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DAMAGE EVOLUTION AND FRACTURE OF VISCOELASTIC COMPOSITES UNDER TIME-VARYING LOADS

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1. Introduction and Problem formulation

Symmetric $[0_n, 90_m]_s$ cross-ply laminate with transverse cracks in 90-layers is shown in Fig. 1. Layers in the (x,y,z) -system are homogeneous, orthotropic and linearly viscoelastic with constitutive relations given by,

$$\sigma^i(t) = \int_0^t E^i(t-\tau)\frac{d\,\varepsilon^i}{d\tau}d\tau \qquad (1)$$

where the superscript i =0, 90 designate the layer and ε^i, σ^i and E^i denote the strain, stress and stiffness tensors, respectively. In general, stresses and strains are functions of the position ξ characterized by dimensionless coordinates x/d and z/d. In the following, stress, strain and stiffness symbols without the superscript stand for averages over the entire laminate. Lower index, if given, specifies the component under consideration. For simplicity residual thermal stresses are not included in analysis. It is also assumed that all stresses arising during the formation of cracks have relaxed and the laminate before the displacement application is stress free.

Plane stress formulation is used and the only applied loading is time dependent displacement in x-direction, see Fig.1, $u_x(t) = \varepsilon_x(t)\, l_0$.

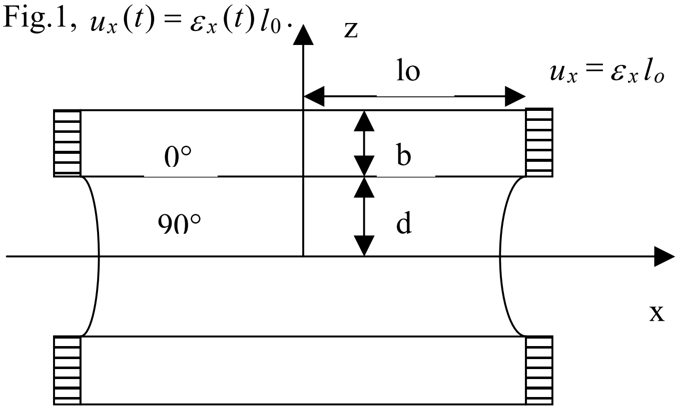


Fig. 1 Schematic showing the cross ply-laminate with cracks in the 90-layers.

For the assumed constant spacing of cracks the solution must satisfy:

1. Stress equilibrium equations

$$\frac{\partial \sigma^i_{kl}}{\partial x_l} = 0 \qquad (2)$$

2. Strain-displacement relationships

$$\varepsilon_{kl}^{i} = \frac{1}{2}\left(\frac{\partial u_k^i}{\partial x_l} + \frac{\partial u_l^i}{\partial x_k}\right) \tag{3}$$

3. Boundary conditions.

Based on symmetry considerations only a quarter of the full repeating unit (Fig. 1) is used. Symmetry conditions are on sides $x = -l_0, z \in [d,h]$ and $x \in [-l_0, 0], z = 0$. Traction free-conditions are on $z = h = d+b$ and on the crack surface $x = -l_0, z \in [0,d]$, applied constant displacement in x-direction, on $x = 0, z \in [0,h]$.

4. All stresses and strains at $t \leq 0$ are zero.

2. Theoretical analysis

The expressions needed for calculation of stress-strain dependence in an arbitrary point for a general loading ramp are derived using Laplace transform technique and applying the linear viscoelastic correspondence principle. This principle states that the solution of a given viscoelastic problem in Laplace domain may be obtained using the solution of the corresponding elastic problem. The only modification is that instead of elastic constants the Carson transforms of the corresponding relaxation functions are used. We denote by $\overline{f}(\xi, s)$ the Laplace transform of an arbitrary stress-strain state characteristic $f(\xi, t)$ and the Carson transform of the set of relaxation functions by $\widetilde{E}(s)$. Here s is Laplace parameter. Due to linearity $\overline{f}(\xi, s)$ is proportional to the applied average strain $\overline{\varepsilon_x}$:

$$\overline{f}(\xi, s) = \Psi\left(\xi, \widetilde{E}^k(s)\right)\overline{\varepsilon_x}(s) \tag{4}$$

Considering relaxation test with unit applied strain, $\overline{\varepsilon_x} = \frac{1}{s}$ and denoting all time dependent functions with index R we have,

$$\overline{f}_R(\xi, s) = \Psi\left(\xi, \widetilde{E}^k(s)\right)\frac{1}{s} \tag{5}$$

The inversion to time domain in this case becomes trivial if we realize that in relaxation test all stress-strain state characteristics are monotonous functions of time with a small curvature in log$t$ scale and, hence, satisfy Schapery's conditions for simple transformation [1]:

$$f_R(t) = \left(s\overline{f}_R(s)\right)_{s=0.56/t} \tag{6}$$

leading to

$$f_R(\xi, t) = \Psi\left(\xi, E^k(t)\right) \tag{7}$$

Function Ψ is given by the linear-elastic solution using values of elastic constants equal to relaxation moduli in the particular instant t.

The expression for the general strain ramp, Eq. (4), may now be rewritten in terms of functions in relaxation test:

$$\overline{f}(\xi, s) = \Psi\left(\xi, \widetilde{E}^k(s)\right)\frac{1}{s}s\overline{\varepsilon_x}(s) = \overline{f}_R(s)s\overline{\varepsilon_x}(s) \tag{8}$$

Inverse transformation to time domain leads to

$$f(\xi, t) = \int_0^t f_R(\xi, t-\tau)\frac{d\,\varepsilon_x}{d\tau}d\tau \tag{9}$$

Stress state characteristics for laminates with cracks subjected to given strain ramp are

1. Macro-response (laminate stress) of the laminate

$$\sigma_x(t) = \int_0^t \sigma_{xR}(t-\tau)\frac{d\,\varepsilon_x}{d\tau}d\tau \tag{10}$$

2. Stress in defined points ξ in layers where fracture may occur:

$$\sigma_x^i(\xi,t)=\int_0^t \sigma_{xR}^i(\xi,t-\tau)\frac{d\,\varepsilon_x}{d\tau}d\tau \qquad (11)$$

3. Average crack opening displacement normalized by the 90-layer half-thickness *d*: (Since the presented calculations are for open cracks, this parameter is needed to check the validity of the method.)

$$u_{an}(t)=u_a(t)/d \qquad u_a=\frac{1}{d}\int_0^d u_x(z)dz \qquad u_{an}(t)=\int_0^t u_{anR}(t-\tau)\frac{d\,\varepsilon_x}{d\tau}d\tau \qquad (12)$$

From Eqs (10)-(12) the response of a damaged laminate to an arbitrary strain ramp can be easily calculated if the corresponding functions are known for relaxation test. The procedure to determine the characteristics in relaxation test follows. According to Eq. (7) functions from relaxation test may be obtained by solving a sequence of elastic problems corresponding to varying parameter t. It may be done using FE method or by developing approximate analytical models. In this paper the macro-response of the damaged laminate (relaxation of laminate stress and average crack opening) is calculated using closed form expressions for elastic case [2,3].

Average crack opening displacement may be calculated using the simple power law which, based on FE parametric analysis, was obtained in [3] as

$$u_{anR}(t)=0.01\frac{E_x(t)}{E_{x0}(t)}u_n(t) \quad \text{with} \qquad u_n(t)=A+B\left(\frac{E_x^0(t)}{E_x^{90}(t)}\right)^{-n} \qquad (13)$$

Here $A=0.955$, $B=0.9102+0.3413\dfrac{d-b}{b}$ and $n=-0.0169\left(\dfrac{d}{b}\right)^2-0.0041\dfrac{d}{b}+0.9074$

The $\dfrac{E_x(t)}{E_{x0}(t)}$ for normalized crack density $\bar{\rho}=d/2l_0$ may be calculated using the following [2],

$$\frac{E_x(t)}{E_{x0}(t)}=\frac{1}{1+2\dfrac{E_x^{90}(t)d}{E_x^{90}(t)d+E_x^0(t)b}u_n(t)\bar{\rho}} \qquad (14)$$

Laminate stress: $\sigma_{xR}(t)=0.01E_x(t)$ (15)

Stress in location ξ at a given instant of time t is obtained solving the elastic problem by FEM, using elastic constants equal to the relaxation moduli at this instant of time.

3. **Numerical example**

We consider three-step loading as shown in Fig.2. In all steps the strain rate is constant, ε_0/t_1, and strain is first linearly increasing until $\varepsilon_0=1\%$, then decreasing to $\varepsilon_2=0.05\%$ and finally increasing again. Results are presented for CF/EP [0/90$_2$]s laminate.

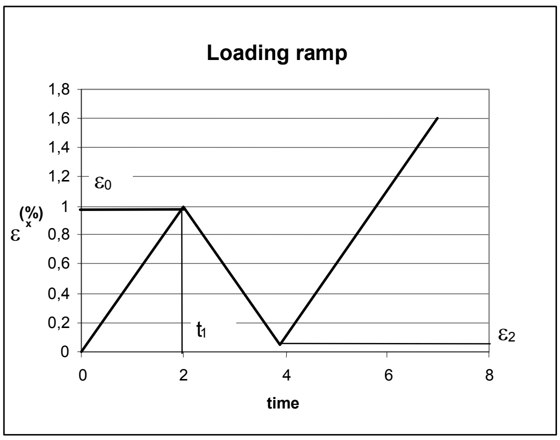


Fig.2 Three-step loading ramp with the same ramp rate in all steps applied to laminates.

Relaxation moduli of unidirectional lamina are shown in Fig 3. In relaxation test with the applied strain $\varepsilon_x = 1\%$, the laminate stress relaxation follows Eqs (15). For a given crack density we use Eq.(14) for relaxation modulus. This expression includes the crack opening displacement $u_n(t)$ which is calculated using the power law Eq.(13). The calculated laminate stress and COD relaxation curves are presented in Fig.4. Fifth order polynomials are used to fit the calculated data. These fitting polynomials are used in Eqs. (10)-(12) for modeling stress response in the loading case shown in Fig.2.

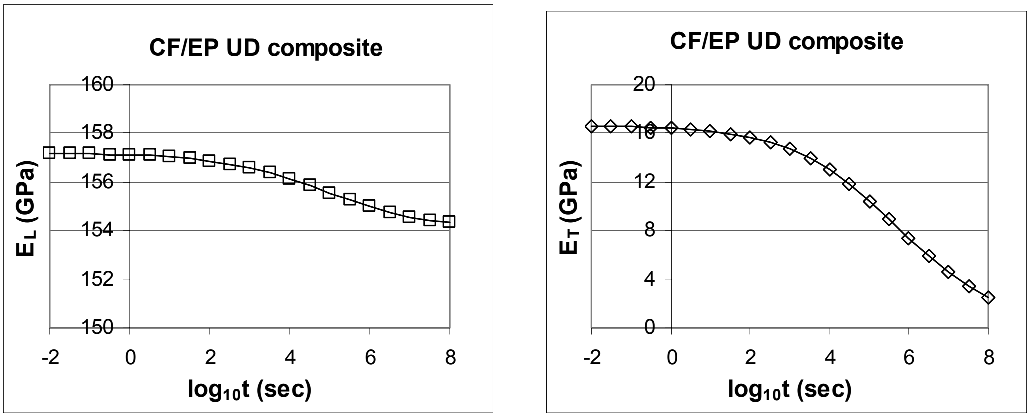
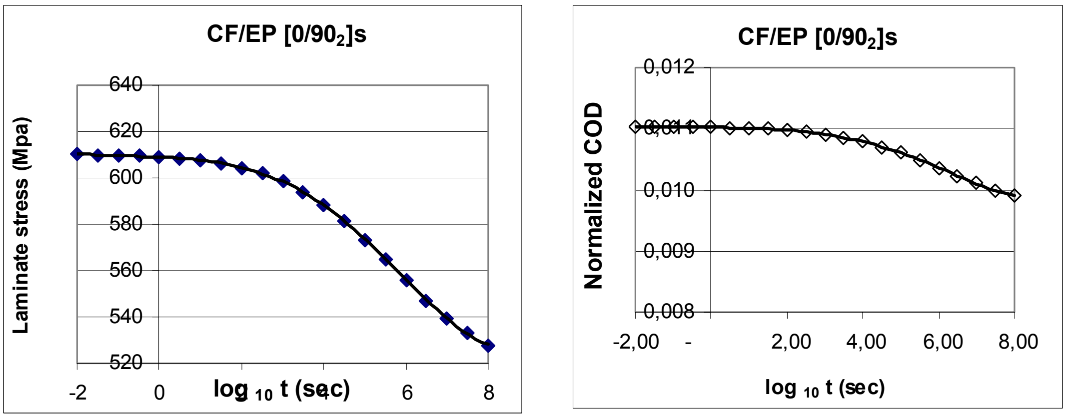


Fig.3 Relaxation functions of the unidirectional CF/EP composite



a) b)

Fig.4 Macro-response of [0/90-2]s CF/EP laminate in relaxation test. Applied strain $\varepsilon_x = 1\%$ *. Laminate with transverse crack spacing* $l_0/d = 5$*. a)* $\sigma_{xR}(t)$ *; b)* $u_{anR}(t)$

As a critical site for further damage development we consider a) x=0, z=0 where the next transverse crack is expected, and b) the fibers in the 0-layer which are located at the tip of the transverse crack for possible fiber fracture. Stress relaxation at these points is shown in Fig.5. The polynomial fit to all time dependent functions in relaxation test is used to calculate the response to strain ramp shown in Fig.2. Expressions (10)-(12) are used.

Before the response of the damaged laminate was simulated, the crack opening was inspected to insure that cracks remained open at all times. If due to hysteresis cracks would close, the used analysis becomes invalid. The calculated macro-response of the damaged laminate is shown in Fig.6. Note, that in case of fast loading (t_l=10) the loading, unloading and reloading curves almost coincide. For the low strain rate (t_l=2e7), the unloading curve is below the loading curve and following reloading leads to slope which is higher than the initial loading slope, thus building a hysteresis loop.

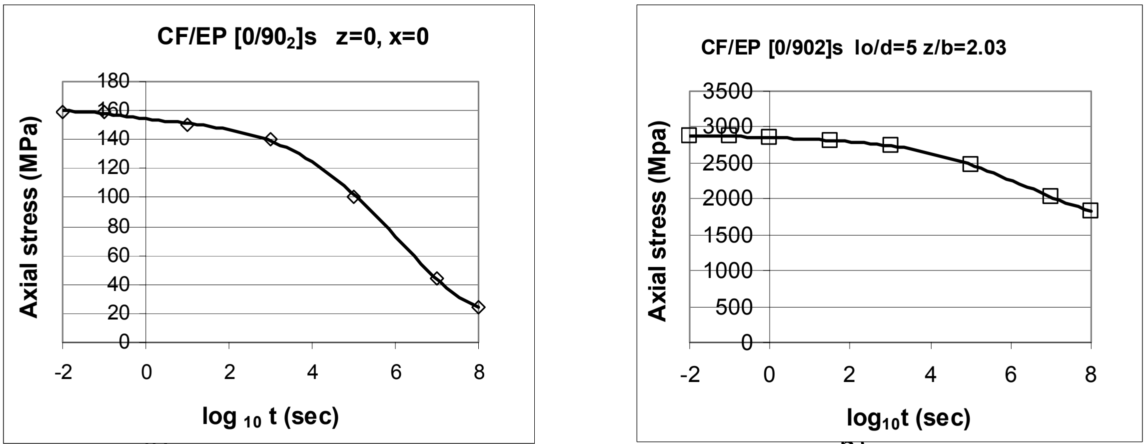
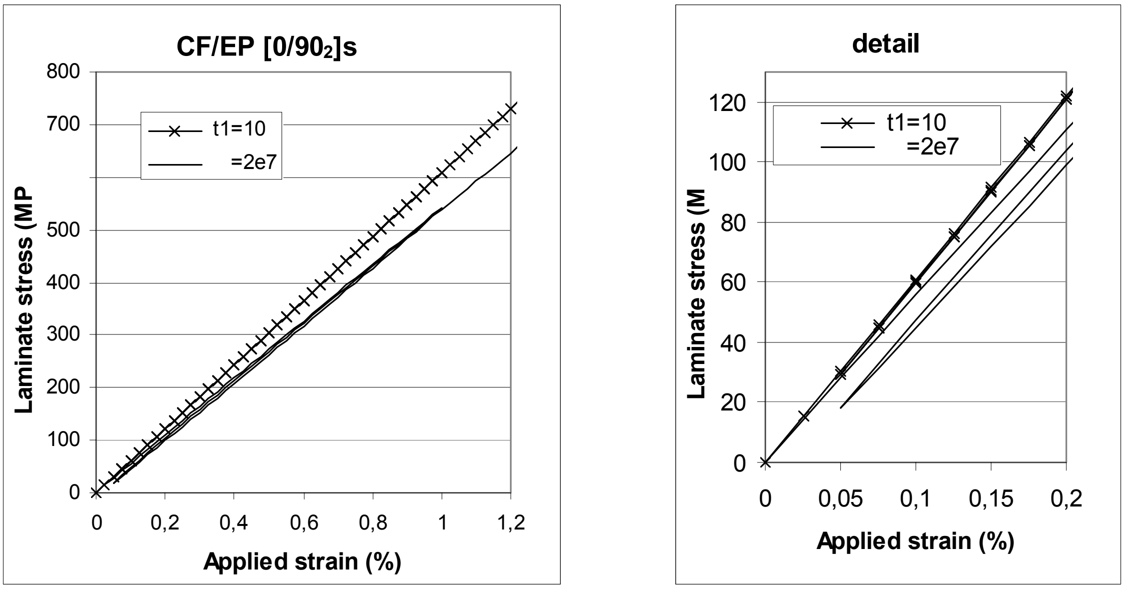
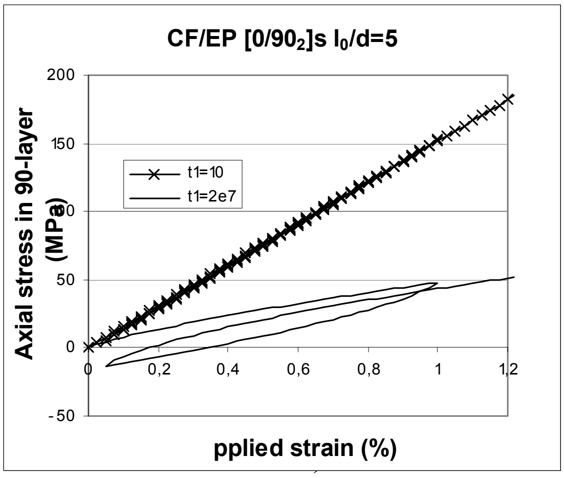


Fig.5 Axial stress $\sigma_x^i(t)$ in CF/EP [0/90$_2$]s laminate. Relaxation test at applied strain $\varepsilon_x = 1\%$. Crack spacing $l_0/d = 5$. a) $\sigma_x^{90}(t)$ in 90-layer at x=0, z=0; b) $\sigma_x^0(t)$ in 0-layer (average over the first closest the crack fiber).



a) b)

Fig. 6 Stress-strain curve of [0/90$_2$]s CF/EP laminate with crack spacing $l_0/d = 5$) obtained using the loading ramp shown in Fig.2. Strain rates t1=10 and t1=2e7 are used. a) the whole loading-unloading-reloading curve; b) detail at small strains.



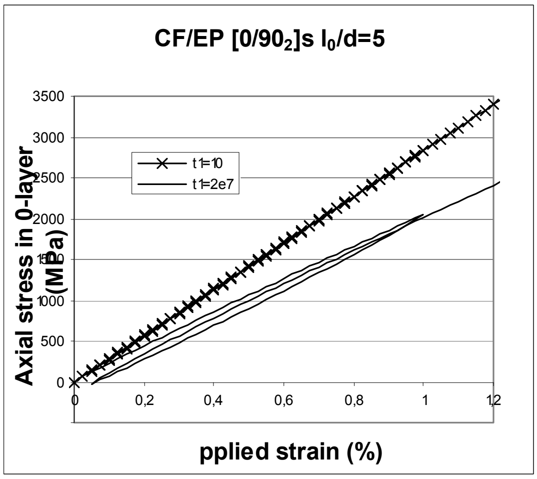


Fig. 7 a)Stress σ_x^{90} in the middle of 90-layer (x=, z=0) ; b) Average stress σ_x^{0} in the closest fiber to the crack tip (x=0, z/b=2.03) in 0-layer.

Fig.7a shows the x-axis stress response at mid-distance between two preexisting cracks due to the applied strain ramp. The hysteresis loop is remarkably large leading to compressive stresses (remember that the applied strain is positive at all times). The fact that stress at the considered point is compressive, while cracks are still open, is remarkable. Loading rate has a huge effect on the obtained stress level. Finally, Fig 7b shows the stresses in the 0-layer. The position considered is at the tip of the transverse crack. This stress, which is the average over the closest fiber diameter (layer thickness), gives an indication of possible fiber fracture due to stress concentration at the crack tip. Stresses are very high, approaching the fiber strength. They are much higher than predicted by commonly used analytical stress models. Slower loading allows for stress relaxation and stresses are lower.

4. Conclusions

The stress response of a damaged laminate to an arbitrary applied strain ramp may be predicted by simple integration, provided the corresponding stress response in relaxation test is known.

The time dependence of stresses in relaxation test is obtained using Schapery's inversion method for Laplace transforms. To accomplish this, a sequence of elastic problems must be solved.

5. References

1. R. A. Schapery, *Approximate methods of transform inversion for viscoelastic stress analysis"*, Proceedings of the 4th U.S. National Congress of Applied Mechanics, ASME, 18-21 June, 1962, Berkeley, CA, pp. 1075-1086.
2. R. Joffe and J. Varna*, "Analytical modeling of stiffness reduction in symmetric and balanced laminates due to cracks in 90° layers"*, Composites Science and Technology, 59, 1999,1641-1652.
3. Varna J., Joffe R. and Krasnikovs A., " *COD based simulation of transverse cracking and stiffness reduction in [S,90n]s laminates*," Composites Science and Technology, 2001, in press.