite medium. The FEM is used to perform the stress analysis for the uncracked body only. The solution for the structural component containing the crack is obtained in an iteration procedure, which alternates between FEM solution for the uncracked body and the SGBEM solution for the crack in an infinite body. Numerical procedures, and the attendant Java code, are developed for the evaluation of crack tip parameters and fatigue crack growth modeling.

The SGBEM, originated by Bonnet et al. [2], is a way of satisfying the boundary integral equations of elasticity in a Galerkin weak form, as opposed to the method of collocations that is generally used to satisfy the integral equations in the traditional BEM. The SGBEM is characterized by weakly singular kernels. After a special transformation that removes the singularity from the kernels, the boundary element matrices can be integrated with the use of conventional Gaussian quadrature. The crack is modeled as a distribution of displacement discontinuities with the crack surface discretized by quadratic eight-node boundary elements. Quarter-point singular elements are placed near the crack front. With the use of the SGBEM/FEM alternating procedure, the crack tip parameters for planar and non-planar cracks in infinite media, and for embedded and surface cracks in finite bodies, can be calculated.

More specifically, for an infinite three-dimensional body containing a non-planar crack of arbitrary geometry, consider that a distributed load is applied at the crack surface. The crack can then be described by a distribution of displacement discontinuity for which the following weakly-singular boundary integral equation is valid for the crack; c.f., [2-4]:

$$-\int_S\int_S D_\alpha u_i^*(\mathbf{z}) C_{\alpha i\beta j}(\xi-\mathbf{z}) D_\beta u_j(\xi)\, dS(\xi)\, dS(\mathbf{z}) = \int_S u_k^*(\mathbf{z}) t_k\, dS(\mathbf{z}) \qquad (1)$$

where $S = S_+$ is one of crack surfaces; u_i are displacement discontinuities for the crack surface; u_i^* are the components of a continuous test function; and t_k are crack face tractions. Using Eqn. (1), the SGBEM models an arbitrary non-planar crack in an infinite body under external loading. The FEM solution for an uncracked finite body then enables a solution for a finite body with a crack to be obtained by superposition. While this can be done with a direct procedure, the alternating method advanced by Atluri [5] provides for a more efficient solution without the need for assembling the joint SGBEM-FEM matrix.

The basic steps of the SGBEM-FEM alternating iteration procedure are (1) using FEM, obtain the stresses at the location of the hypothetical crack in a finite uncracked body that is subjected to given boundary conditions, (2) using SGBEM, solve the problem of a crack, the faces of which are subjected to the tractions found from FEM analysis of the uncracked body, (3) determine the residual forces at locations corresponding to the outer boundaries of the finite body that result from the displacement discontinuities at the crack surface, (4) using FEM, solve a problem for a finite uncracked body under residual forces from SGBEM analysis, and (5) obtain the stresses at the location of the crack corresponding to FEM solution. Steps 2 to 5 are repeated until the residual load is sufficiently small. Usually, less than 10 iterations are enough for convergence. Then, by summing all the appropriate contributions, the total solution for a finite body with the crack is obtained. This procedure is described in detail by Nikishkov et al. [6].

Having the converged solution, the next step is to compute the crack tip parameters associated with fatigue crack growth. For simplicity, consider mode I fracture for which the SGBEM/FEM alternating procedure solution is used to evaluate:

$$K_I = \frac{E}{(1-\nu^2)} \frac{u_3}{4\sqrt{2r/\pi}} \tag{2}$$

where, as usual, K_I is the mode I stress intensity factors; E is the elastic modulus, ν is the Poisson's ratio, r is the distance from a point on the crack surface to the crack front, and u_3 is the normal component of the displacement discontinuity at that point. For modeling fatigue crack growth it is only necessary to add another element layer to the existing crack model. To advance a point at the front of a nonplanar crack it is necessary to know the direction and extent of crack growth. Cherepanov's formulation [7] of the J-integral has been found to provide the most effective criterion for fatigue crack growth according to which the crack grows in the direction of the vector $\Delta\vec{J}$ with the crack growth rate determined by the relative magnitude of ΔJ using a conventional fatigue crack growth relationship (e.g., from the NASGRO database).

The procedure for the advancement of the front of a nonplanar crack is (1) using the SGBEM-FEM alternating method, solve the problem for the current crack configuration and determine ranges for the stress intensity factors for the element corner nodes located at the crack front, (2) for each corner node determine the crack front coordinate system by averaging the coordinate axis vectors determined at the corner points of two neighboring boundary elements, (3) for each corner node, calculate the crack advance Δa and the crack growth direction, (4) move each corner node in the local crack front coordinate system and transform the movement to the global coordinate system, (5) find the locations of crack front midside nodes, using cubic spline interpolations for corner nodes from several neighboring elements, and (6) shift the quarter-point nodes of the previous crack front elements to midside positions on the element sides normal to the crack front. After terminating the crack growth procedure, the total number of cycles N is calculated as a sum of the ΔN_i's.

This algorithm has been implemented as a Java code because its numerous attractive features (e.g., object-oriented nature, simplicity, reliability and portability) despite its somewhat slower speed in comparison to C and Fortran. A comparison of finite element codes written in C and Java shows that in many cases Java provides comparable performance as the C language [8]. While the manual tuning that is required for Java requires some additional effort, the use of Java leads to an overall development time reduction in comparison to other languages because of easier programming and debugging.

VALIDATIONS OF THE METHODOLOGY

To assess the accuracy of the methodology, the Java SGBEM-FEM alternating code, displacement discontinuity finite element alternating method, (DDFEAM) was applied to the solution of complex crack shape problems taken from the open literature. For these comparisons, 8-node quadrilateral boundary elements were used for the crack surface discretization, and the Gaussian integration rule was used with three points in each of the four directions employed for computing boundary element matrices for regular and singular cases. Quarter-point singular elements were placed at the crack front. The finite element models consisted of 20-node brick-type finite elements. The following open literature solutions were examined:

- Penny-shaped crack under tensile and shear loading -- compared with exact solutions given by Sneddon [9] and by Kassir and Sih [10].
- Inclined elliptical crack under tension – compared with exact solution for 45^o inclination given by Kassir and Sih [10].
- Circular arc crack under tension – compared with exact solution by Cotterell and Rice [11].
- Spherical penny-shaped crack under internal pressure and tension – compared with numerical solution given by Xu and Ortiz [12] and by Li, Mear and Xiao [13].
- Embedded circular crack in a cylindrical bar and in a cube – compared with numerical solution given by Li, Mear and Xiao [13].
- Semi-elliptical surface cracks – compared with numerical solution given by Wu [13].
- Inclined semi-circular surface crack in a plate – compared with numerical solutions given by Shivakumar and Raju [14], and by He and Hutchinson [15].

Good to excellent agreement was obtained in all cases. A detailed description of these comparisons can be found in the paper of Nikishkov et al. [6].

APPLICATION OF THE METHODOLOGY

As a first step in testing the practicality of the methodology for a representative drive system component of a rotorcraft. The analysis that was made was for a small planar crack in a much larger body under mixed-mode loading conditions. The initiating defect was taken to be an elliptical crack inclined at 45^o to the direction of a remote applied tensile loading. The minor/major semi-axis ratio a/c was taken as 0.5. The crack was discretized by 68 quadratic boundary elements. For simplicity, a Paris Law fatigue relation model was used in which $C = 1.49 \cdot 10^{-8}$ and $m = 3.321$ to represent 7075 Aluminum. The elliptical initial crack was analyzed and the stress intensity factors K_{I}, K_{II} and K_{III} calculated for each of the nodes along the crack front. Then, in accord with the calculated J-integral vector orientation and magnitude at each individual point, the crack front was advanced to new positions via scaling to the maximum crack advance $(\Delta a)_{max}$. A new layer of elements was then generated between the old and the new crack front lines, and the process repeated. The stress intensity factors that were determined inn this process were normalized to the reference value $K_o = \sigma\sqrt{\pi a}$. The values along the crack periphery that resulted from each of six individual crack advance increments, each with $(\Delta a)_{max}/a = 0.1$, are shown in Figure 2. A three-dimensional view of the crack after all six increments of growth have been completed is presented in Figure 3.

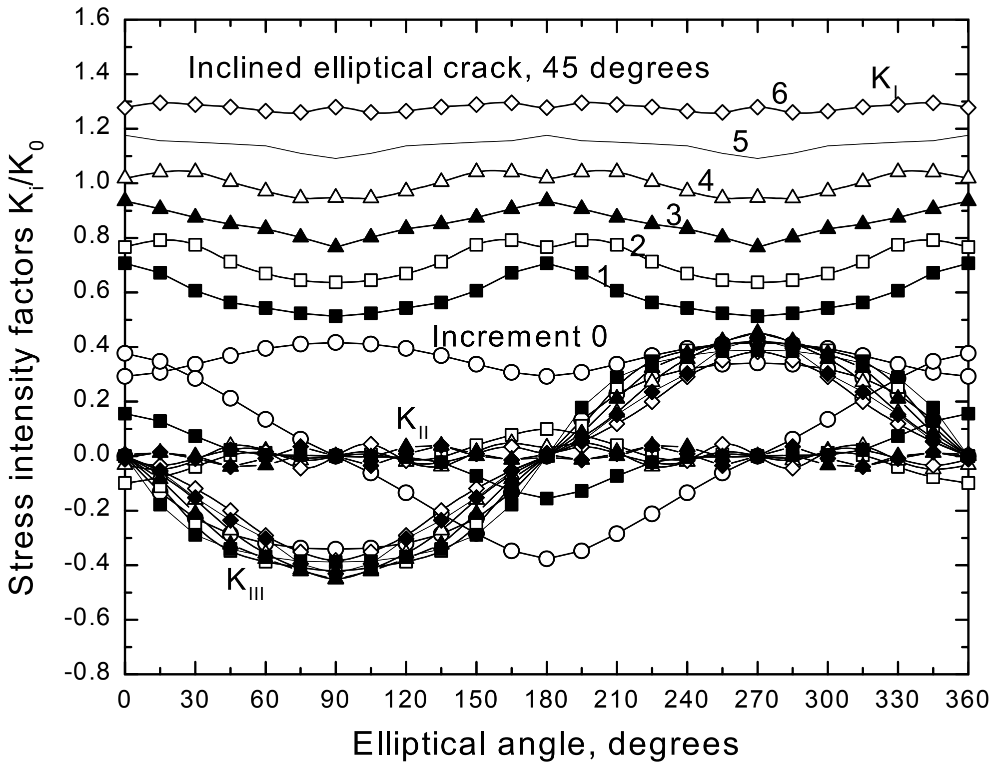

Figure 2: Calculated results for the stress intensity factors along the front of the initial crack, and along each of six subsequent crack fronts, for simulated fatigue crack growth from an initial elliptical crack oriented at 45^o to the direction of a remote tensile loading

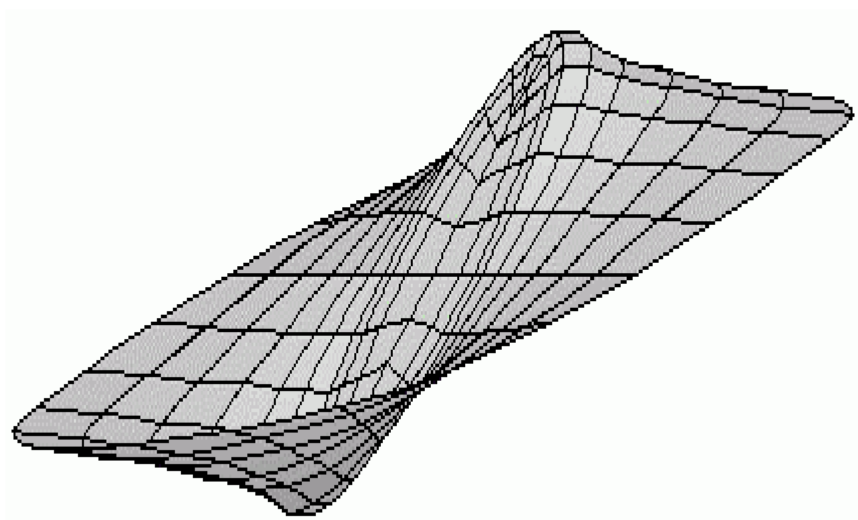

Figure 3: Three-dimensional view of the crack face after six crack growth increments

SUMMARY AND CONCLUSIONS

To help meet the demanding conditions associated with rotorcraft damage tolerance (RCDT), an SGBEM-FEM alternating method has been developed for predicting fatigue crack growth from non-planar cracks. The accuracy of the procedure was demonstrated by critical comparisons with a variety of solutions for complex cracks. To demonstrate its potential for attacking practical problems, fatigue crack growth from an inclined elliptical initial crack was calculated. While the results presented in Figures 2 and 3 are certainly in good qualitative agreement with observations of fatigue crack growth (e.g., crack growth takes place with $K_{II}=0$), it is not possible to directly assess the computed results using experimental data. However, some checks can be made by comparing with other numerical solutions. Such comparisons show that the distributions of the stress intensity factors along crack front during crack growth are similar to those obtained by Mi and Aliabadi [16], while the shape of the final crack is similar to crack shapes obtained both by them and by Forth and Keat [17]. Hence, while the progress that has been described in this paper is still at a preliminary stage, it is believed that an excellent start has been made towards overcoming the full range of the research challenges that need to be met for the implementation of a practical RCDT approach.

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