ORAL REFERENCE: ICF10058OR

CURVED CRACK PATH PREDICTION BY THE MVCCI-METHOD AND EXPERIMENTAL VERIFICATION FOR A SPECIMEN UNDER LATERAL FORCE BENDING

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ABSTRACT

In this paper it is shown that the curved crack path simulation can be improved considerably in accuracy by using a new predictor-corrector procedure that results in a piece by piece parabolic approximation of the simulated crack path in combination with the **M**odified **V**irtual **C**rack **C**losure **I**ntegral (MVCCI) method. In order to show the superiority of the proposed crack path simulation method in relation to the well established basic strategies, experiments of non-coplanar fatigue crack growth are carried out with a special specimen under lateral bending. In all cases considered the computationally predicted crack trajectories show an excellent agreement with the different types of curved cracks obtained experimentally.

KEYWORDS

Crack path simulation, curved increment, predictor-corrector procedure, virtual crack closure integral method

INTRODUCTION

Failure of structures and components is often caused by cracks that frequently originate and extend in regions characterised by complicated geometrical shapes and asymmetrical loading conditions. In such cases the developing crack paths are found to be curved. Several simulation methods have been proposed for crack path predictions based on step-by-step analyses by using finite elements or boundary elements (Bergquist and Gnex[1]; Sumi[2,3]; Portela and Aliabadi[4]). In the present paper, attention is focused on a new predictor-corrector procedure that results in an incremental parabolic approximation of the crack path on the basis of quantities which the straightforward Modified Virtual Crack Closure Integral Method can provide (Theilig, Döring and Buchholz[5]). In order to show the significance of the proposed technique computational results are compared with findings from experimental investigations obtained by the aid of a specially designed specimen under lateral force bending.

TWO-DIMENSIONAL CRACK PATH PREDICTION

Consider a crack in a two-dimensional linear elastic body under proportional mixed-mode loading conditions. The stresses ahead of the crack tip are given by

$$\begin{aligned}
\sigma_{11}(x_1,0) &= \frac{k_I}{\sqrt{2\pi x_1}} + T + b_I\sqrt{\tfrac{x_1}{2\pi}} + O(x_1),\\
\sigma_{22}(x_1,0) &= \frac{k_I}{\sqrt{2\pi x_1}} + b_I\sqrt{\tfrac{x_1}{2\pi}} + O(x_1),\\
\sigma_{12}(x_1,0) &= \frac{k_{II}}{\sqrt{2\pi x_1}} + b_{II}\sqrt{\tfrac{x_1}{2\pi}} + O(x_1),
\end{aligned} \tag{1}$$

where k_I and k_{II} are the stress intensity factors (SIFs). T, b_I and b_{II} are the included higher order stress field parameters. It is known that in such a situation the crack will propagate in a smoothly curved manner after an abrupt deflection out of its original plane (Figure 1). For several mixed-mode fracture criteria the initial direction φ_0 depends only on the ratio k_{II}/k_I of the SIFs of the original crack, whereas for others a further dependence on Poisson's ratio ν is found. But for small ratios k_{II}/k_I practically the same values $\varphi_0 = -2\,k_{II}/k_I$ are predicted by all criteria. This direction results in the state of local symmetry at the actual crack tip (K_{II} = 0). The generalisation of the local symmetry criterion can be regarded as the basis for the evolution of the crack path. Therefore the state of stress ahead of the deflected new crack tip exhibits no K_{II} and is given by

$$\begin{aligned}
\sigma_{11}(x_1^*,0) &= \frac{K_I}{\sqrt{2\pi x_1^*}} + T^* + b_I^*\sqrt{\tfrac{x_1^*}{2\pi}} + O(x_1^*),\\
\sigma_{22}(x_1^*,0) &= \frac{K_I}{\sqrt{2\pi x_1^*}} + b_I^*\sqrt{\tfrac{x_1^*}{2\pi}} + O(x_1^*),\\
\sigma_{12}(x_1^*,0) &= b_{II}^*\sqrt{\tfrac{x_1^*}{2\pi}} + O(x_1^*).
\end{aligned} \tag{2}$$

It can be stated that continuous crack deflections can only be caused by the existing non-singular stresses. According to Sumi[2,3] the crack path prediction can be performed by using the first order perturbation solution of a slightly kinked and curved crack.

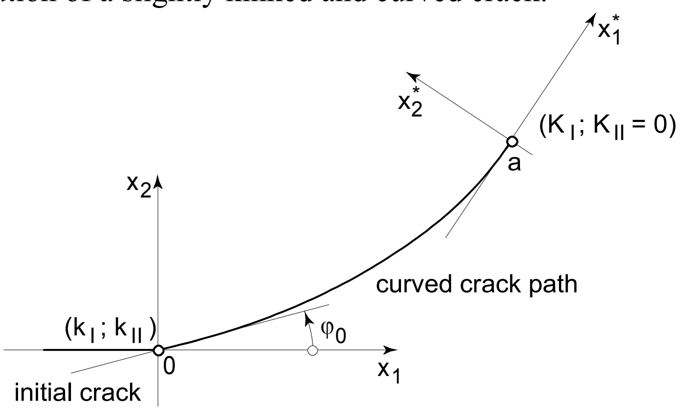


Figure 1: A kinked and curved crack

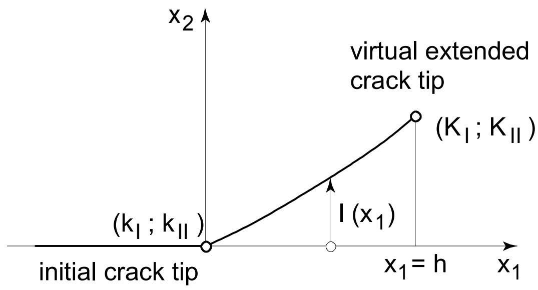


Figure 2: A slightly kinked and curved virtual crack

A virtually extended slightly kinked and smoothly curved crack path profile (Figure 2) is assumed in the form

$$l(x_1) = \alpha x_1 + \beta x_1^{3/2} + \gamma x_1^2 + O(x_1^{5/2}), \tag{3}$$

where α, β and γ are the shape parameters. As the consequence of the crack propagation criterion of local symmetry the SIF K_{II} vanishes along the smooth crack path and the shape parameters of the natural crack geometry are obtained as

$$\alpha = -2k_{II}/k_I\,,\ \beta = \frac{8}{3}\sqrt{\tfrac{2}{\pi}}\frac{T}{k_I}\alpha\,,\ \gamma = -\left(k_{II}\bar{k}_{22} + k_I\bar{k}_{21} + \frac{b_{II}}{2}\right)\frac{1}{k_I} + \left\{\left[k_I\left(2\bar{k}_{22} - \bar{k}_{11}\right) + \frac{b_I}{2}\right]\frac{1}{2k_I} + 4\left(\frac{T}{k_I}\right)^2\right\}\alpha\ .(4)$$

where the quantities $\bar{k}_{11}, \bar{k}_{21}, \bar{k}_{22}$ represent the effects of the far field boundary conditions to the crack growth. If we consider a straight crack under local symmetry at the initial crack tip, i.e. $k_{II} = 0$, we find $\alpha = \beta = 0$. Therefore the parabolic crack profile

$$l(x_1) = \gamma x_1^2\,,\ \gamma = -\left(\frac{b_{II}}{2} + k_I\bar{k}_{21}\right)\frac{1}{k_I} \tag{5}$$

is holding. In this case the crack will propagate without kinking with a continuous deflection. But in the case of a self-similar virtual crack extension of the postulated straight crack the SIF's

$$\bar{K}_I = k_I + \left(\frac{b_I}{2} + k_I\bar{k}_{11}\right)h,\ \bar{K}_{II} = \left(\frac{b_{II}}{2} + k_I\bar{k}_{21}\right)h \tag{6}$$

are obtained. Further $K_I(h) = \bar{K}_I(h)$ is found in consequence of the considered slightly curved crack extension. Finally one gets for a selected increment Δh the following information of the real crack path

$$\Delta\varphi = -2\frac{\Delta\bar{K}_{II}}{k_I}\,,\ \Delta l = -\frac{\Delta\bar{K}_{II}}{k_I}\Delta h\,,\ \Delta a \approx \Delta h\left[1 + \frac{2}{3}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^2 - \frac{2}{5}\left(\frac{\Delta\bar{K}_{II}}{k_I}\right)^4\right]. \tag{7}$$

It is seen that under the local symmetry criterion $K_{II} = 0$ the change of the slope and the locus of the crack tip can be interpreted as the consequence of $\Delta\bar{K}_{II} \neq 0$ for a virtual tangential crack extension Δh (Figure 3).

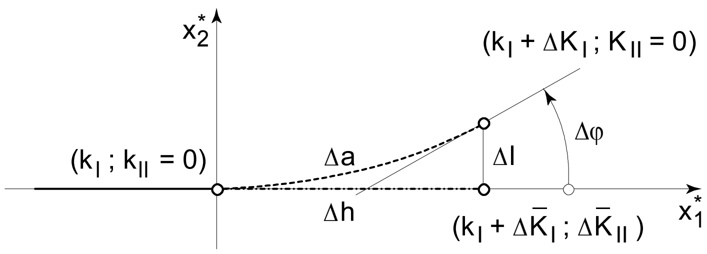


Figure 3: Curved crack propagation

Therefore, in the case of proportional loading conditions the analysis of a smooth crack path can be carried out by a small virtual tangential crack extension as the predictor-step in combination with a finite change of the crack path as the corrector-step. Due to the predictor-step the calculation of $K_I = \bar{K}_I$ and $\Delta\bar{K}_{II}$ is necessary in conjunction with the related tangential crack extension Δh. This can be done by using the finite element method. From Eqs.(7) the need for an efficient numerical mode separation technique in conjunction with the step-by-step analysis can be seen.

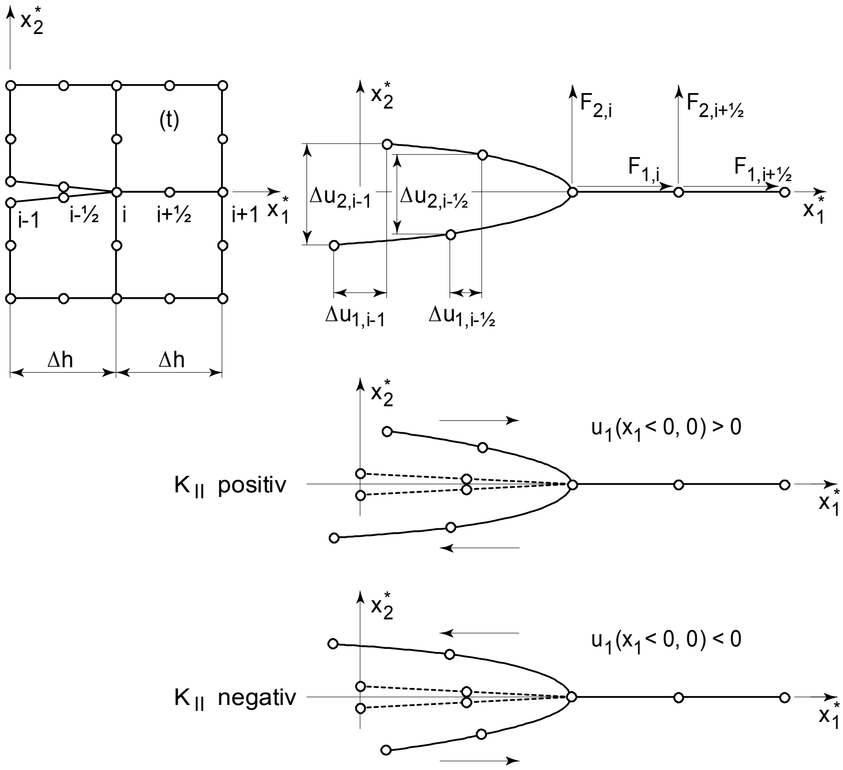


Figure 4: Modified virtual crack closure integral method

With respect to this requirement the MVCCI-method has proved to be highly advantageous, because it delivers the separated strain energy release rates of two modes simultaneously without any additional effort. For 8-noded quadrilaterals at the crack tip (Figure 4), which are necessary to model the parabolic curved increments of the crack path, the following finite element representation of Irwin's crack closure integral relations can be given (Buchholz[6])

$$\overline{G}_I = \frac{1}{2\Delta ht}\left(F_{2,i}\Delta u_{2,i-1} + F_{2,i+½}\Delta u_{2,i-½}\right),\ \Delta\overline{G}_{II} = \frac{1}{2\Delta ht}\left(F_{1,i}\Delta u_{1,i-1} + F_{1,i+½}\Delta u_{1,i-½}\right). \tag{8}$$

CURVED FATIGUE CRACK GROWTH TESTS

In order to evaluate the validity and the efficiency of the proposed higher order crack path simulation method with respect to the well established basic strategies, experiments of non-coplanar fatigue crack growth are carried out with a specially designed specimen under **l**ateral **f**orce **b**ending (LFB)[7]. The LFBH-specimen has been designed with a **h**ole in the centre in order to produce a non-homogeneous stress field (Figure 5).

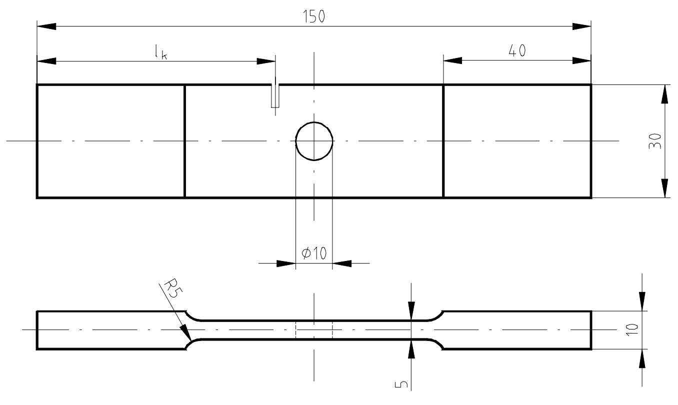
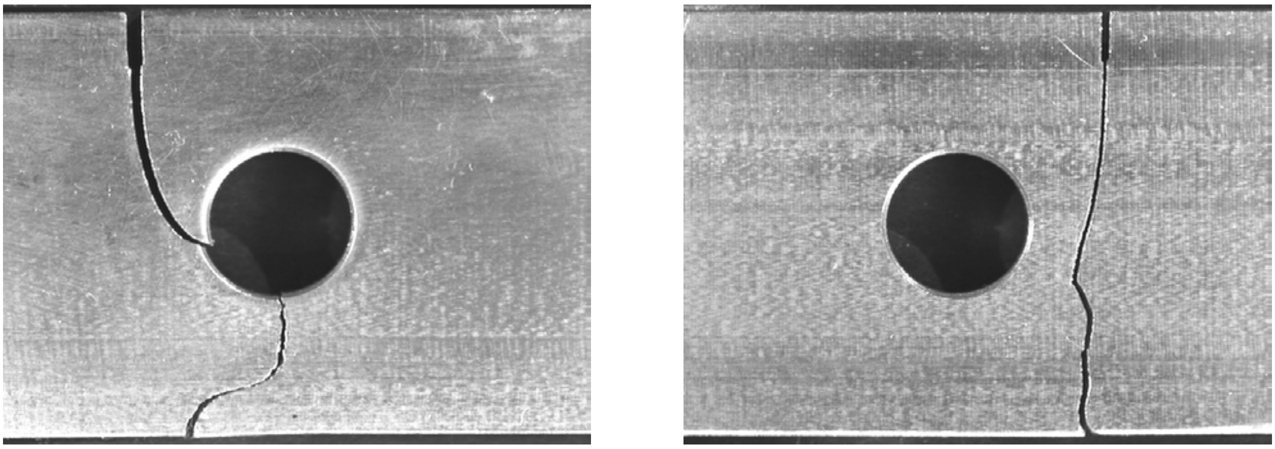


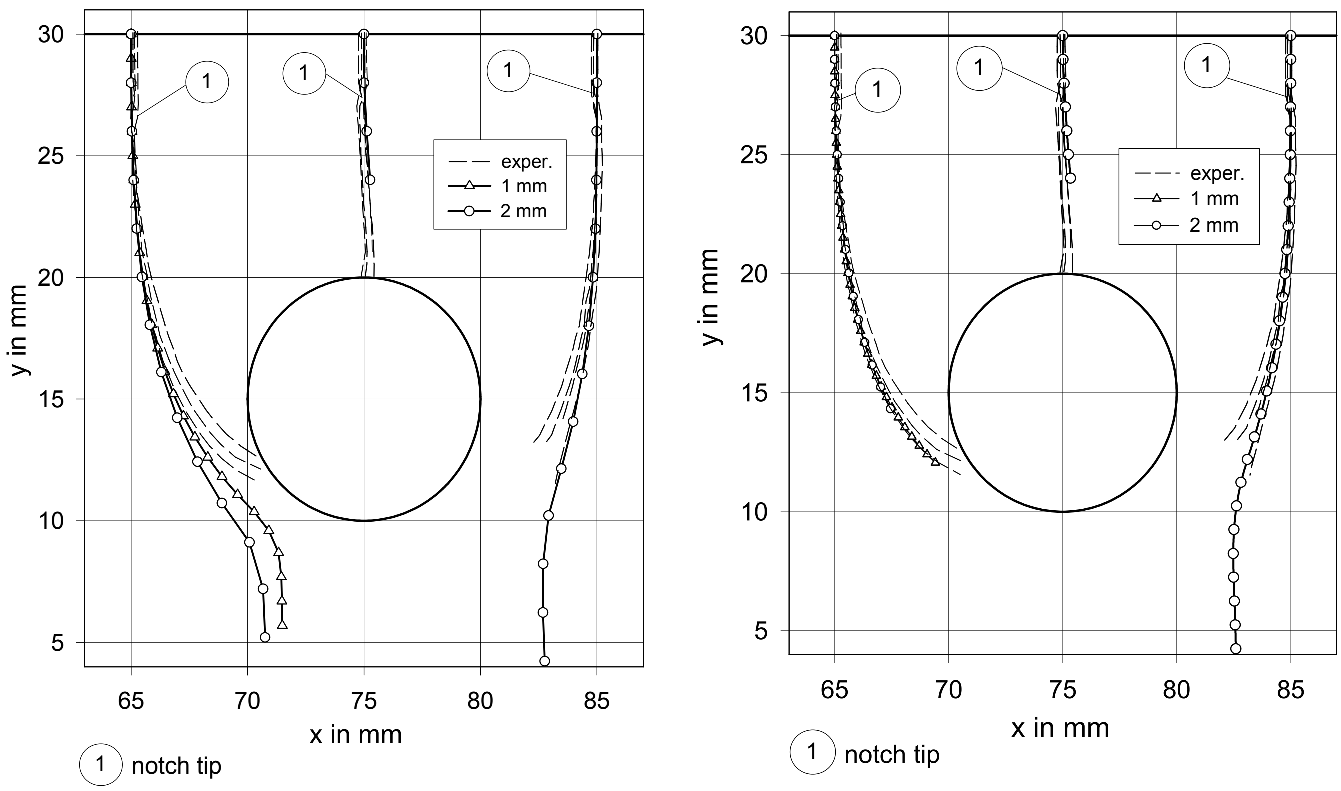
Figure 5: Dimensions and notch position of the LFBH-specimen

Crack initiations from notches at different positions l_K along the tensile loaded edge of these specimens are investigated to produce different crack interactions with the hole. In particular l_K = 65, 75 and 85 mm were selected. The notches have been manufactured with a width of 0.3 mm and a maximum depth of 3 mm. In Figure 6 two broken LFBH-specimens are shown with an experimentally obtained curved fatigue crack path, respectively for two of the three tested notch positions. In the experimental findings given in Figure 7 it can be recognised that the small differences of the notch positions with respect to the bore and the local positions of the pre-cracks in the roots of the notches essentially determine the experimental scattering.



(a) (b)

Figure 6: Experimental fatigue crack paths of the LFBH-specimen with the notch position (a: l_K = 65 mm; b: l_K = 85 mm)



(a) (b)

Figure 7: Simulated and experimentally obtained crack paths of the LFBH-specimen (a: straight incremental steps; b: curved incremental steps with marked mid-side nodes)

NUMERICAL CRACK PATH PREDICTION

For the finite element calculations the model are chosen in accordance with the design of the LFBH-specimen and the available test assembly. After each predictor step of virtual tangential crack extension by Δh a re-meshing is necessary in such a way, that the corrector step is realised in order to model the curved crack surfaces and that together with the following predictor-step new crack tip elements are generated providing additional nodes. All calculations were carried out with the FE-code ANSYS. In Figure 7 the numerical results of the notch position l_K = 65, 75 and 85 mm and the experimental findings are given. For all chosen increments Δh (2 mm, 1 mm) an excellent agreement is found. Additional calculations were carried out without the proposed corrector step (straight increments) in order to verify the improved convergence of the new method. In particular for the critical notch position l_K = 65 mm the new method results in an accurate evaluation of the final fracture mode of the cracked specimen whereas, in this case, the other method fails to simulate the correct crack path.

SUMMARY

This investigation has shown that the new predictor-corrector procedure in combination with the MVCCI-method provides excellent crack path simulation results with 8-noded quadrilaterals and only moderately refined finite-element-meshes around the crack tip. The step-by-step higher order simulation process with a piece by piece parabolic curved approximation of the crack path offers an excellent way for the numerical analysis of fatigue crack growth in complex two dimensional structures under proportional loading conditions. From the excellent agreement of the numerical and experimental results one can also conclude that the applied criterion of local symmetry provides a correct and reliable basis. The proposed predictor-corrector-method in conjunction with the evaluation by the MVCCI-method provides a powerful numerical tool for a general computational approach to the fracture analysis of complex crack configurations and loading conditions.

ACKNOWLEDGMENT

The authors are grateful to Dipl.-Ing. R. Döring, who provided most of the computational results presented in this work.

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