ORAL/POSTER REFERENCE :

COUPLING OF DAMAGE MECHANICS AND PROBABILISTIC APPROACH FOR LIFE-TIME PREDICTION OF COMPOSITE STRUCTURES

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ABSTRACT

We propose a damage model with a probabilistic approach for laminates made of unidirectional fibre reinforced plies. Statistical information is collected through multiple cracking tests. The defects are considered as transverse matrix cracks and we study them by examining $[0_3/90_3]_s$, $[0_2/60_2/-60_2]_s$ and $[0_2/90_2/-45_2/45_2]_s$ laminates.
Parameters of a cumulative distribution function of the failure strength are determined. Probabilistic parameters of the cumulative distribution are chosen to be independent of the ply thickness and multiaxial loading to have intrinsic values for describing the ply. Probabilistic parameters found previously are introduced into a finite element computation of laminates using a Statistical Volume Element (SVE). As experimental results, numerical ones present a dispersion of the failure stress.

KEYWORDS

Carbon/epoxy fabric laminates, composite laminates, defects statistics, crack density, transverse cracking, multiaxial loading, probabilistic failure criteria.

INTRODUCTION

The fracture behaviour of fibre reinforced composites has an inherent variability which results from the presence of defects in the constituents. A probabilistic model including failure criteria, taking into account the presence of defects for predicting the statistical fracture behaviour is proposed for a composite laminate under multiaxial loading.
The studied composite laminate is a stack of plies where each ply is made of unidirectional carbon fibre (T300) embedded in an epoxy matrix (914). Different stackings of composite laminates $[0_3/90_3]_s$, $[0_2/60_2/-60_2]_s$ and $[0_2/90_2/-45_2/45_2]_s$ are subjected to mechanical loads which lead to damage. The damage is known to consist of intralaminar cracks (fibre breaks, axial and transverse cracks) and interlaminar cracks formed by local separation of plies (delamination). We focus on transverse cracks which give a variability of fracture properties.

In order to predict failure stresses in composite laminates, it is necessary to take into account the probabilistic nature of defects. Such an approach consists of two parts: identification of a population of defects and simulation of the statistical behaviour of the material under multiaxial loading. The statistical aspect is introduced in finite element calculation with the Statistical Volume Element (SVE) described by Baxevanakis et al. [1]. The paper is organized in the following way. The first part describes the identification of transverse cracks and a multiaxial fracture criterion. The second part presents the numerical simulation of the fracture of a laminate composite.

TRANSVERSE CRACKING

Description of the test

The aim of the test is to estimate the population of defects that generates transverse cracks during the damage process of composite laminates. The material used in this study is a carbon fibre reinforced epoxy (T300, 914) with the ply properties reported on Tab. 1.

TABLE 1: MECHANICAL PROPERTIES OF CARBON/EPOXY PLY.

Material	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	ν_{LT}	σ_R (MPa)	ε_R (%)
Ply	140	9.5	3.2	0.31	2150	1.1

In a symmetric laminate, an axial load produces an in-plane stress state in off-axis plies consisting of normal stresses parallel and perpendicular to fibres and shear stresses. Following the orientation and the stacking, the stress state varies. To have different stress state we consider three laminates : $[0_3/90_3]_s$, $[0_2/60_2/-60_2]_s$, $[0_2/90_2/-45_2/45_2]_s$ laminates. Samples from the laminates are cut and tested in an Instron testing machine. Axial and transverse strains are measured using strain gauges. Once the edges of the specimens are polished, the specimens are loaded to a selected strain level. The position of every crack is measured in situ by a traveling optical microscope (Fig. 1). The position and the number of cracks are collected for each ply and for each level of deformation until the specimen fails.

Results

To determine the local failure stress state resulting from the global stress applied experimentally, we accurately simulate experiments by introducing cracks exactly at the positions and at the deformation levels they were found experimentally. Figure 2 shows how we calculate the stresses in each ply for a deformation level at which a crack appears.
We choose the damage variable defined by Thionnet [2] as $\Psi = d \times t$ (d : number of cracks per unit length and t : the thickness of the ply) and we define a quadratic criterion $\sigma_{eR} = \sqrt{\sigma_T^2 + a\tau_{LT}^2}$ (σ_T is the transverse stress and τ_{LT} is the shear stress in the local reference and a is a coefficient representing the effect of defects on shear) for describing the defect population in a ply (Fig. 3 a). These variables are chosen to describe the population of defects independently of the ply thickness and multiaxial loading. First from the curves of Fig. 3 a, we determine $a = 0.45$, this value is consistent to what is observed in literature for a deterministic mesoscopic Tsai-Hill criterion.
Next, we gather the information on defects coming from different plies. This is possible only if transverse cracks were observed just before the occurrence of the delamination because our pseudo-tridimensional calculation does not take into account this phenomenon. This is the reason why Figure 3 b) shows the addition of two populations in $[0_3/90_3]_s$ and $[0_2/60_2/-60_2]_s$ laminates. The population of cracks of $[0_2/90_2/-45_2/45_2]_s$ laminate is removed because the delamination appears at the same time as transverse cracks. Then, the possible model of distribution for the transverse cracks is a sigmoidal distribution represented by the Eq. 1 :

$$\Psi(\sigma_R) = A(1 - e^{(\frac{\sigma_R}{\sigma_\circ})^m}) \tag{1}$$

with $A = 0.47$ is the maximum number of defects (nondimensional),
$\sigma_\circ = 64.82$ scale parameter (MPa),

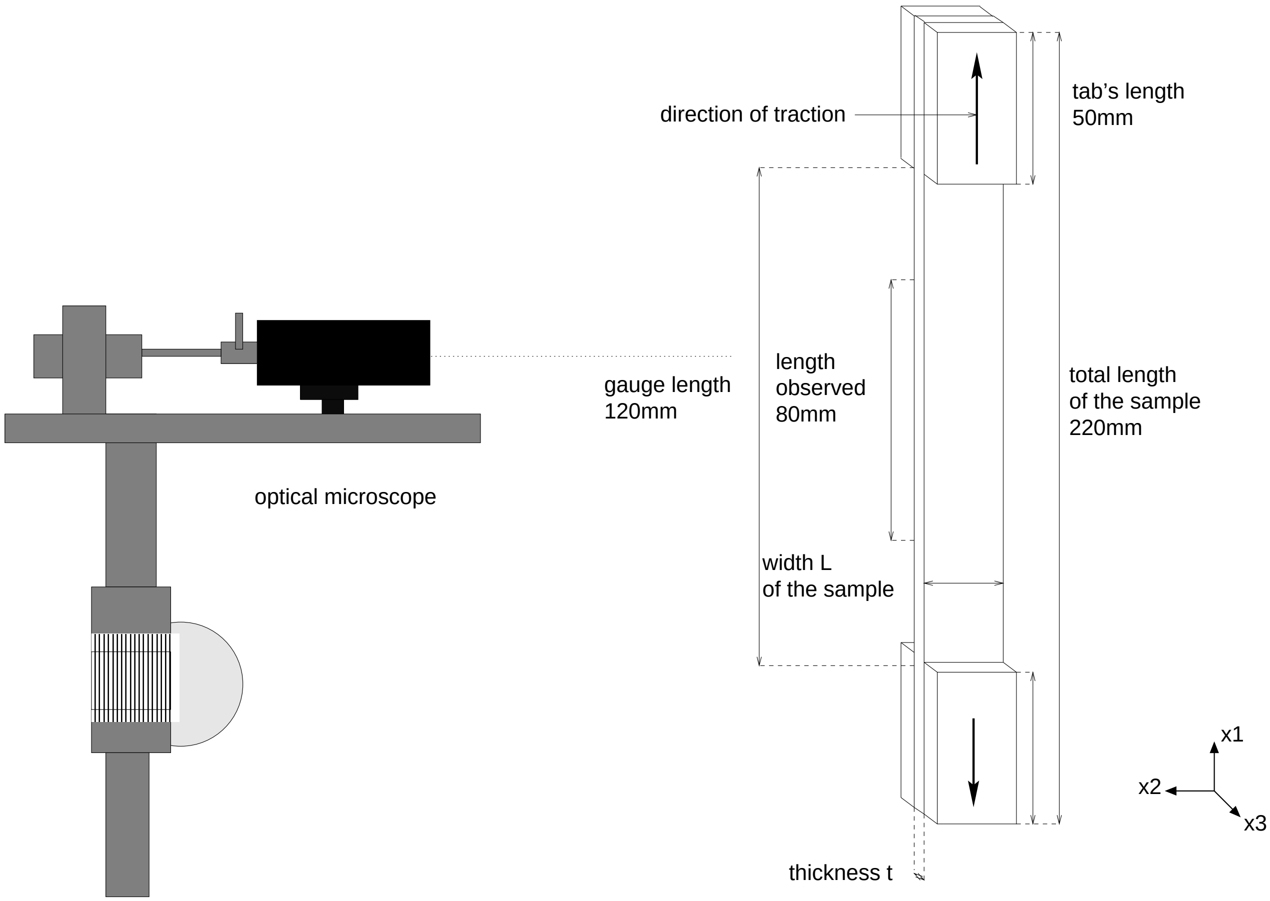

Figure 1: Counting cracks on the sample during the tension.

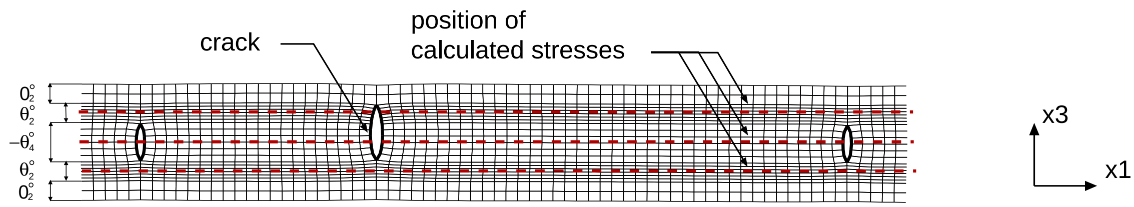

Figure 2: Mesh of the thickness of the laminate with cracks (pseudo-tridimensional calculation)

$m = 5.39$ a shape parameter (nondimensional).

SIMULATION OF THE FRACTURE OF A LAMINATE COMPOSITE

Statistical Volume Element (SVE)

The Statistical Volume Element (SVE) gives the possibility to introduce the stochastic aspect in the numerical simulation. It is defined as having one critical defect which is in our case a crack. During experiments, we observed that the geometry of the crack had the thickness of the ply (0.246 mm to 0.738 mm) and the width of the sample (25 mm). So two dimensions of the SVE are directly defined. The third dimension of the SVE is the intercrack spacing at saturation. Only the $[0_3/90_3]_s$ laminate has its cracks at saturation (1.41 mm), laminates with off-axis plies undergo delamination. Jeulin [3] assumes that defects are distributed according to a Poisson point process and that SVE breaks with the weakest link assumption described by Weibull [4] so its probability to break is given by Eq. 2:

$$P_r(\sigma_R) = 1 - e^{-\int_{SVE} \Psi(\sigma_R) dv} \tag{2}$$

Knowing $\Psi(\sigma_R)$ from the experimental study (Eq. 1) and using a uniform random variable P_r between 0 and 1, we associate to each SVE a fracture value σ_{RSVE} obtained by Eq. 3 :

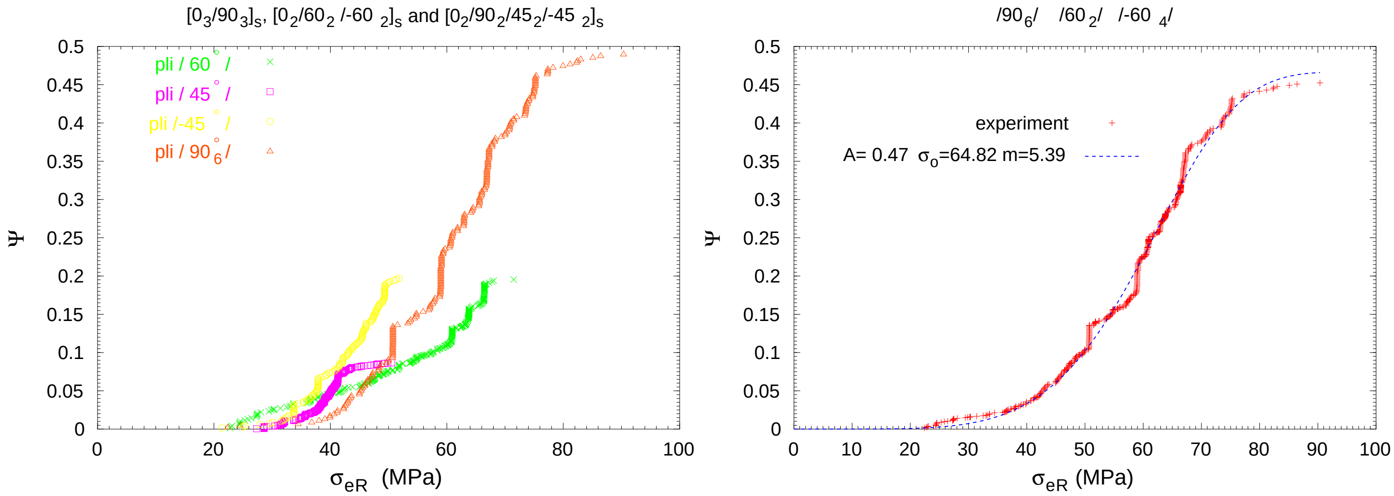

Figure 3: a) Cumulative damage variable Ψ for cracks in different laminates. b) Cumulative damage variable Ψ for cracks in $[0_3/90_3]_s$, $[0_2/60_2/-60_2]_s$ laminates.

$$\sigma_{RSVE} \quad = \quad \sigma_\circ \left[-ln \left(1 + \frac{ln(1-P_R)}{AV_{SVE}} \right) \right]^{\frac{1}{m}} \tag{3}$$

Damage model for numerical simulation

Figure 4 shows two meshes of a plate representing the sample studied experimentally : a finite element mesh and a statistical mesh. The finite elements are used to calculate under classical lamination theory [5] the stress and displacement states. The statistical mesh is made of SVE's which are introduced to give the failure stress variability of the sample. The different gray levels represent the values of the fracture criterion σ_{RSVE} associated with each SVE. The two meshes are superimposed to give the input for numerical calculation.

The appearance of damage in the laminate is simulated as follows. If in one SVE, the criterion σ_{eSVE} which is the mean of the criterion σ_{eiel} of all finite element which belong to the SVE reaches σ_{RSVE} according to Eq. 4, the damage of these elements is simulated numerically by a stiffness reduction described by Renard and Thionnet [6].

$$\sigma_{eSVE} = \sum \frac{v_{iel}\sigma_{eiel}}{V_{SVE}} > \sigma_{RSVE} \tag{4}$$

Numerical results

A numerical simulation is made on $[0_3/90_3]_s$ and $[0_2/60_2/-60_2]_s$ laminates with the model described previously, to compare it with experimental results. For both laminates we have a low variability on behaviour laws so the stress-strain curves are less interesting than the density of cracks represented in Fig. 5, where is plotted the damage variable (also called the cumulative density Ψ) against the criterion σ_{eR} ($\sigma_{eR} = \sigma_{RSVE}$ for numerical results). In the case of $[0_3/90_3]_s$ laminate, the SVE breaks in the same way as experimentally as shown in the Fig. 5 a).

In the case of $[0_2/60_2/-60_2]_s$ laminate (Fig. 5 b), the experimental and numerical curves are not similar, since we do not reach experimentally the saturation, the calculation being done with the size of the SVE of $[0_3/90_3]_s$ laminate. Numerically, by using the size of SVE of $[0_3/90_3]_s$ laminate, we introduce more cracks than there are in reality. In addition, the parameters used for the statistical model penalize this laminate configuration.

CONCLUSION

We proposed experimental and numerical schemes for the determination of defects population parameters in a ply. At the same time, we defined a multiaxial criterion $\sigma_e = (\sigma_T^2 + a\tau_{LT}^2)^{1/2}$, with $a = 0.45$. More

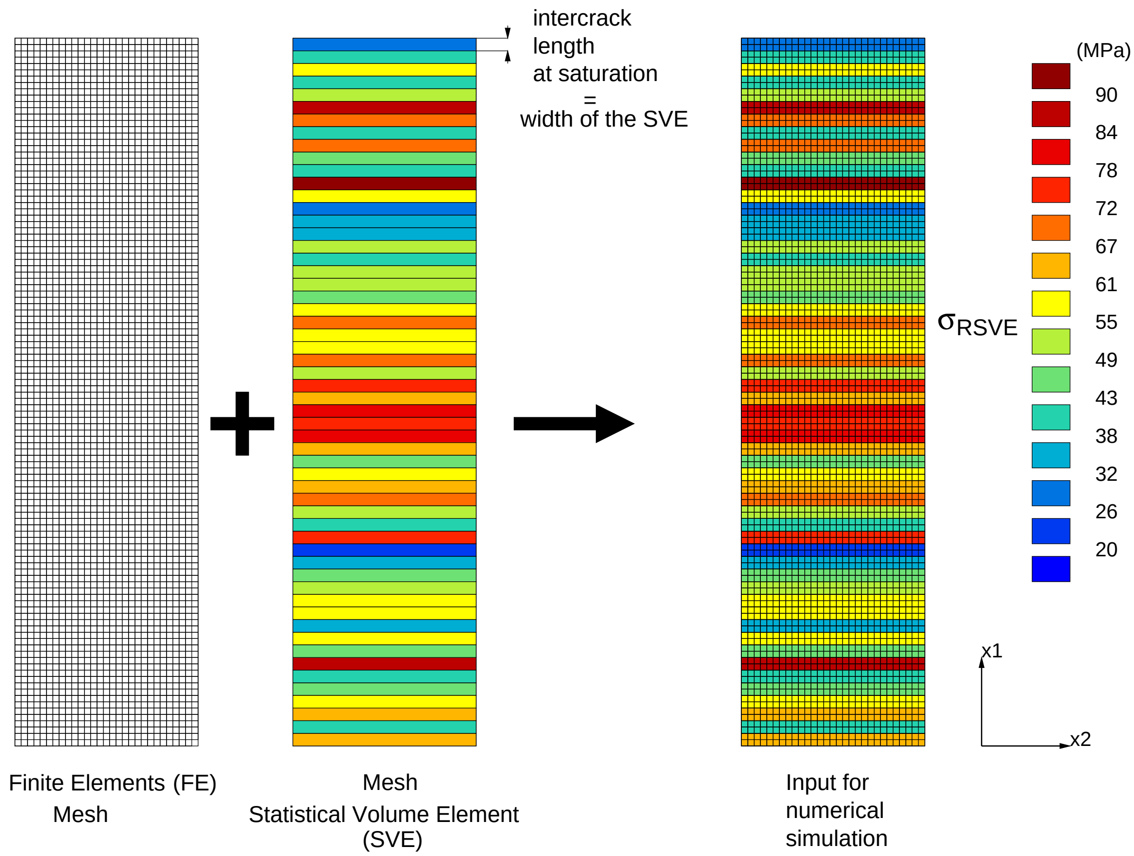

Figure 4: Input for the numerical simulation

we have information on defects more the parameters of population and the parameter a will be precise and realistic. Next, we built a probabilistic damage model by introducing stochastic failure stresses with the SVE. Numerical results $[0_3/90_3]_s$, gave a cumulative density similar to the experimental one. Concerning $[0_2/60_2/-60_2]_s$ laminate, the cumulative density was higher because of the choice of the size of the SVE. To model the cumulative density more precisely, we have to account for the tridimensional stress state, in order to include the delamination phenomenon so that the classical laminate theory assumptions could not be considered anymore.

The ongoing work is to apply the model for high stresses gradient zones (notched plates). Figure 6 shows the numerical input. The failure criterion is changed. Eq. 5 takes into account the fact that we have a higher probability to break in the vicinity of the circular hole.

$$\sigma_{eSVE} = \sigma_\circ \left[-ln \left[\sum_{iel}^{N} \frac{v_{iel} e^{-\left(\frac{\sigma_{iel}}{\sigma_\circ}\right)^m}}{V_{SVE}} \right] \right]^{\frac{1}{m}} > \sigma_{RSVE} \tag{5}$$

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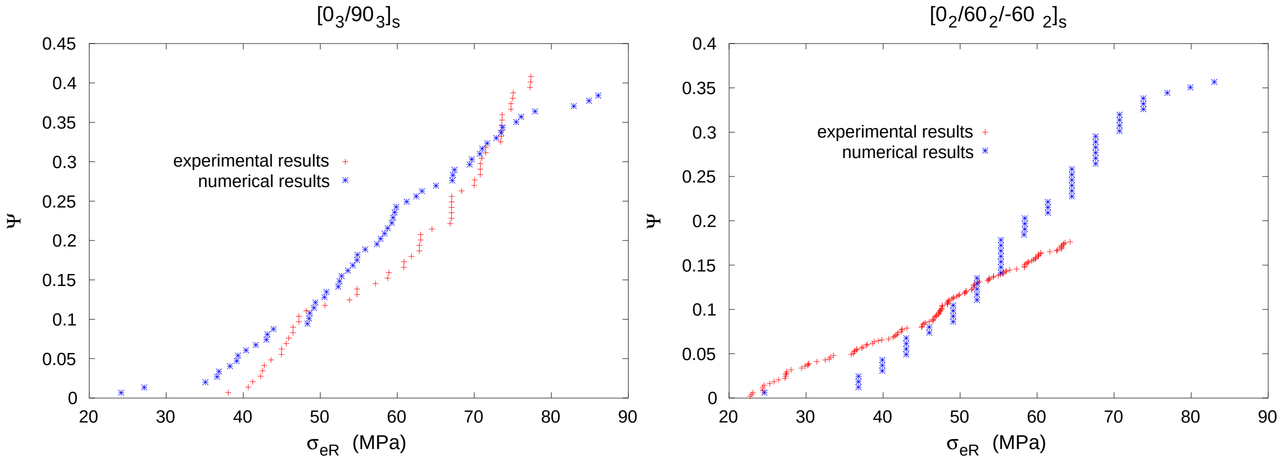

Figure 5: Comparison of numerical and experimental results for a) $[0_3/90_3]_s$ and b) $[0_2/60_2/-60_2]_s$ laminates.

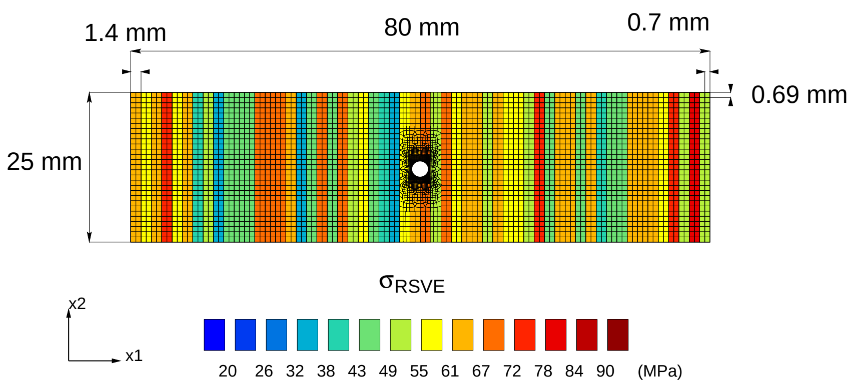

Figure 6: Example of realization of σ_{RSVE} for a notched plates.

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