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COMPARISON OF EXPERIMENT AND THEORY FOR CRACK TIP FIELDS IN DUCTILE SINGLE CRYSTALS

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ABSTRACT

We compare extremely detailed experimental studies of "plane strain" crack tip deformation fields for two symmetric crack orientations in a ductile single crystal of low hardening copper with new asymptotic analytical solutions employing single crystal elasto-plasticity. The experimental studies were motivated by the pioneering analysis of Rice [1] of crack tip fields in nonhardening ductile single crystals. Rice showed that, in contrast to crack tip fields in isotropic (polycrystalline) ductile materials, the single crystal crack tip fields consist of angular sectors of constant Cartesian stress components that are joined by rays of stress and radial displacement discontinuity. Rice's solutions assume yield is attained at *all* angles about the crack tip, which *requires* radial shear bands of both slip and kink type. The new experiments confirm several of Rice's predictions; however, there are several important differences. Our experimental observations and measurements show: an *absence* of kink-type shear bands; some sector boundary locations differing significantly from Rice's predictions; different near-tip fields for a 90° crack orientation change (in contrast to the theoretical prediction); and angular regions exhibiting no evidence of plastic slip and very low strain (as measured by Moiré microscopy). Based on these observations, we have derived new asymptotic analytical solutions that relax Rice's assumption that yield is attained at all angles about the crack tip; this permits derivation of solutions that do not require rays of kink-type shearing and that possess near-tip sub-yield angular sectors. Direct comparison of these solutions with the experimental observations and measurements show that the new asymptotic solutions agree quite well with the experiments. The result is enhanced fundamental understanding of ductile single crystalline crack tip fields as well as quantitative predictive capability.

KEYWORDS

crack tip fields; single crystal plasticity; optical interferometry; asymptotic analysis.

INTRODUCTION

Nonlinear fracture mechanics in its current state deals largely with isotropic materials; a main application is ductile polycrystalline materials whose grains are sufficiently small and randomly oriented that a macroscopic isotropic continuum theory suffices. However, structural components are increasingly being fabricated in single crystal form, for reasons including the avoidance of grain boundary defects and superior creep resistance. The fracture behavior of ductile single crystalline materials is as yet not well understood. Also, the fracture of more commonly employed polycrystalline materials involves, at the microscale, crack growth through (single crystalline) grains or along grain boundaries. To understand and predict the fracture behavior of such materials from a fundamental perspective, it is necessary to understand and be able to characterize the stress and deformation fields present at the tip of a crack in a single crystal.

In a pioneering paper, Rice [1] published an asymptotic study of crack tip stress and deformation fields for plane strain tensile cracks in elastic-ideally plastic single crystals. He showed that, unlike crack tip fields in *isotropic* elastic-ideally plastic materials, the single crystal crack tip fields in at-yield regions are comprised exclusively of angular sectors of constant Cartesian components of stress. For the stationary crack case, he showed that these are joined necessarily by stress and displacement discontinuities when, as he assumed, the stress state is at yield at all angles about the crack tip. Rice's solutions address the specific cases of a crack on the (0 1 0) plane pointing in the [1 0 1] direction, and a crack on the (1 0 1) plane pointing in the [0 1 0] direction, for both FCC and BCC crystals that flow according to the critical resolved shear stress (Schmid) criterion. Saeedvafa and Rice [2] extended Rice's [1] asymptotic analysis to incorporate Taylor power-law hardening.

The analyses of Rice and co-workers just summarized employed continuum elastic-plastic modeling of ductile single crystals, and analyzed crack tip fields via asymptotic analysis within an infinitesimal displacement gradient formulation. Rice et al. [3] performed full-field "small strain" numerical finite element calculations, using continuum crystal modeling; these solutions were in accord with Rice's [1] asymptotic analytical ones, confirming that the latter have a significant radius of validity. Mohan et al. [4] and Cuitino and Ortiz [5] employed finite deformation continuum theory to analyze (numerically) these crack tip fields, accounting for the full three-dimensional crystal geometry.

Several extremely careful and fascinating experimental studies of "plane strain" tensile crack tip fields have recently appeared, notably those of Shield and Kim [6], Shield [7], Crone and Shield [8] and Bastawros and Kim [9]. Also, very recently, numerical studies of plane strain tensile crack tip fields using discrete dislocation dynamics to model ductile materials containing substantial initial distributions of dislocations and dislocation sources have been conducted, the most recent by Van der Giessen et al. [10].

The solutions of Rice [1] involve rays of concentrated plastic shearing, of both slip (parallel to slip systems) and kink (perpendicular to slip systems) type, emanating from the crack tip. The recent experimental studies confirm Rice's predictions of the presence of discrete sectors near the crack tip, and also exhibit rays of slip-type concentrated plastic shearing. However, they do not appear to show kink-type concentrated plastic shearing. This motivated Drugan [11] to construct asymptotic solutions for stationary crack tip fields in elastic-ideally plastic ductile single crystals that do not contain kink-type rays of concentrated plastic shearing.

In the present paper, we provide direct comparisons of certain of Drugan's [11] solutions to the experimental measurements of Crone [12] and Crone and Shield [8] for cracks having two different orientations in FCC ductile single crystalline copper.

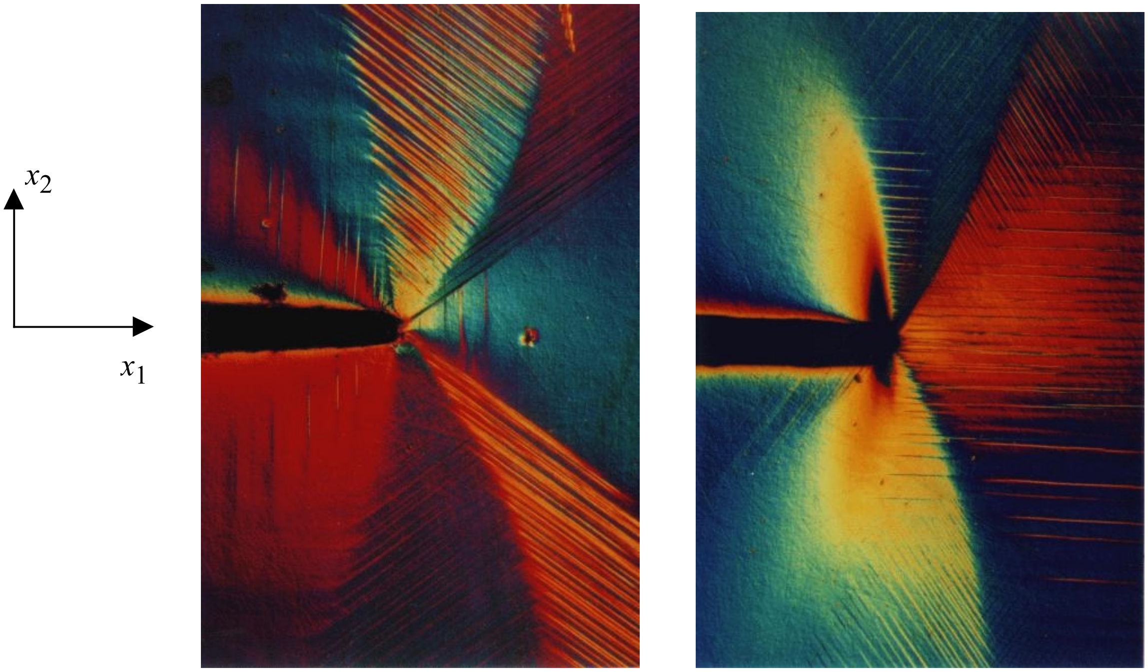
COMPARISON OF EXPERIMENT AND THEORY

Experimental Results

Single crystals of copper were grown with the Bridgman technique [13] and prepared as four-point-bend specimens [8]. The two crystallographic orientations were investigated with the interferometric method of Moiré microscopy to obtain detailed information about the surface strains [14] and the optical method of differential image contrast (DIC) to obtain general information about the surface deformations. Samples having a notch on the (101) plane and its tip along the $[10\bar{1}]$ direction are identified as Orientation I, while samples with a notch on the (010) plane and its tip along the $[\bar{1}01]$ direction are identified as Orientation II.

The formation of persistent strain localization bands, observed optically on the sample surface during testing and after unloading as shown in Figure 1, provide insight into the active slip systems within a sector and delineate the sector boundary angles. Slip band observations, refined by detailed Moiré microscopy strain measurements, suggest that certain sectors of the near-tip field may remain elastic. Although the details of the Moiré results [7, 8] are not presented here, they inform the discussion that follows.

The theoretical slip plane trace angle between the slip plane trace and the x_1 axis (see Figure 1) is of particular interest for comparison to the persistent strain localization band angles observed in experiments. Because FCC copper slips on {111} planes in <110> directions, the slip plane trace angles are 35°, 90°, and 145° from the x_1 axis for Orientation I. The related kink-like shear trace angles are 55°, 125°, and 180°. The slip and kink angles for Orientation II are interchanged. All of the persistent strain localization bands observed on the sample surface occur at orientations corresponding to the plane strain slip systems available. Thus all of the persistent strain localization bands observed are categorized as slip bands. No evidence of kink-type shear bands was observed in either orientation. If kink is not exhibited in these orientations, then the angles at which two plastic sectors may adjoin are greatly diminished.



Orientation I

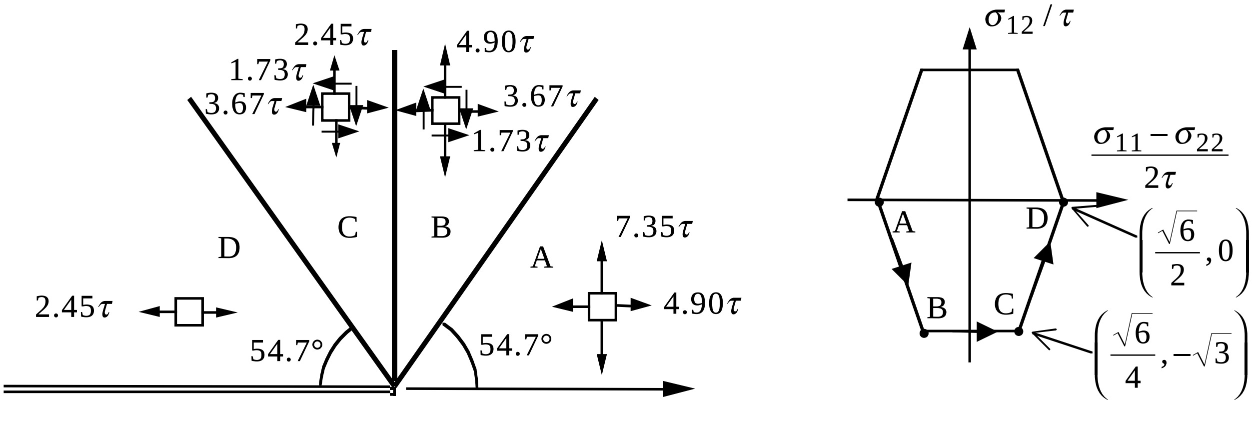
Orientation II

Figure 1. Optical micrographs were taken using a DIC microscope of FCC copper bend samples with Orientation I (left) [7] and Orientation II (right) [8]. The notch enters from the left. The lines in the sectors emanating from the notch tip are slip lines. Changes in color/shade indicate small changes in surface tilt. The black regions very near the tip are regions with larger out of plane deformation and thus larger tilt. The field of view is 2.7 by 1.8 mm.

Figure 1 shows slip bands at 90° (for Orientation I) and 180° (for Orientation II) occur ahead of the notch. However, Rice [1] proved that in nonhardening material, plastic sectors must have constant Cartesian stress components, and since we must have $\sigma_{12} = 0$ on $\theta = 0$ (i.e., the crack plane ahead of the tip) from symmetry, one expects $\sigma_{12} = 0$ in the entire sector ahead of the crack. However, it was observed during the experiments that these slip bands did not form until the final stages of loading; therefore, we hypothesize that they are the result of material hardening. Indeed, the results of Saeedvafa and Rice [2] show that even a low level of hardening permits nonzero σ_{12} everywhere in the front sector except at $\theta = 0$. Thus, for comparison to the perfectly plastic crack tip field solutions, the slip bands directly ahead of the notch will be ignored.

Rice's Fully Plastic Asymptotic Solutions

Rice's [1] solutions are illustrated in Figure 2 for the crack orientations shown in Figure 1. The near-tip sector assembly, the stress and displacement jump locations, and the stress field, are all identical for the two orientations. The solutions have rays of stress and displacement discontinuity at the following angles, measured counterclockwise from the crack plane ahead of the crack tip: θ = 54.74°, 90°, 125.3°. Importantly, the character of the deformation fields differs between the two orientations: In the Orientation II case, the rays at θ = 54.74° and 125.3° are sites of radial displacement discontinuities (concentrated plastic shearing) produced by slip on the crystal's slip plane traces, while θ = 90° is the site of a radial displacement discontinuity produced by a kinking mode of concentrated shear, since this direction is perpendicular, not parallel, to one of the crystal's slip plane traces. The situation is reversed for Orientation I: θ = 90° is the site of a slip-type concentrated shearing, while θ = 54.74° and 125.3° are sites of kink-type concentrated shearing. A comparison of Figure 2(a) with Figure 1 shows that neither of the experimental images appears to match completely Rice's [1] solution. [Since the experiments used notches, sharp-crack asymptotic solutions are expected to apply outside a radius of 2-3 times the notch radius.] Orientation II does seem to agree with Rice's solution for, say, $0 \leq \theta < 90°$, but the experimental image shows slipping on both of Sector B's slip line traces significantly beyond θ = 90°, as opposed to changing to Sector C behavior at θ = 90° as predicted by the analytical solution. Orientation I looks quite different from Rice's solution, except perhaps for the apparent sector boundary at $\theta \approx 54°$, but the experiments show no evidence of kink-type plastic shear there, as the analytical solution predicts.



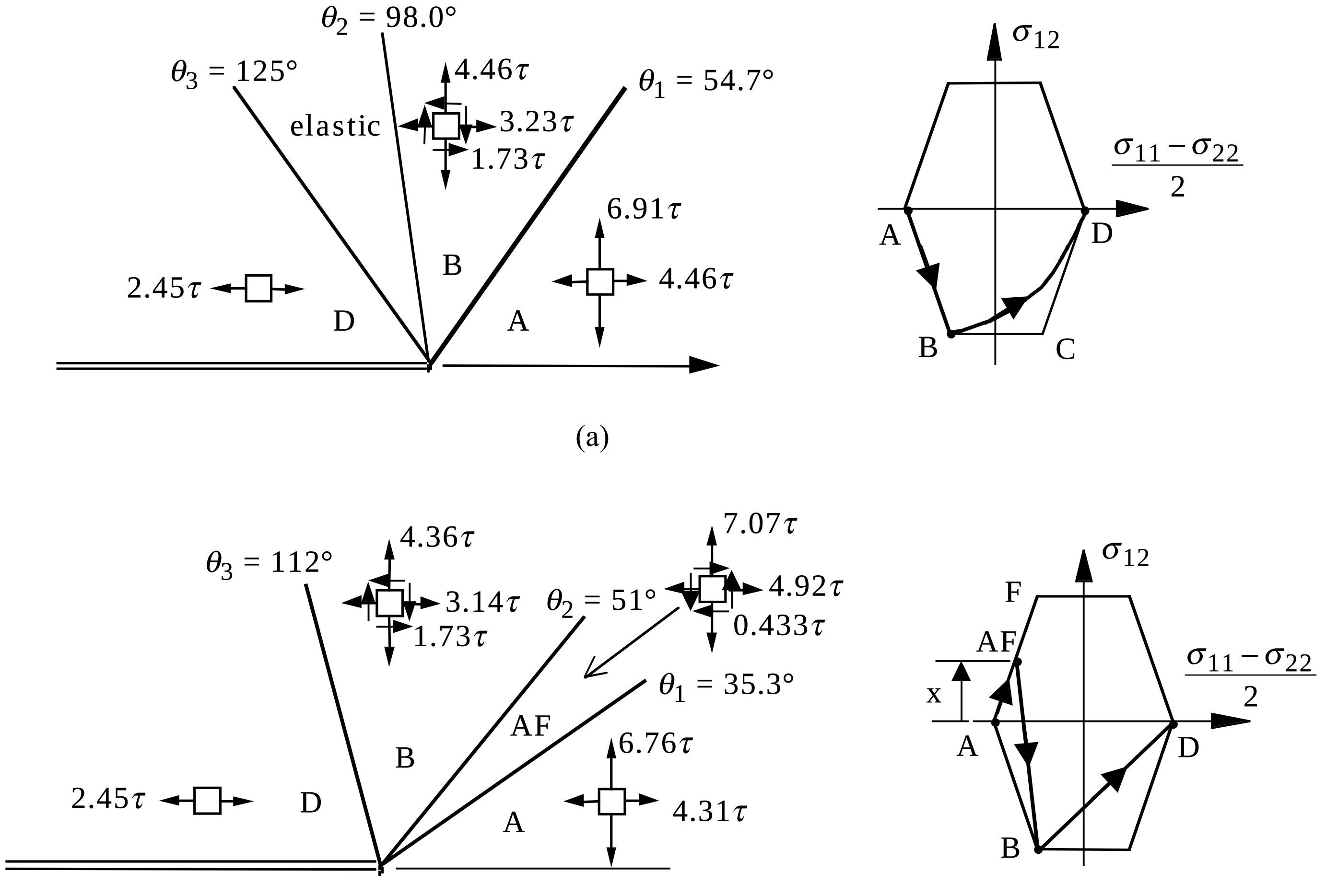
(a) (b)

Figure 2. (a) Rice's [1] fully plastic stationary crack near-tip solutions for FCC crystals for the crack orientations discussed. The dark rays are sites of stress and displacement discontinuity; the angular sectors have constant Cartesian stress components corresponding to the points labeled on the yield surface shown in (b); τ is the critical resolved shear stress on {1 1 1}<110>.

New Elastic-Plastic Asymptotic Solutions Without Kink-Type Shear

Based in part on the observations and comparisons described above, Drugan [11] derived new asymptotic near-tip solutions that do not contain kink-type concentrated plastic shearing for cracks in nonhardening ductile single crystals. In contrast to Rice's [1] solutions, these new solutions necessarily contain sub-yield regions near the crack tip (treated as isotropic for simplicity). Here we select two of Drugan's [11] solution families that appear to agree well with the experimental images of Figure 1. These solutions are illustrated in Figure 3. Significant constraints exist on permissible near-tip elastic-plastic solutions; see Drugan [11].

We begin with Orientation II, as this appears to agree reasonably with Rice's [1] solution for $0 \leq \theta < 90°$; however, for $\theta > 90°$, the experiments of Crone and Shield [8] clearly show the persistence of a Type B sector beyond 90°, after which there is an angular span of no apparent slip activity, and then finally some single slipping adjacent to the crack flank, as can be seen in the lower half of Figure 1. A simple solution of Drugan's [11] that is in accord with these observations is that illustrated in Figure 3(a). A constant stress at-yield sector of A type directly ahead of the crack joins by a stress and slip-type displacement jump to a B-type sector at $\theta = 54.7°$; this B-type sector extends until 98°, at which angle it joins, via full stress and displacement continuity, a sub-yield elastic sector. This extends to $\theta = 125°$, where it joins, with full stress and displacement continuity, a D-type sector, which persists to the crack flank. The sub-yield sector has Cartesian stress components that vary with angle, as illustrated in the stress plane. (This is actually one extreme member of a *family* of closely-related solutions, in which the angular extent of the elastic sector decreases until it becomes an elastic stress jump at $\theta = 112°$, which is the other extreme member of the family; see Drugan [11].)



(a)

(b)

Figure 3. Drugan's [11] solutions that show good agreement with the experiments on:
(a) Orientation II; (b) Orientation I.

Orientation I differs significantly from Rice's [1] solution even ahead of the crack tip. As Figure 1 illustrates, the Crone [12] and Shield [7] measurements indicate a slip-type sector boundary at $\theta \approx 35°$, another sector boundary at about $\theta \approx 54°$, and a third at about $\theta \approx 111°$. A solution family of Drugan's [11] that agrees well with this experimental image is shown in Figure 3(b). This has a Sector A directly ahead of the crack tip, joined by a slip-type stress and displacement jump at $\theta = 35.3°$ to a Sector AF (whose stress state lies somewhere along the yield surface segment joining vertices A and F). This sector then joins a B-type plastic sector via an elastic stress jump. We illustrate the specific example having $x = 1/4$ in the stress plane of Figure 3(b), for which the solution shows $\theta_2 = 51°$. Sector B then persists until $\theta = 112°$, at which angle it joins, via an elastic stress jump, a sector of D type. See Drugan [11] for details of the calculation. Figure 3 shows the stress states in each of the near-tip constant stress sectors. We emphasize that the experimental results do not provide definitive information about the near tip field behavior beyond about $\theta \approx 111°$, and thus that other asymptotic analytical solutions are possible having, for example in the solution of Figure 3(b), the material from the Sector B boundary all the way to the crack flank below yield. Further discussion of the possibilities, and constraints on these, is given in Drugan [11].

CONCLUSIONS

The combined investigation of new asymptotic analytical solutions and detailed experimental studies of "plane strain" crack tip deformation fields for symmetrically oriented cracks in a ductile FCC single crystal has produced interesting and encouraging new results. The *absence* of kink-type shear bands, and angular regions exhibiting no evidence of plastic slip and very low strain, in the experimental observations and measurements have guided the development of new analytical solutions employing single crystal elasto-plasticity. Direct comparison of these solutions with the experimental observations and measurements show that two new solutions agree quite well with the experiments on two crack orientations. Thus it appears that asymptotic crack tip field analysis within a "small strain" formulation of continuum single crystal elastoplasticity is capable of characterizing actual single crystal crack tip fields. The result is enhanced fundamental understanding of ductile single crystalline crack tip fields and quantitative predictive capability.

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