CONSTRUCTION OF STRAIN-LIFE DIAGRAM BASED ON VARIABLE AMPLITUDE FATIGUE DATA

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ABSTRACT

A method is proposed that constructs a total strain range versus fatigue life curve ($\Delta\varepsilon_t$-N_f curve) based on variable amplitude loading data. In this method material parameters in the $\Delta\varepsilon_t$-N_f curve are determined by the least squares method on the assumption that fracture or crack initiation is occurred when the summation of life fractions estimated using Miner’s rule reaches unity. Computer simulation of random loading fatigue test was first conducted to assure that the method is applicable to a wide variety of real materials. Constant and variable amplitude axial loading tests were carried out on annealed 0.37% carbon steel specimens. During the fatigue test, varying strain was measured by a strain gage and reduced to a frequency distribution of strain range using the rainflow method. The $\Delta\varepsilon_t$-N_f curve based on initiation of a crack was reasonably estimated by the proposed method. The procedure for obtaining the $\Delta\varepsilon_t$-N_f curve directly from an actual component subjected to service loading is also discussed.

KEYWORDS

Fatigue damage, Variable amplitude data, Miner’s rule, Rainflow method, Inverse problem, Strain-life curve.

INTRODUCTION

One of the best ways for preventing fatigue failure of machines or structures is to monitor the fatigue damage accumulated in them. To do this it is necessary to measure the variable amplitude strain signal under the service condition where the component is used, and then to estimate the fatigue life by proper fatigue damage analysis. Service load fatigue phenomenon is so complicated that many problems have yet to be solved. Although study for understanding the essence of fatigue phenomena is important, development of practical technique for fatigue life estimate will become more important in the future.

Murakami et al. [1] have recently developed a small compact strain histogram recorder named Mini Rainflow Corder (MRC). In this device the rainflow method [2, 3] is used to decompose complex strain history into discrete strain ranges, related to fatigue damage, and the frequency is stored in the form of a histogram. In contrast, the authors [4] have developed user-friendly software that assists fatigue damage evaluation by analyzing the data obtained using MRC.

In usual fatigue life analysis, S-N curve or strain-life curve is necessary as basic material properties under cyclic loading. These relationships are usually obtained in a constant amplitude load or displacement fatigue test where number of cycles to complete fracture of specimens or initiation of a crack with a definite length is examined. In general such a test requires an expensive testing machine and a skilled task. The purpose of this study is to propose a method to construct a basic relation of strain range versus cycles to failure from variable amplitude fatigue data. It is expected that using this method the relation can be obtained in an easy test with an inexpensive testing machine. In addition, this method may have a promising future in that field data reflecting particular effects of load spectrum, geometry of component, environment or other effects unforeseen in laboratory tests can be collected from actual components subjected to service load.

PROPOSED METHOD

Phenomenon on service load fatigue is not sufficiently elucidated now, and a unified method for damage or life assessment has not been established yet though many methods have been proposed. In this study, therefore, only well-known fatigue theories will be used to propose a new method. Usual process of fatigue damage assessment with MRC may be described as follows:

1. Strain range versus cycles to failure relationship (strain-life diagram) is obtained in a constant amplitude fatigue tests using small specimens in laboratory.
2. Varying strain signal is measured through a strain gage in a component or structure in service and the rainflow cycle counting is instantaneously made using MRC. The reduced strain history data is stored in MRC as frequency distribution of strain ranges, in which the maximum strain range is 7000×10^{-6} and it is divided into 256 discrete levels with the interval of 27×10^{-6} strain.
3. Fatigue damage summation is performed using Miner’s rule. If consideration for stress concentration is necessary, local strain approach is employed with the aid of Neuber’s rule, finite element analysis, etc.

If this process is grasped as a forward analysis, it is expected that a strain-life diagram may be reconstructed by a back analysis using output information of material; that is, frequency distribution of strain ranges, number of cycles to failure, etc.

For this analysis several assumptions are made in this study. First, it is assumed that strain-life diagram is stated as

$$\Delta\varepsilon = f(N_f), \tag{1}$$

where $\Delta\varepsilon$ is strain range, N_f is number of cycles to failure and $f(N_f)$ means a function of N_f. Failure means complete fracture, initiation of a crack with a given length, etc. Although the type of function can be arbitrarily determined depending on the purpose of life estimate, in this study the following equation is assumed:

$$\Delta\varepsilon_t/2 = \mathrm{A}\ (2N_f)^{\mathrm{B}}, \tag{2}$$

where A and B are material constants and $\Delta\varepsilon_t/2$ is total strain amplitude. Total strain may be convenient for practical use because it is directly measured by a strain gage. To deal with plastic strain we need the relation between stress and strain, which is usually changed with load cycling.

Second, it is assumed that a complex strain history is reduced by a certain cycle counting technique into a relation between strain range and its frequency. The rainflow algorithm [3] is used in MRC to provide the histogram.

Finally, it is assumed that fatigue damage is cumulated according to Miner’s rule [5] expressed by

$$D = \sum d_i = \sum 2n_i / 2N_i\ , \tag{3}$$

where D : damage, i : level of strain range (i = 1~256 in MRC), d_i : damage fraction at level i, $2n_i$: number of reversals at level i and $2N_i$: number of reversals to failure when strain range of level i is cycled alone. Failure is assumed to occur when the summation of damage fractions, D, equals 1. Load sequence effects and mean stress effects are not taken into account in this analysis.

The following equation is obtained substituting Eqn. 2 into Eqn. 3 and setting D = 1.

$$\sum[2n_i(\Delta\varepsilon_i / 2\mathrm{A})^{-\frac{1}{\mathrm{B}}}]-1=0\,. \tag{4}$$

If giving linear forms by doing Taylor expansion for the left-hand side of Eqn. 4 as a function of variables A and B, the unknown values A and B will be computed by the least squares method. In this analysis, the solution of (A, B) is given as values converged by iteration starting the computation from a given set of initial values (A_0, B_0).

SIMULATION TESTS

The calculation ability of the proposed method can be examined by the following simulation test:

1. The values of (A, B) in Eqn. 2 are initially given as a correct solution.
2. A strain range level is randomly generated on a computer. For this strain range, damage fraction is calculated using Eqn. 2 and then summed up with the previous damage D using Eqn. 3. The number of reversals for the corresponding level is increased by 1.
3. If this work is continued until D = 1, a histogram of strain range frequency will be obtained.
4. For two or more sets of histograms obtained as above, a solution of (A, B) is given by applying the proposed method. The solution is compared with the correct solution initially given.

The calculable region of (A, B) is dependent on the initial values (A_0, B_0). The best values of (A_0, B_0) were obtained to be (0.002, – 0.6) through trial and error. Figure 1 shows the calculable region examined using (A_0, B_0) = (0.002, – 0.6) for the worst condition that the region becomes narrowest. The region is appeared to be broad enough to cover the regions corresponding to material constants of the Coffin-Manson relationship and Basquin's equation expected for a wide variety of real materials.

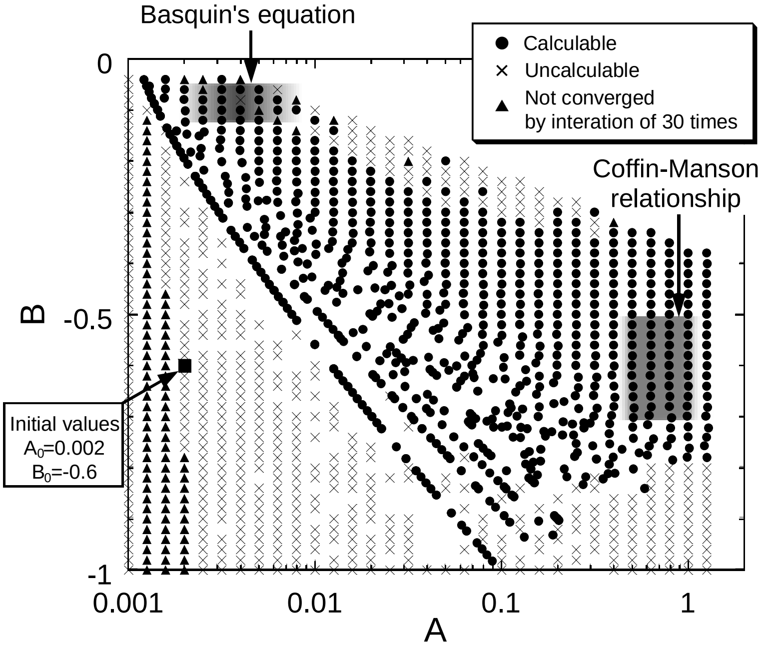


Figure 1 : Calculable region of (A, B) examined using (A_0, B_0) = (0.002, – 0.6)

FATIGUE TESTS

Material used is annealed 0.37% carbon steel. The chemical composition (wt %) is 0.37C, 0.21Si, 0.65Mn, 0.019P, 0.017S, 0.13Cu, 0.06Ni, 0.14Cr. The lower yield strength is 328 MPa, the tensile strength is 586 MPa, the reduction of area is 50.7 % and the Vickers hardness is 160. Specimen geometry is shown in Figure 2. The stress concentration factor K_t is 1.10. The specimens were electropolished to remove a surface layer of 40 μm in diameter. Axial load fatigue tests were conducted under load control using a 100 kN digitally-controlled servo-hydraulic testing machine, operating at 1-10 Hz. The alignment of specimen and machine axes was adjusted to minimize bending for each test using four strain gages at the positions indicated in Figure 2. The nominal stress amplitudes in constant load tests were 270, 290, 310 and 330MPa. The tests were performed under fully-reversed loading (R= –1). Stress is the nominal stress defined by the cross section area at notch root. It follows that the true stress has a tensile mean stress even for R = –1. The variable amplitude tests were carried out by block load tests using the stress histories shown in Figure 3. Note that there are three hysteresis loops with different stress and strain ranges in a block, as shown by schematic relationship between stress and strain. During the fatigue test the strain frequency was measured using an MRC. Notch root strain was numerically calculated using Neuber's rule, stress concentration factor K_t and the cyclic stress-strain curve estimated from the monotonic curve. Plastic replicas were taken during the tests to monitor crack growth.

RESULTS AND DISCUSSION

Figure 4a shows an example of the histogram of strain range frequency measured using MRC in block load tests. Figure 5 shows the strain-life diagrams constructed based on the definition that D = 1 when a 1000 μm

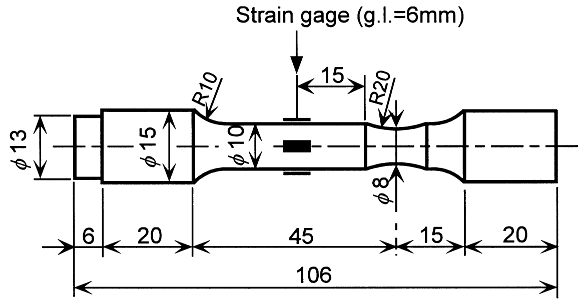
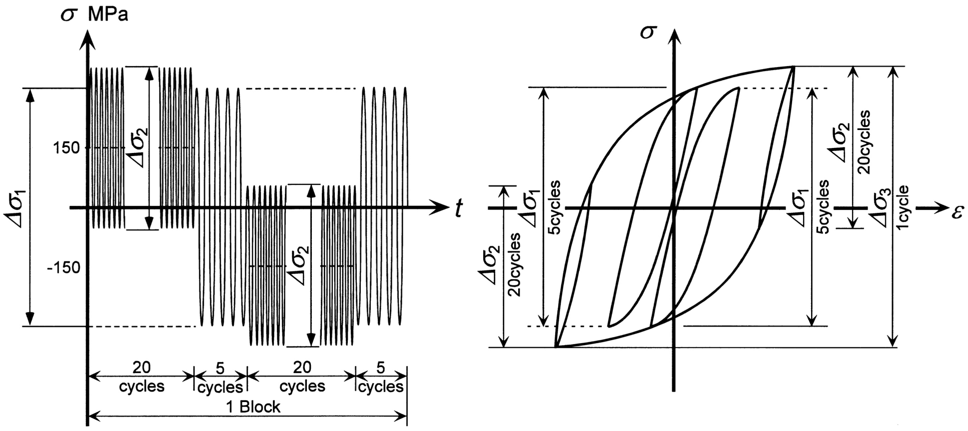


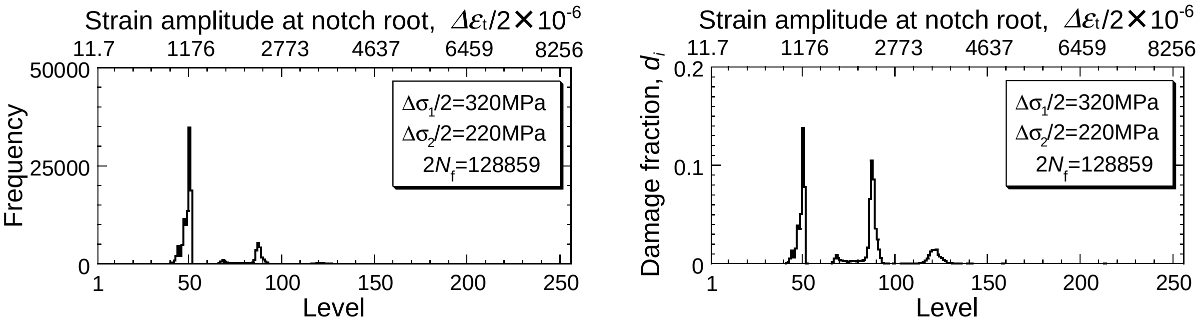
Figure 2 : Specimen geometry



History	A	B	C	D
$\Delta\sigma_1/2$ (MPa)	340	320	300	280
$\Delta\sigma_2/2$ (MPa)	240	220	200	180

Figure 3 : Condition of variable amplitude load tests

crack is initiated. For small specimens shown in Figure 2, this definition is virtually equivalent to that for complete fracture. In this analysis damage below the fatigue limit was taken into account. The curve for variable amplitude is very close to that for constant amplitude. Table1 shows damage ratios for fatigue test results, which were computed using reconstructed curves. All ratios fall between 0.744 and 1.16. It may be concluded that the strain-life diagrams are well constructed from the variable amplitude data. Figure 4b shows histogram of damage fraction for the same data. The histogram has three populated groups, which may correspond to strain ranges of three hysteresis loops, see Figure 3. The scatter in the distribution may be mainly due to cyclic softening and hardening of material.



(a) Strain range frequency (b) Damage fraction

Figure 4 : Example of histogram obtained in block load test

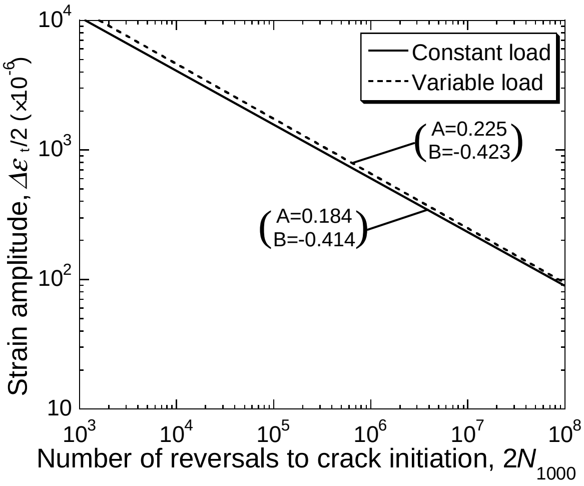


Figure 5 : Constructed strain-life diagrams

TABLE 1

DAMAGE RATIOS OF FATIGUE TEST RESULTS

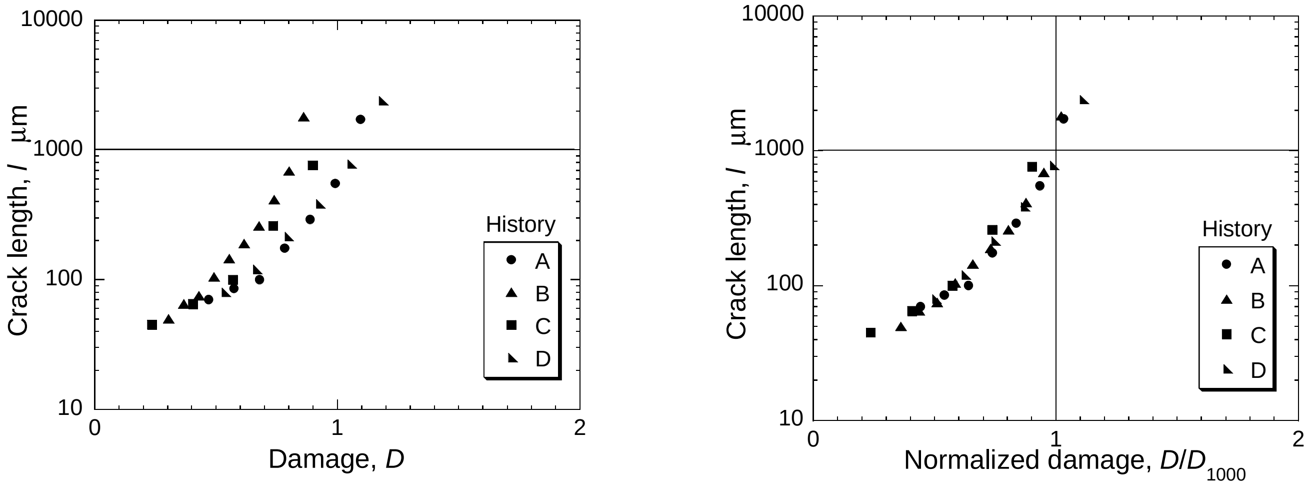
(a) Constant amplitude test

Stress amplitude (MPa)	330		310		290		270	
Damage ratio	1.03	1.04	1.16	0.744	0.795	0.842	1.08	1.14

(b) Variable amplitude test

History	A	B	C	D
Damage ratio	1.06	0.842	0.995	1.07

The present method can be used also for the case that $D \neq 1$. Therefore, if certain physical damage observed in an actual component is correlated with D in the midst of its fatigue process, the data of strain-life relationship will be collected directly from the component subjected to service load. Although the term "fatigue damage" is often used in different ways by different researcher [6], in this study the length of a main crack leading the specimen to fracture was used as a measure of fatigue damage. Figure 6a shows the relationship between surface crack length and damage D for the case of variable amplitude load tests. The value of D for a given crack length was calculated using the reconstructed strain-life curves and the strain range histogram corresponding to the length. Figure 6b shows the relationship between crack length and damage D normalized by the damage D_{1000} corresponding to the length of 1000 μm. All plots are within a narrow band. The values of D for cracks observed in a component may be defined using this relation.



(a) Damage ratios

(b) Damage ratios normalized by D_{1000}

Figure 6 : Relationship between surface crack length and damage D

Several methods for constructing an S-N curve from variable amplitude fatigue data have previously been proposed. Nakamura et al. [7], Dowling [8] and Kikukawa et al. [9] used equivalent stress or plastic strain range to correlate the data between variable and constant amplitude loading. It is assumed in those methods that the value corresponding to B in Eqn. 2 equals the value obtained in constant amplitude fatigue tests. In contrast, the present method does not require such assumption. In addition it is applicable for the case that Eqn. 1 has two or more material constants.

CONCLUSIONS

A method for constructing a strain-life relationship from variable amplitude data was proposed. Computer simulation and fatigue tests were carried out to assure that the method is applicable to a wide variety of materials and complex strain history data. An idea for applying this method to an actual component subjected to service load was also presented.

REFERENCES

1. Murakami, Y., Morita, T. and Mineki, K., (1997) *J. Soc. Mater. Sci., Japan*, **46**, 1217.
2. Matsuishi, M. and Endo, T., (1968) *Preliminary Proc. of the Kyushu District Meeting, Japan Soc. Mech. Engrs*, pp. 37-40.
3. Anzai, H. and Endo, T., (1979) *Int. J. Fatigue*, **1**, 49.
4. Miyata, H. and Endo, M., (1999) *Preliminary Proc. of the Kyushu District Meeting, Japan Soc. Mech. Engrs*, **998**-3, pp. 25-26.
5. Miner, M. A., (1945) *J. Appl. Mech., Trans. ASME*, **12**, A159.
6. Lankford, J., Davidson, D. L., Morris, W. L. and Wei, R. P. (Eds.), (1982), *Fatigue Mechanisms: Advances in Quantitative Measurement of Physical Damage, ASTM STP 811*, ASTM, Philadelphia.
7. Nakamura, H. et al., (1973) *Trans. Japan Soc. Mech. Engrs*, **39**, 472.
8. Dowling, N. E., (1988) *Int. J. Fatigue*, **10**, 179.
9. Kikukawa, M., Jono, M. and Song, J., (1972) *J. Soc. Mater. Sci., Japan*, **21**, 753.