CRITICAL LENGTH OF SHORT FATIGUE CRACKS

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ABSTRACT

A model is presented that predicts the fatigue limit of a metal by determining the critical crack length. The threshold stress range for short fatigue crack growth is related to the strain intensity factor range by taking into consideration surface strain distribution and crack closure. In particular the surface strain concentration factor has been carefully evaluated. This factor decreases, together with an increase in crack closure, as crack length increases within the short crack range. The resulting threshold stress for crack growth increases to a maximum that corresponds to the fatigue limit stress. This occurs at the critical crack length. In addition to successfully predicting the fatigue limit stress, the model is capable of determining the crack initiation stress range and depth of non-propagating cracks as a function of material, grain size and stress ratio.

KEYWORDS

Critical crack length, threshold stress, short fatigue cracks.

INTRODUCTION

On cycling a polycrystalline metal each surface grain will experience a different amount of strain according to its orientation relative to the loading axis. Large, favourably oriented grains represent preferred sites for crack initiation because of localized slip. With increasing depth, the constraints and strain compatibility requirements become more severe, leading to a lower local strain range. The initial high local strain range, $\Delta\varepsilon$, decreases, approaching the nominal strain range, Δe. The strain concentration factor, Q_ε, decreases with the projected crack length, a, according to [1]:/

$$Q_\varepsilon = \Delta\varepsilon/\Delta e = 1+q \exp [a(-\alpha/D)] \qquad (1)$$

where q is a constant, α is a material constant depending upon deformation character and represents the ease of cross-slip. The function for the decay of Q_ε gives an average continuous description of short crack behaviour allowing a relatively simple model to be applied although it is recognized that short crack behaviour may be discontinuous and strongly affected by microstructural features such as grain boundaries [2]. The depth at which $Q_{\text{å}}$ is effectively equal to unity determines the extent of the surface-affected zone, L_i. When a = 0, $Q_{\text{å}}$ =1+q represents the strain concentration factor at the free surface, $Q_{\text{åS}}$. Originally, Abdel-Raouf et al [1] concluded that q=5.3, based on the probability of slip at the surface. This gave a value of $Q_{\text{ås}}$ = 6.3. Reconsidering this original work, $Q_{\text{åS}}$ yielded a value of approximately 8.7. Using a slightly different probabilistic approach, the strain concentration factor at the surface was found to vary between 6.1 and 7.2 [3].

Another approach has been to consider the strain in Persistent Slip Bands (PSBs). In this case, the values for the strain concentration factor at the free surface ranged between 4.7 and 10.0 [4,5] and the weighted average value for eleven different set of data was 8.4.

By means of the Neuber approach $Q_{\text{ås}}$ varied between 5.9 and 8.1 if the cyclic yield limit were considered as the applied stress. The average values for twelve steels and seven aluminum alloys were 6.9 and 7.0 respectively, indicating material independence[6]. Hence, for simplification, Q_{eS} will be applied using a single value of 7.0.

INTRINSIC THRESHOLD STRESS RANGE

In the absence of crack closure, the intrinsic strain intensity factor range, can be expressed as follows:

(2)

where E is the modulus of elasticity and F is the geometrical crack factor.

When the nominal applied strains are elastic, ÄeE = ÄS_i where is the intrinsic component of the applied stress range, the intrinsic stress intensity factor range simplifies to:

(3)

At high stress ratios (R= minimum/maximum stress ≥ 0.6) ÄK_i can be assumed to be the intrinsic threshold stress intensity factor ÄK_{ith}. According to DuQuesnay [7] no closure effects were present at a stress ratio of R = 0.6 for aluminum alloy Al 2024-T351 with ÄK_i = 2.2 MPa $m^{1/2}$.

The intrinsic threshold stress range, ÄS_{ith} can be calculated at any crack depth using Eq. 3. As expected, a linear relationship with a slope of -0.5 exists between log ÄS_{ith} and log a for long cracks (a ≥ L_i) when $Q_{\text{å}}$ = 1, since linear elastic fracture mechanics (LEFM) applies. However, for short cracks (a < L_i), the curve deviates from linearity and their behaviour is under microstructural control.

The maximum value for ÄS_{ith} represents the nominal stress range required to maintain continuous crack propagation, i.e. the fatigue limit of the material for the intrinsic condition when closure is absent. For the Al2024-T351 alloy, the crack initiation stress range, ÄS_c, had a value of 96 MPa at the minimum crack depth of 3 ìm and the maximum value for ÄS_{ith} was determined to be 110 MPa which is in good agreement with the experimental value of 125 MPa. This occurred when a_c = 190 ìm (3.8D). Since the calculated fatigue limit ÄS_{FL} is slightly smaller than the actual fatigue limit stress at the stress ratio of R=0.6 the crack may not have been fully open.

Figure 1: Nominal threshold stress range, as a function of crack depth and stress ratio for Al 2024-T351.

a_e = a-0.4D, q = 6.0, á = 1.0, D = 50 ì m, = 2.2MPa $m^{1/2}$,
k=20 mm^{-1}, F (a=3ì m), = 1.12 and F (a=200ì m) = 0.72

CLOSURE

The stress intensity factor range (ÄK = K_{open} - K_{min}) required to open a closed crack increases with crack depth to a steady-state level, representative of long crack development. Hence at lower stress ratios when closure is present, the threshold stress intensity factor range must include a crack opening component in addition to the intrinsic component. This is achieved by introducing the closure development factor, H_{cl} representing the ratio of the total stress range to the open portion of the stress range, which increases the threshold stress intensity factor range, ÄK_{th}, and is expressed by [2, 8, 9]:

(4)

or with the corresponding expression for stress:

(5)

H_{cl} is given by:

(6)

The factor è′ is expressed by:

(7)

where k is a material constant describing the rate of crack closure development and a_c is an effective crack depth

a_c=a-0.4D since closure starts to build up about half-way into the surface grain [2].

For small cracks, H_{cl} is approximately unity and Eq. (4) yields $\Delta K_{th} = \Delta K_{ith}$, indicating that the crack is fully open. For long cracks, however, the steady state value of H_{cl} is invariant with crack length. Its magnitude increases as the stress ratio decreases.

The values of K_{th} at steady-state are listed in Table 1 for the corresponding stress ratios.

FATIGUE CRACK MODEL

Combining Eqs. (3) and (5) leads to the final equation describing the variation of the threshold stress range with crack length in the short and long fatigue crack regime:

$$\Delta S_{ith} = H_{cl}\, \Delta K_{ith} /(Q_{\varepsilon}\, F \qquad) \tag{8}$$

The three mechanisms involved in the present model are incorporated in Eq. (8) and are as follows: i) the closure parameter H_{cl} ii) the inherent strain concentration factor Q_{ε} and iii) the LEFM-contribution. The crack length appears three times in Equation (8) through H_{cl}, Q_{ε} and a. It is the only unknown variable. Stress ratio is taken into account by H_{cl}.

FATIGUE LIMIT PREDICTION

The experimental data for 2024-T351 aluminum alloy is available from previous work [7, 8, 9] and applied to the present model. The important mechanical and microstructural material properties are given in Table 1.

The plot of Eq. (8) versus crack depth is seen in Figure 1. This illustrates the relationship between the nominal threshold stress range and the crack depth for three different stress ratios. The threshold stress range has a local maximum value, representing the stress range required for continuous crack growth, which defines the fatigue limit. For stress ratios less than 0.6, the magnitude of the threshold stress range at the fatigue limit increases with decreasing stress ratio due to an increase in the contribution of crack closure.

TABLE 1
Material Properties of 2024-T351 Aluminum Alloy [7, 8, 9]

Material Property	Stress Ratio	Value
ΔK_{th} for long-crack propagation	R = -1	4.4 MPa $m^{1/2}$
ΔK_{th} for long-crack propagation	R = 0	3.4 MPa $m^{1/2}$
ΔK_{th} for long-crack propagation	R = 0.6	2.2 MPa $m^{1/2}$
ΔS_{FL} at 2×10^7 cycles	R = -1	246 MPa $m^{1/2}$
ΔS_{FL} at 2×10^7 cycles	R = 0	170 MPa $m^{1/2}$
ΔS_{FL} at 2×10^7 cycles	R = 0.6	125 MPa $m^{1/2}$
Grain size in crack growth direction	-	50 μm

Table 2 summarizes the predicted fatigue limit stress range, the critical crack length at the local maximum, a_c, the experimental fatigue limit stress range and the relative deviation. The critical crack is about four grain diameters in length. The predicted and experimental values of the fatigue limit stress are in good agreement. However, the results are very sensitive to the closure parameter. A small deviation in the experimentally determined threshold stress intensity factor ranges can lead to a large scatter in the prediction.

TABLE 2
Predicted Fatigue Limits for Different Stress Ratios

R	a_c [ìm]	$ÄS_{FL}$[MPa]	Experimental $ÄS_{FL}$ [MPa]	Relative deviation [%]
0.6	188 (3.8D)	110	125	-12.0
0	194 (3.9D)	167	170	-1.76
-1	198 (4.0D)	214	246	-13.01

The fatigue limit stress range may be predicted using an alternative approach based on the experimentally determined $ÄK_{th}$ (Table 1) without having to consider the closure parameter. Since, at a given stress ratio, $ÄK_{th}$ accounts for closure development at the steady-state level and the critical crack length at the fatigue limit stress is known from the previous analysis, the following expression leads to the fatigue limit stress range for that stress ratio:

(9)

The critical crack depth a_c is taken as 4D. For the stress ratios of R=0.6, R=0 and R=-1 the fatigue limit stress ranges are then determined to be $ÄS_{FL}$ = 111 MPa, $ÄS_{FL}$ = 171 MPa and $ÄS_{FL}$ = 221 MPa, respectively. These values agree with those obtained in the previous section. It is important to note if the extreme values of 5 and 8 for q are considered, then the variation in $ÄS_{FL}$ is only in the range of 5% for a given stress ratio.

If a specimen were cycled with a stress range larger than $ÄS_c$ yet lower than $ÄS_{FL}$, the crack will grow to a depth corresponding to the threshold stress range given in Fig. 1 and become non-propagating. Due to the difference in closure levels the crack would stop growing at a shorter depth if the stress ratio were lower. The model is capable of predicting the depth of non-propagating cracks. This has been observed in smooth specimens cycled at low stress ratios [2] where there is a larger difference between $ÄS_c$ and $ÄS_{FL}$.

SUMMARY

The fatigue limit stress can be predicted accurately with the current model. The information required is the average grain size, the intrinsic threshold stress intensity factor range and the closed portion of the stress range for a long crack at a given stress ratio. The threshold stress curve may then be plotted against crack depth and its maximum corresponds to the fatigue limit stress range. The model is capable of predicting the development of non-propagating cracks when cycled at constant amplitude stress ranges lower than the fatigue limit.

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