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DAMAGE ACCUMULATION AND FATIGUE CRACK GROWTH CHARACTERISTICS UNDER MIXED-MODE LOADING CONDITION

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ABSTRACT

Using thin-walled cylindrical specimens subjected cyclic tension and torsion, fatigue crack growth tests under mixed-mode loading conditions were conducted. The characteristics of the fatigue crack growth rate (FCGR) were identified. By comparing the experimental results with the results of FE analyses, it was found that the characteristics of the FCGR were related to fatigue damage accumulation before initial crack growth. Based on these results, fatigue life prediction under mixed-mode loading was proposed. Damage analyses is indispensable for characterization of the fatigue crack growth and fatigue life prediction.

KEYWORDS

mixed-mode loading, fatigue crack growth rate, plastic deformation, finite element analyses, fatigue damage, life prediction

INTRODUCTION

Many studies have been conducted on fatigue crack behavior under mixed-mode loading conditions [1-9]. Among several experimental methods, a method in which combined tension and torsion are applied to a thin-walled cylindrical specimen has the advantage to achieve mixed-mode loading ranging pure mode-I to mode-II [5-9]. Using this method, fatigue crack growth tests were conducted and the characteristics of crack growth behavior were identified. Moreover, in order to discuss the effect of fatigue damage on mixed-mode crack growth, elastic-plastic FE analyses were carried out. Fatigue life prediction was also discussed.

CRACK GROWTH TESTS UNDER MIXED-MODE LOADING

The experiments on fatigue crack growth behavior under mixed-mode loading conditions were conducted [5-9]. Tensile and torsional stresses in the same phase were applied to thin–walled hollow cylindrical specimens of 5083P-O aluminum alloy. Using this method, mixed-mode fatigue load ranging from mode-I to mode-II can be applied to an initial crack, which was introduced in the circumferential direction of the specimen. Details of the experimental procedure are described elsewhere [5-7].

After fatigue crack initiation, the crack propagates in the direction perpendicular to the maximum principal stress, σ_1 (called ϕ_1-direction), as shown in Figure 1 [5-9]. The characteristics of fatigue crack growth rate (FCGR), db/dN, in region II were found to be divided into two regions, that is, region IIa followed by region

IIb, as shown in Figure 2. In each region, the FCGR can be expressed in the form of Paris law :

$$\frac{db}{dN} = A\Delta K_{Ib}{}^{m} \tag{1}$$

where, ΔK_{Ib} ($= \Delta\sigma_1\sqrt{\pi b}$, $\Delta\sigma_1$: maximum principal stress range) is the stress intensity range corresponding to the crack length, b, projected on the line of the ϕ_1-direction. In each region, m and A of Eqn. 1 are constants depending on loading condition. The value of m shown in Figure 3 can be expressed as:

$$\text{Region IIa} \qquad m = \begin{cases} 3.05 + 6.89(\sigma_2/\sigma_1) & (0.0 \le |\sigma_2/\sigma_1| \le 0.15) \\ 1.66 - 2.10(\sigma_2/\sigma_1) & (0.15 \le |\sigma_2/\sigma_1| \le 1.0) \end{cases} \tag{2a}$$

$$\text{Region IIb} \qquad m = \begin{cases} 3.11 - 7.31(\sigma_2/\sigma_1) & (0.0 \le |\sigma_2/\sigma_1| \le 0.47) \\ 9.34 + 5.58(\sigma_2/\sigma_1) & (0.47 \le |\sigma_2/\sigma_1| \le 1.0) \end{cases} \tag{2b}$$

where, σ_2/σ_1 is the ratio of the minimum and maximum principal stresses, which is a function of mixed-mode condition, $\Delta K_{II0}/\Delta K_{I0}$. It should be noted that the values of m are independent of stress ratio, R ($=\sigma_{min}/\sigma_{max}$).

The effect of the stress ratio, R on the FCGR is significant under mixed-mode loading condition, as shown in Figure 2. It was found that the FCGR in region IIa is accelerated by the static tensile stress component $\sigma_{st.}$($=\sigma_{min.}$) of mixed-mode fatigue loading, while the FCGR in region IIb is not affected by the R-ratio [9] . By considering these effects, the fatigue crack growth law can be expressed as the following equation [9]:

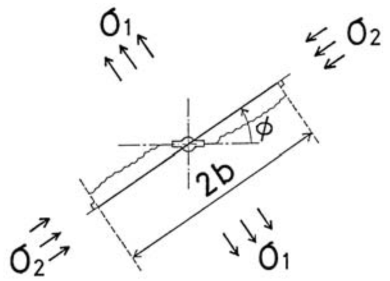


Figure 1: Crack length, b, projected to the line perpendicular to the σ_1 .

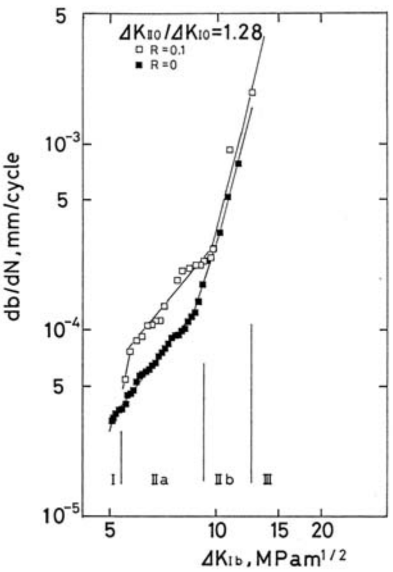


Figure 2: Characteristics of the db/dN under mixed-mode conditions.

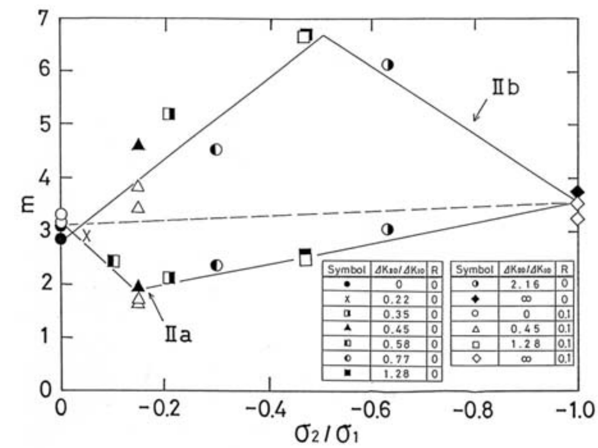


Figure 3: The effect of mixed-mode loading condition on the power coefficient, m, in Paris law in regions of IIa and IIb.

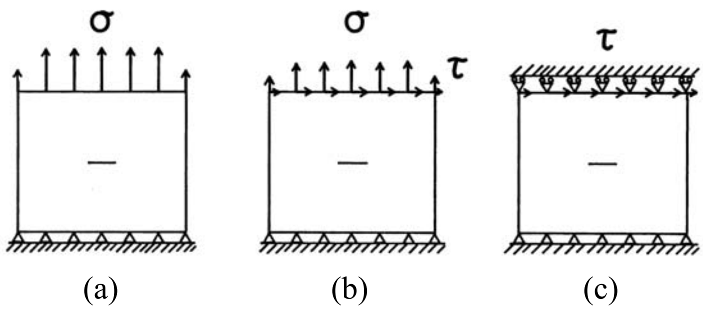


Figure 4: Model for mode I, mixed-mode and mode II conditions for the elastic-plastic FE analyses : (a) $\Delta\tau/\Delta\sigma = 0$ (b) $\Delta\tau/\Delta\sigma = 0.45, 1.28$ (c) $\Delta\tau/\Delta\sigma = \infty$.

$$\frac{db}{dN} = \left(A^{R=0}\right)^{1/r^*} \Delta K_{Ib}{}^{m} \tag{3}$$

$$r^* = 1 + \frac{1}{7}(\sigma_{st.}/\Delta\tau)^{1/2} \qquad \text{(Region IIa)} \tag{4a}$$

$$r^* = 1 \qquad \text{(Region IIb)} \tag{4b}$$

where, $A^{R=0}$ is the value of A for R=0. In Eqn.3, r^* is a parameter which represents the effect of the R-ratio. The relationship between constants $A^{R=0}$ and m is given by:

$$\text{Region IIa} \qquad A^{R=0} = 2.16 \times 10^{-5} \times 0.230^{m} \tag{5a}$$

$$\text{Region IIb} \qquad A^{R=0} = 3.64 \times 10^{-6} \times 0.147^{m} \tag{5b}$$

By substituting Eqns. 2, 4 and 5 into Eqn. 3, the fatigue crack growth rate under mixed-mode loading conditions can be formulated as a function of ΔK_{Ib}, σ_2/σ_1 and $\sigma_{st.}/\Delta\tau$. In the crack growth law, constant m represents the effect of mixed-mode condition, $\Delta K_{II0}/\Delta K_{I0}$, while r^* represents the effect of the R-ratio.

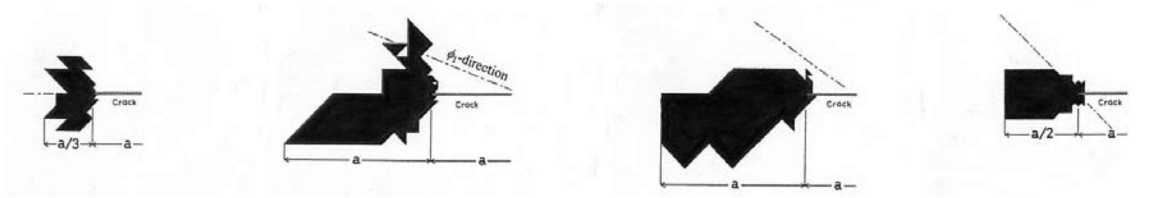
EFFECT OF DAMAGE DUE TO PLASTIC DEFORMATION

The dominant mechanical factor of the FCGR under mixed-mode loading is now investigated. Mixed-mode crack growth has different characteristics from mode-I crack growth in which the direction of the maximum tangential stress, $\sigma_\theta^{\max}$ in the vicinity of the crack is the same as the direction of σ_1. Therefore, mode-I fatigue crack propagates in the same direction as it initiates, and the FCGR accelerates monotonically even though the effect of the global damage contributes it. On the contrary, under mixed-mode loading, as the direction of $\sigma_\theta^{\max}$ is not the same as the direction of σ_1, the crack growth direction shifts as the crack propagates [5-9]. Therefore, the crack growth behavior changes during its propagation. This transition of the crack growth is affected by the accumulated damage due to cyclic loading. It has been shown that plastic deformation increases with increase of the contribution of the shearing stress component [10]. Hence, analyses of fatigue damage are indispensable to understand crack growth behavior under mixed-mode loading.

To clarify the effect of the damage due to plastic deformation on the FCGR, elastic-plastic FE analyses were conducted under cyclic loading conditions. Plastic stress-strain relation is assumed to be linear hardening. Total strain is given by sum of the elastic strain and the plastic strain. As the yielding condition, the von Mises criterion was employed. The original elastic-plastic FE program developed by Yamada *et.al.* [11] was modified so that cyclic loading can be applied to a model. Analyses were carried out for $\Delta\tau/\Delta\sigma$= 0, 0.45, 1.28, ∞. The applied equivalent stress range, $\Delta\sigma_{eq}$ was kept constant throughout the analyses. Cyclic tensile and shear stresses were applied to center cracked models (shown in Figure 4), which simulate the situation where mixed-mode loading is applied to a crack before the initiation. Material constants of 5083 aluminum alloy were used for the analyses. The analyses were conducted up to 10 cycles, under plane stress condition.

From the FE analyses, the shapes of plastic zone for each condition were obtained, as shown in Figure 5. Using these results, the length, r_ϕ of the plastic zone in the ϕ_1-direction was determined (see Figure 6). The value of r_ϕ takes the maximum value for $\Delta\tau/\Delta\sigma$ =0.45 and its value is small under mode-I and mode-II conditions. For large value of r_ϕ, the crack closure might become eminent because the plastic zone around a crack could contribute plastic-induced crack closure. Experimentally, the crack closure under mixed-mode loading was more substantial than that under mode-I condition [7]. From this point of view, the characteristics of a/r_ϕ are in good agreement with the value of m in region IIa shown in Figure 3. Therefore, the deceleration of the FCGR in region IIa can be attribute to the effect of plastic-induced crack closure.

For quantifying the fatigue damage, the value of plastic strain energy, W_p was calculated. Figure 7 shows the change of the W_p against number of the loading cycles. It was shown that the value of W_p takes the maximum value for $\Delta\tau/\Delta\sigma$=1.28. The values of the W_p for the first cycle are presented in Figure 8. This result is in good agreement with the characteristics of the m in region IIb, as shown in Figure 3. Therefore, acceleration



(a) $\Delta\tau/\Delta\sigma$=0 (b) $\Delta\tau/\Delta\sigma$=0.45 (c) $\Delta\tau/\Delta\sigma$=1.28 (d) $\Delta\tau/\Delta\sigma$=∞

Figure 5: Plastic zone ahead of a crack tip at maximum load for N=1, where a is half of the crack length.

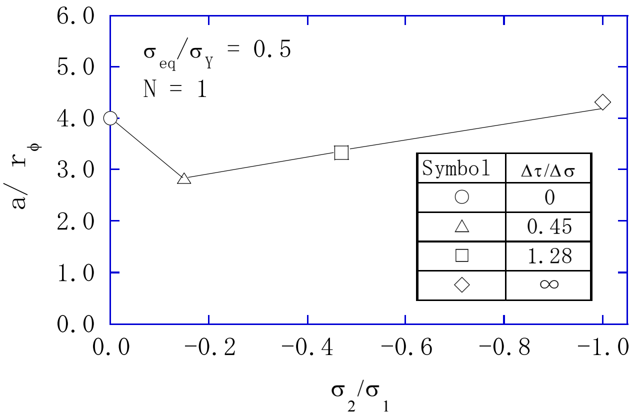


Figure 6: Effect of mixed-mode loading on the length, r_ϕ , of the plastic zone along the direction of the main crack growth.

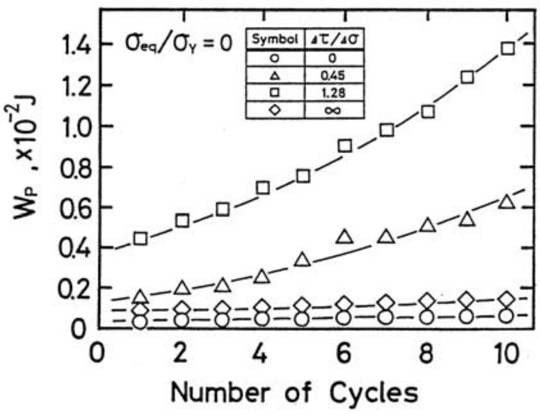


Figure 7: Accumulation of the plastic strain energy, W_p due to cyclic loading under mixed-mode conditions.

of the FCGR in region IIb might be dominated by the accumulation of the plastic strain energy, which is measure of the fatigue damage due to cyclic loading. This result indicates that fatigue damage around the initial crack has significant effect on the fatigue crack growth behavior under mixed-mode loading condition.

The effect of plastic deformation around an inclined main crack is also examined. Under mixed-mode conditions, after the fatigue crack initiates, it changes the growth direction to that nearly perpendicular to the maximum principal stress, σ_1 (see Figure 2). After this transition, the biaxial stress conditions are developed, where σ_1 perpendicular to the crack plane and the compressive principal stress, σ_2 parallel to the crack plane are applied to the inclined main crack. The FE analyses were also conducted for this case. Biaxial stresses $\Delta\sigma_1$ and $\Delta\sigma_2$ are applied to the same FE model. The values of $\Delta\sigma_1$ and $\Delta\sigma_2$ are determined so that the equivalent stress, $\Delta\sigma_{eq}$ takes the same value as that for the previous analyses. From the analyses, the characteristics of the W_p under biaxial loading are found to be very small compared with that obtained by the application of $\Delta\sigma$ and $\Delta\tau$ to the initial crack model. From these results, the fatigue damage accumulated around the main crack is found to be very small compared with that accumulated around the initial crack.

The FE results presented in this work reveal the effect of mixed-mode loading on fatigue damage accumulation. The characteristics of the FCGR, which is represented by the constants m and A in Eqn. 3, are closely related to formation of the fatigue damage around the initial crack due to mixed-mode loading.

FATIGUE LIFE PREDICTION UNDER MIXED-MODE LOADING CONDITIONS

Here, life prediction based on fatigue crack growth law is discussed. The experimental results of the fatigue failure life, N_f tends to increase as increase of $\Delta K_{II0}/\Delta K_{I0}$ [12]. However, other effects including the effects of the stress ratio, R and the initial stress intensity range, ΔK_{Ib0} might exist. Ratio of crack initiation life, N_i to the failure life, N_f is shown in Figure 9. Except shearing stress condition, crack initiation life, N_i is

approximately 10 percent of the failure life, N_f. Therefore, the life for crack propagation is significant within the total life and N_f could be estimated by the crack propagation life using the fatigue crack growth law.
It seems that life prediction based on fatigue crack growth law is complicated under mixed-mode conditions. The region II of the stable crack growth can be divided into two regions, that is region IIa followed by region IIb. In each region, the FCGR expressed by Eqn. 3 shows different characteristics, as shown in Figure 3. Although the fatigue failure life, N_f could be predicted by integrating the crack growth law of Eqn.3 from the beginning to the end of the test, this procedure includes consideration of the crack growth law during both regions IIa and region IIb. Moreover, this method requires the experimental results of the stress intensity range, ΔK_{Ibt} at the transition from region IIa to region IIb, and the stress intensity range, ΔK_{Ibf} just before the final failure. The relationship between ΔK_{Ibt} and ΔK_{Ibf} for various mixed-mode conditions is presented in Figure 10. This relationship indicates that ΔK_{Ibt} has the similar effect of mixed-mode loading as ΔK_{Ibf}. This result could be related to the similarity between N_f and the crack initiation life, N_i presented in Figure 9.

It should be noted that the FCGR during region II is closely related to formation of the fatigue damage around an initial crack due to mixed-mode loading. Experimentally, fatigue failure life, N_f is proportional to the crack initiation life N_i as shown in Figure 9. Therefore, the crack propagation at the first stage is important for life prediction. From this point of view, crack propagation life until the transition from region IIa to region IIb is to be examined. By integrating the crack growth law of Eqn. 3 between the beginning of the test and the transition, the following expression can be obtained:

$$N_t = \int_0^{N_t} dN = \int_{b_0}^{b_t} \frac{db}{(A^{R=0})^{1/r^*}\,\Delta K_{Ib}{}^{m}} \tag{6}$$

where, N_t is the crack propagation life and b_t is the half-length of the crack until the transition from region IIa to region IIb, respectively. By integrating, the following expression was obtained:

$$N_t = \frac{2}{\Delta\sigma_1{}^2\pi}\frac{(\Delta K_{Ib0}{}^{2-m} - \Delta K_{Ibt}{}^{2-m})}{(A^{R=0})^{1/r^*}(m-2)} \tag{7}$$

The value of N_t is now compared with the experimental results. By substituting Eqns. 2(a), 4(a) and 5(a), and the experimental results of ΔK_{Ibt} into Eqn. 7, the value of N_t can be determined for each condition. The comparison between the experimental results of the fatigue failure life, N_f and the value of N_t is presented in Figure 11. It was found that the N_f is approximately proportional to the N_t. Hence, the total life, N_f seems to be controlled by the FCGR of region IIa, which is closely related to formation of the fatigue damage around the initial crack. The failure life, N_f under mixed-mode loading can be predicted by following expression:

$$N_f \approx \frac{3}{\Delta\sigma_1{}^2\pi}\frac{(\Delta K_{Ib0}{}^{2-m} - \Delta K_{Ibt}{}^{2-m})}{(A^{R=0})^{1/r^*}(m-2)} \tag{8}$$

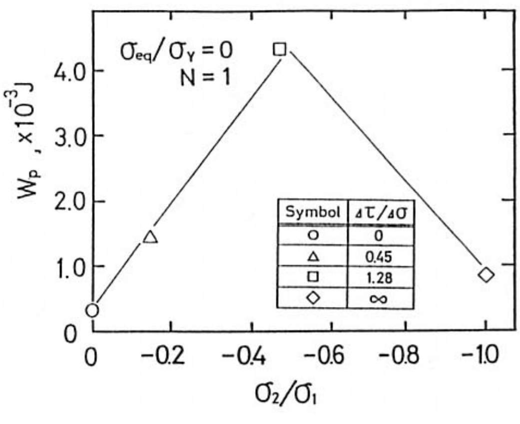


Figure 8: Effect of mixed-mode loading on the plastic strain energy, W_p, at the end of first unloading process.

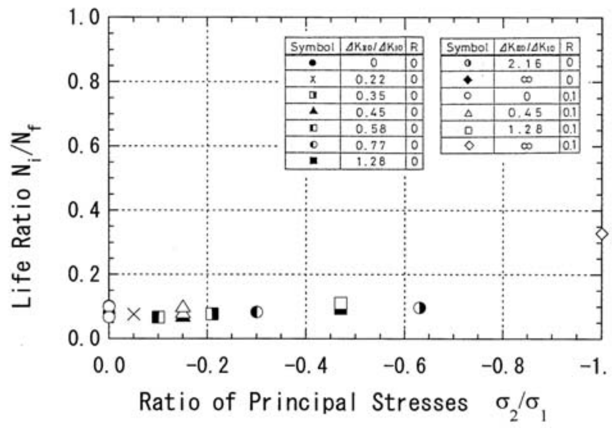


Figure 9: The characteristics of the life ratio, N_i/N_f under mixed-mode conditions.

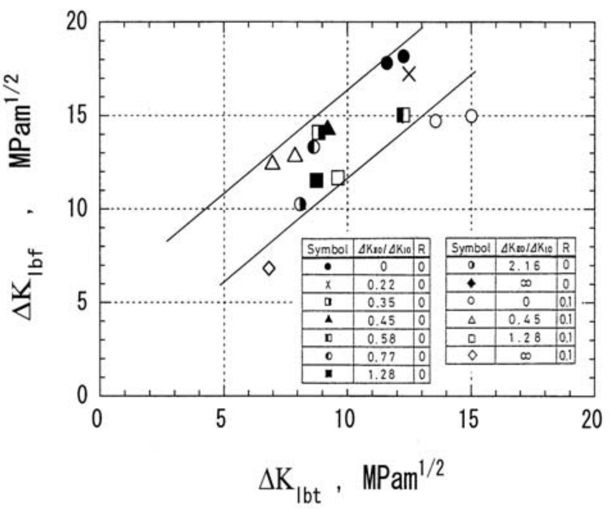


Figure 10: The relation between ΔK_{Ibf} and ΔK_{Ibt} under mixed-mode loading conditions.

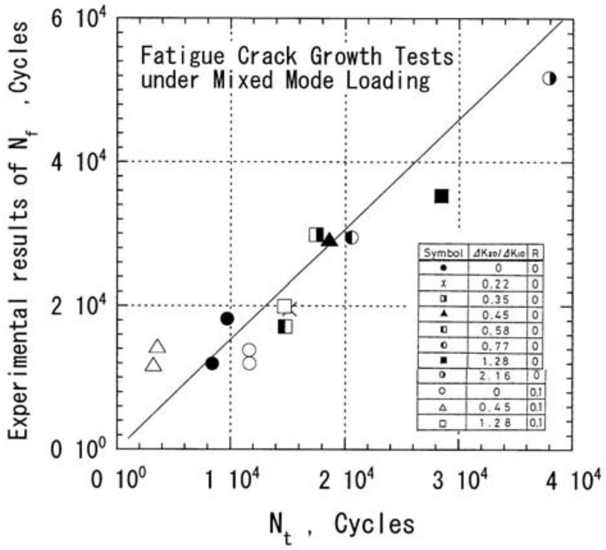


Figure 11: The prediction of the fatigue failure life, N_f under mixed-mode conditions.

In this prediction, the effect of the fatigue damage on N_f is represented by the constants, $A^{R=0}$ and m, which can be described as Eqns. 2a and 5a. The effect of the R-ratio on N_f is represented by the value of r^*.

CONCLUTION

When shearing stress is applied to an initial crack, plastic damage accumulated before initial fatigue crack growth affects significantly the characteristics of fatigue crack growth rate. Under mixed-mode loading conditions, the life for the initial crack growth dominates fatigue failure life. Damage analyses is indispensable to characterize the fatigue crack growth and to predict failure life.

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