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DETERMINATION OF DIFFERENT FRACTURE MODES STRESS INTENSITY FACTOR WITH VIRTUAL CRACK EXTENSION METHOD

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ABSTRACT

There are many methods for determination of stress intensity factors K_I, K_{II}. Evaluation of stress intensity factors determination using maximum energy release rate theory and complex J integral is the main purpose of the paper. A number of numerical analyses using the Compact Tension Shear (CTS) specimen were performed for determination of stress intensity factors. Virtual extension method (VCE) in framework the finite element method was used for crack propagation analysis. Calculated crack propagation angles with VCE method were compared to experimental results and crack propagation angles calculated using a maximum tangential stress criterion. Accuracy of determination crack propagation angles using virtual crack extension method was evaluated for different fracture mode.

KEYWORDS

Mixed Mode fracture, CTS specimen, Virtual Crack Extension method, Stress Intensity Factor

INTRODUCTION

For general, cracked structures it is necessary to consider the combined effects of mode I, II and III loading in linear elastic fracture investigations. In fact, mode III is largely separable and can be dealt with in an independent manner, but the combined effect of modes I and II, under tensile and shear loading, presents difficulties in analysis. Several mixed-mode fracture criteria exist, and they can be generally divided into two groups, depending on their scopes. Some criteria are concerned only with the local information at or around the crack tip (local approach) whereas others consider the global or total information about the whole body containing the crack (global approach). In the local approach, one needs to choose a parameter (or physical quantity) that measures the severity experienced by the local material particles at or around the crack tip. Widely used parameters include the maximum principal stress, the maximum circumferential stress ($\sigma_{\theta max}$) and the minimum strain energy density (S_{min}). The local approach appears to be based on a choice of the parameter through intuition. In contrast, the global approaches are based on the total potential (strain) energy of the system. The fundamental physical quantity in the global approach is the strain energy release rate G, which is the sole fracture parameter that governs the behaviour of the crack. G represents the strain energy that is lost by the system through unit surface extension of the crack. Richards [1] showed the most accurate criterion for crack propagation on CTS specimen is MTS criterion.

VIRTUAL CRACK EXTENSION METHOD (VCE)

The Virtual Crack Extension method, originally proposed by Hellen [2], is based on the criteria of released strain energy dV per crack extension da

$$G = -\frac{\mathrm{d}V}{\mathrm{d}a} \tag{1}$$

which serves as a basis for determination of the combined stress intensity factor around the crack tip

$$K = \begin{cases} \sqrt{E \cdot G} & \text{plane stress;} \\ \sqrt{\dfrac{E \cdot G}{(1-\nu^2)}} & \text{plane strain.} \end{cases} \tag{2}$$

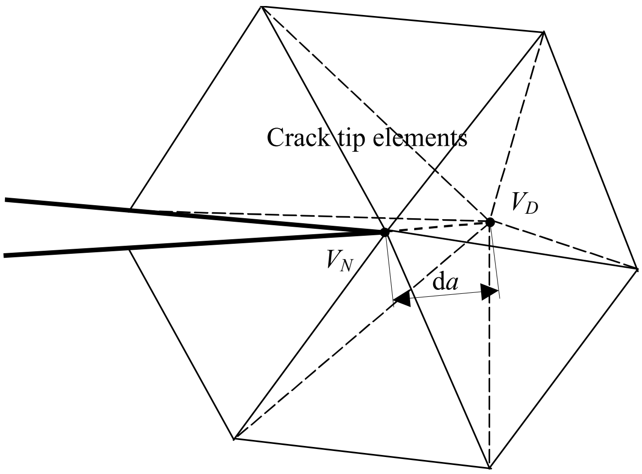

Figure 1: Initial and extended crack tip configuration

If V_C is the strain energy obtained for all degrees of freedom not present in the crack tip elements, and V_N is the energy in the crack tip elements when the tip is not extended, while V_D is the energy in these elements when the tip is extended, Figure 1, then the total energies of the initial and altered bodies, V_N^T and V_D^T, respectively, are equal to

$$V_N^T = V_C + V_N \quad \text{and} \quad V_D^T = V_C + V_D \tag{3}$$

Thus for a virtual crack extension δa it follows

$$\frac{\mathrm{d}V}{\mathrm{d}a} = \frac{V_D^T - V_N^T}{\delta a} = \frac{V_D - V_N}{\delta a} \tag{4}$$

which is clearly independent of V_C. It follows that only strain energies V_N and V_D in the crack tip elements need to be calculated for every possible crack extension. This results in a very efficient method for determination of the instantaneous energy release rate and thus the stress intensity factor for any given crack extension. Following the same argument, the energy release rate G and the stress intensity factor K can be easily determined for several different possible crack extension directions for a cluster of points on an arc around the initial crack tip with radius da, see Figure 2a

$$\left(\frac{\mathrm{d}V}{\mathrm{d}a}\right)^j = \frac{V_D^j - V_N}{\mathrm{d}a^j}. \tag{5}$$

Assuming the validity of the maximum energy release criterion, the crack will propagate in direction corresponding to the maximum value of $(\mathrm{d}V/\mathrm{d}a)^j$, *i.e.* in the direction of the maximum stress intensity factor

K^j. Computational procedure is based on incremental crack extensions, where the size of the crack increment is prescribed in advance. The virtual crack increment should not exceed 1/3 of the size of crack tip finite elements. For each crack extension increment the stress intensity factor is determined in several different possible crack propagation directions and the crack is actually extended in the direction of the maximum stress intensity factor, which requires local remeshing around the new crack tip. The incremental procedure is repeated until full fracture occurs or until the stress intensity factor reaches the critical value K_c, when full fracture is imminent. For improved numerical results, special fracture finite elements are used in the first circle of elements around a crack tip, with ordinary elements elsewhere, Figure 2b. In these special fracture finite elements, the displacements are proportional to the square root of the distance from the tip. Since the tip stresses are singular, they are not calculated at the crack tip node.

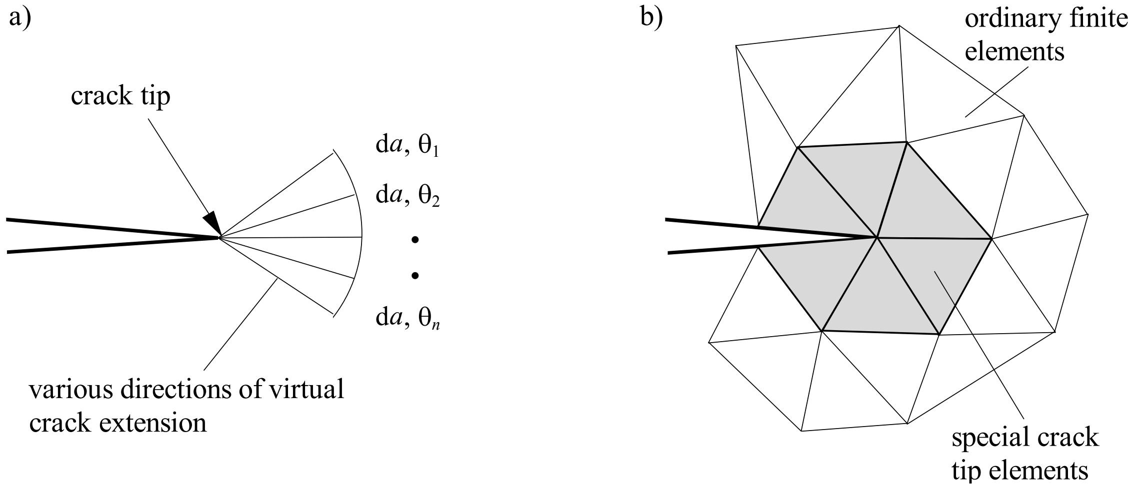

Figure 2: Virtual crack extensions of the crack tip

Following the above procedure, one can numerically determine the functional relationship $K=f(a)$ and the critical crack length a_c at $K=K_c$ from the computed values of K at discrete crack extensions a.

DETERMINATION OF CRACK PROPAGATION

Maximum Energy Release Rate using the Complex J Integral (MERRJ)

The maximum energy release rate criterion is based on the assumption that the energy release rate may be expressed as a function of the J_1 and J_2 integrals [4]. This theory is of particular practical interest since it compliments the finite element VCE method for mixed mode situations. Since J is equivalent to G for the linear elastic case, the values of stress intensity factors are

$$K_I^2 = \frac{2E}{(1+\nu)(1+\kappa)}\left(J_1 + \sqrt{J_1^2 - J_2^2}\right) \text{ and } K_{II}^2 = \frac{2E}{(1+\nu)(1+\kappa)}\left(J_1 - \sqrt{J_1^2 - J_2^2}\right) \quad (6)$$

where J_1, J_2 are energy release rate for crack extensions parallel and perpendicular to the crack, $\kappa = 3 - 4\nu$ for plain strain and $\kappa = (3-\nu)/(1+\nu)$ for plain stress. The maximum energy release rate is for a crack extending at the angle θ_0:

$$\theta_0 = \arctan\left(\frac{2K_I K_{II}}{K_I^2 + K_{II}^2}\right) \quad \text{or} \quad \theta_0 = \arctan\left(\frac{J_2}{J_1}\right) \quad (7)$$

to the plane of the crack and has magnitude

$$G(\theta_0) = \frac{(1+\nu)(1+\kappa)}{4E}\sqrt{K_I^4 + 6K_I^2 K_{II}^2 + K_{II}^4} \quad (8)$$

The Maximum Tangential Stress criterion (MTS)

Erdogan and Sih [3] used the stress equations for determination of direction of crack propagation. The crack propagates in direction of maximum tangential stresses calculated on a circle of sufficiently small radius around the crack tip. Angle of crack propagation θ_0 is determined with:

$$\tan\frac{\theta_0}{2} = \frac{-2K_{II}}{K_I + \sqrt{(K_I)^2 + 8(K_{II})^2}} \tag{9}$$

For opening-mode loading $(K_I \neq 0, K_{II} = 0)$, equation (9) yield $\theta_0 = 0$, while for sliding-mode loading $(K_I = 0, K_{II} \neq 0)$, it results in $\theta_0 = -70{,}6^\circ$.

NUMERICAL ANALYSES

Different crack propagation methods were evaluated for the CTS specimen shown on Figure 3.

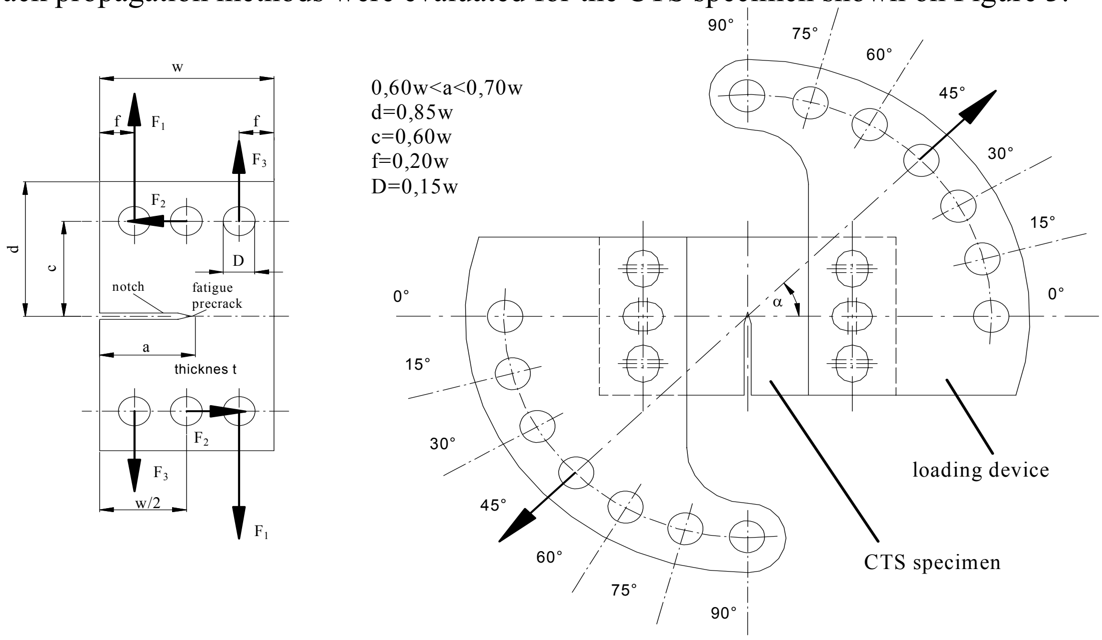

Figure 3: CTS specimen with loading device

A 2,5 mm fatigue pre-crack is on the end of the 52,5 mm long notch. The CTS specimen is loaded with a static load of 15 kN. In computational analysis this load is replaced with three equivalent nodal forces in x-y direction as shown on Figure 3. Different load cases for load angles between 0° and 90°, with a step of 15°, were used to simulate different fracture mode conditions. Pure Mode I condition was simulated with load angle of 0° while pure Mode II was simulated with load angle of 90°. The mixed mode conditions are simulated using load angles between 15° and 75°.

RESULTS

Figure 4 shows distribution of strain energy release rate G around the crack tip. The curve is seen to be sinusoidal, showing clearly that the directions of maximum G and minimum G are opposite. There are two directions of no energy release. Between them the energy release rate is negative, therefore crack extension is physically impossible in these directions. The value of G depends primary on K_I, resulting in highest value of G at pure Mode I, while G has the lowest value at pure Mode II.

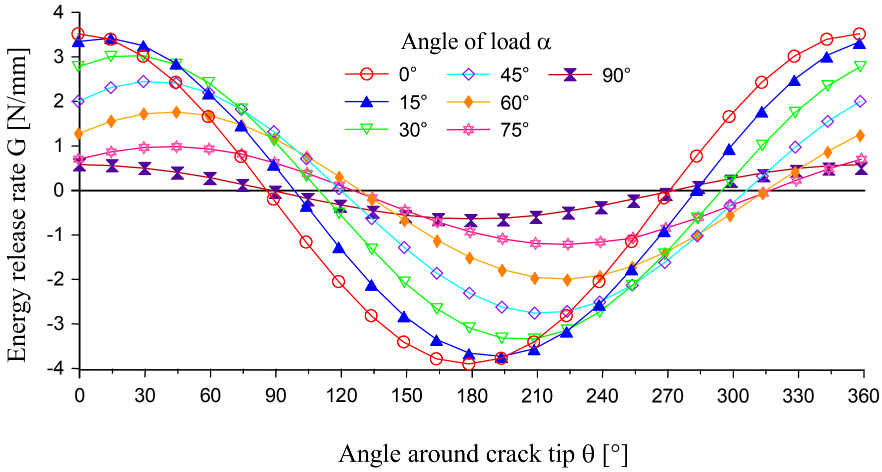

Figure 4: Plot of G against angle θ

The stress intensity factors K_I, K_{II}, shown in Table 1, were determined from Eqn. 6 using VCE method for determination of J_1 and J_2. At the start of crack propagation a kink in crack path is observed under mixed mode loading. The results in Table 1 are therefore given for a loaded initial crack configuration. In experimental testing [6] it has been observed that the crack propagation angle is θ_0 = 24° for load angle α = 30°, θ_0 = 46,2° for α = 60° and θ_0 = 52° ± 2° for α = 75°.

TABLE 1: STRESS INTENSITY FACTORS K_I, K_{II}

	0°	15°	30°	45°	60°	75°	90°
K_I	549,85	531,29	476,67	389,79	279,05	138,32	0,09
K_{II}	0,19	56,49	109,24	153,73	186,20	217,28	222,58

The results of computational analyses show a reasonable agreement between VCE method and MERRJ criterion for crack propagation is shown on Figure 4:

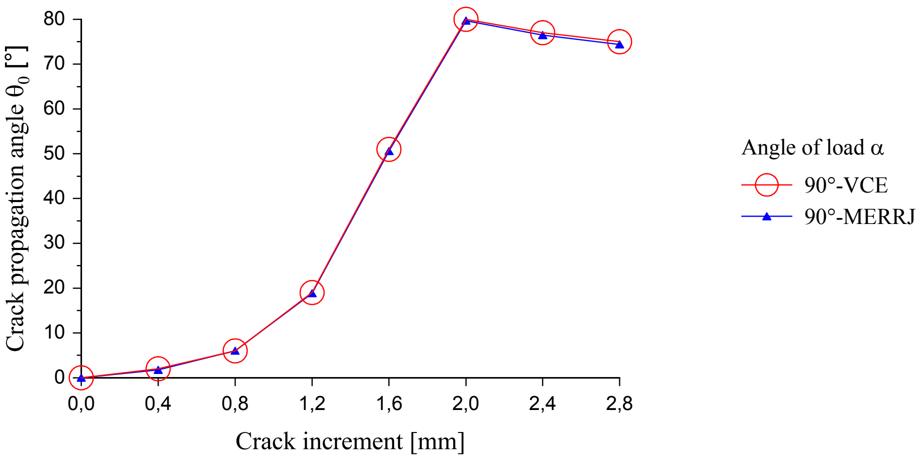

Figure 4: Plot of θ_0 against crack increment

Crack propagation angles calculated using VCE method are shown on Figure 5 while crack propagation angles calculated using MTS criterion are shown on Figure 6. Comparison between experimental crack propagation angle θ_0 and calculated crack propagation angle θ_0 shows that the MERRJ criterion is less accurate when $K_{II}>K_I$ as crack does not kink immediately. Crack propagates to the experimental value after a few increments.

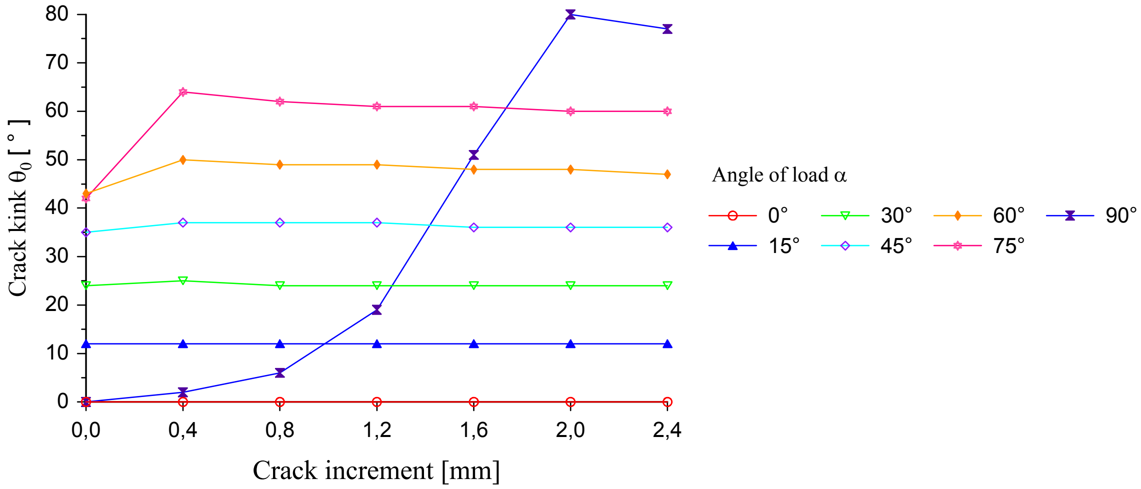

Figure 6: Crack propagation angle for MERRJ criterion

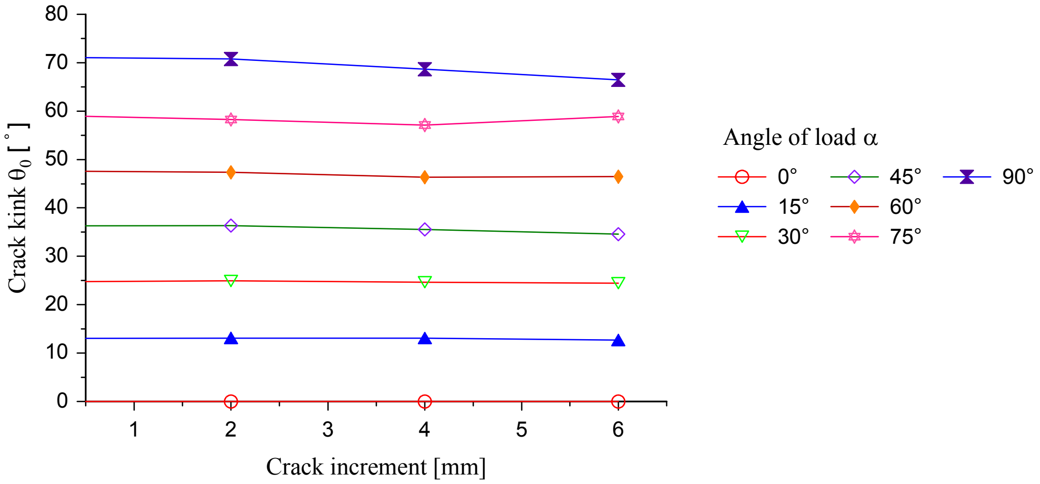

Figure 7: Crack propagation angle for MTS criterion

CONCLUSIONS

There are several criterions for determination of stress intensity factor $K_{I,}$ K_{II} and crack propagation angle. Determination of stress intensity factor $K_{I,}$ K_{II} using VCE method was evaluated. It can be observed that for cases where K_{II} is dominant the VCE method is less accurate. Therefore special care should be considered using this method for determination of crack propagation angle.

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