BIFURCATION ASSESSMENT OF MIXED CRACK IN ELASTIC-PLASTIC MATERIALS

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ABSTRACT

It is well kwon that the near-tip asymptotic stress field of an elastic-plastic crack in plane strain can be defined by two parameters, the *J*-integral and the plastic mixity parameter M^p (Shih, [5]). These two parameters for general yielding cracks can rapidly be calculated using a recently developed method based on the calculation of two associated *J*-integrals, J^{*I} and J^{*II} (Li, [3]). Many experimental studies showed that the crack growth under mixed mode loading can take place either in cleavage manner or in slip manner (Tohgo and Ishii [1], Aoki et al. [2]). Therefore, it is necessary to study the competition between these two kinds of crack growth in the frame of the J-M^p system.

In this paper, we carry out detailed numerical calculations of crack-tip fields in elastic-plastic materials in order to assess the bifurcation angle based on the calculations of the parameters *J* and M^p. Some specimens used by Aoki et al. [2] have been analysed by finite element modelling. The loading varies in order to produce fracture range from Mode I to Mode II. By using the numerical method developed recently (Li et al [3][4]), the *J*-integral and the plastic mixity parameter M^p were calculated and the theoretical HRR near-tip fields were obtained. The bifurcation angles were estimated then according to the maximum $\sigma_{\theta\theta}$ (circonferential opening stress) and the maximum $\tau_{r\theta}$ (shear stress) rules. The initial crack in the specimen was supposed to extend a small length in the two possible bifurcation angles. The competition between the cleavage growth and the slip one of cracks is discussed comparing with experimental tests carried out by Aoki et al. [2].

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ABSTRACT

Many experimental studies showed that the crack growth could take place either in cleavage manner (Tensile-type fracture) or in slip manner (Shear type fracture), see Tohgo and Ishii [1], Aoki *et al*. [2]. The purpose of this work is to study the competition between these two kinds of crack growth. We carry out the detailed numerical calculations of crack-tip fields in order to assess the bifurcation and propagation of the cracks in elastic-plastic materials. Some specimens used by Aoki et al. [2] have been analysed by finite element modelling. The loading varies in order to produce complete fracture range from Mode I to Mode II. By using the numerical method developed recently, the *J*-integral and the plastic mixity parameter M^p were calculated and the theoretical HRR near-tip fields were obtained. The bifurcation angles were estimated then according to the maximum $\sigma_{\theta\theta}$ (circonferential opening stress) and the maximum $\tau_{r\theta}$ (shear stress) rules. The initial crack in the specimen was supposed to extend a small length in the two possible bifurcation angles. The competition between the cleavage growth and the slip one of cracks is discussed comparing with experimental tests carried out by Aoki *et al.* [2].

KEYWORDS

crack bifurcation, crack propagation, mixed mode, stress analyses, *J*-Integral, energy release rate, elastic-plastic behaviour

1. INTRODUCTION

An elastic-plastic material can present two failure mechanisms. The first one, cleavage failure, is due to maximum circonferential opening stress $\sigma_{\theta\theta}$ or to the maximum energy release rate G (which can be presented as the *J*-integral). The failure occurs when the maximum $\sigma_{\theta\theta}$ (at certain distance r_c from the crack tip), G or J reach their critical values, respectively, σ_c, G_c or J_c. The second failure mechanism, slip failure, is due to very high local strains involving slip bands in the direction of the maximum shear stress $\tau_{r\theta}$. The failure mechanism occurs when $\tau_{r\theta}$ at certain distance r_c reaches τ_c. In order to analyse these two failure mechanisms, one has to know the near-tip stress field. The ratio $\frac{\sigma_{\theta\theta}}{\tau_{r\theta}}$ near the crack tip compared to $\frac{\sigma_c}{\tau_c}$ will determine the type of failure. Shih [5] showed that the near-tip asymptotic stress field of an elastic-plastic crack in plane strain can be defined by two parameters, the *J*-integral and the plastic mixity parameter M^p. A numerical method has been recently developed in order to determine these two parameters (Li *et al* [3][4]). This method is founded on the basis of the calculation of two associated *J*-integrals, J^{*I} and J^{*II} by introducing two auxiliary fields, a symmetric one and an anti-symmetric one with respect to the crack axis. Some studies have shown the validity of this method [4][6].

In the present work, we determine, using our numerical method, the near-tip stress fields in the specimens tested under mixed loading by Aoki et al. [2]. The experimental results are interpreted by the use of calculation results. The competition between the tensile-type fracture and the shear type fracture is discussed.

2. METHOD OF EVALUATION OF THE PARAMETERS *J* AND M^p

Shih [5] showed that, for a mixed mode crack lying in a power-law hardening material, the stresses, strains and displacements fields near the crack tip are dominated by the HRR singularity, and can be characterized by two parameters, the *J*-integral and a mixty parameter M^p. The later is defined as follows:

$$M^p = \lim_{r \to 0} \frac{2}{\pi} \tan^{-1} \left| \frac{\sigma_{\theta\theta}(\theta = 0)}{\sigma_{r\theta}(\theta = 0)} \right| \qquad (1)$$

The method to evaluate the parameter M^p has been reported in [3] [4] that we resume briefly. First, one defines an associated *J*-integral, the *J**-integral as follows:

$$J^* = \int_{\Gamma} \left(w^* n_1 - \sigma_{ij} n_j \frac{\partial u^*_i}{\partial x} \right) ds \qquad (2)$$

where Γ is an arbitrary path around the crack tip; σ_{ij} are the stress components of the actual field; u^*_i are the displacement components of an auxiliary field; w^* is the associated energy density defined as:

$$dw^* = \sigma_{ij} d\varepsilon^*_{ij} \qquad (3)$$

The auxiliary field can be constructed in terms of the actual field. Following the approach of Ishikawa *et al.* [8], one can decompose the actual field into symmetrical and anti-symmetrical parts with respect to the crack axis:

$$u^{*M}_i(x,y) = \frac{1}{2}\left[u_i(x,y) + (-1)^{i+M} u_i(x,-y) \right] \quad i = 1,2;\ M = I, II \qquad (4)$$

With these two auxiliary fields, we obtain two associated integrals J^{*I} and J^{*II}. It is clear that J^{*I} and J^{*II} are path independent. An equivalent elastic mixity parameter M^{*e} can be defined from J^{*I} and J^{*II}, namely:

$$M^{*e} = \frac{2}{\pi} \tan^{-1} \sqrt{\frac{J^{*I}}{J^{*II}}} \qquad (5)$$

By carrying out an asymptotic analysis near the crack tip, one can find the relationship between the M^{*e} and M^p. This relationship was given in [3]. Moreover, one can calculate the *J*-integral from J^{*I} and J^{*II}:

$$J = J^{*I} + J^{*II} \qquad (6)$$

This method is valid for any yielding cases.

3. EXPERIMENTAL RESULTS

An experimental investigation has been carried out by Aoki *et al* (1990 [2]). They used A5083-O aluminium alloy that the mechanical properties are given in Tab.1. A compact-tension-shear specimen attached to a special device, developed by Richard and Benitz [7], was employed (Fig.1). A mixed or pure mode fracture is obtained by applying the load in different hole numbers. A Mode I loading was performed using the No. 1 and No.1' holes in the loading device shown in Fig. 1.(b), and a Mode II loading was carried out using the No. 7 and No. 7' holes. A fatigue crack was introduced up to $a_0 / w \approx 0.5$ (a_0 is pre-cracked length, w is specimen width) (Fig.1.(a)).

Young's modulus E (GPa)	68.65
0.2% Yield strength σ_0 (MPa)	142.1
Tensile strength σ_B (MPa)	308.6
Ultimate strength σ_f (MPa)	284.2
Reduction of area Ψ (%)	35.01

Tab.1 Mechanical properties of aluminium alloy A5083-O

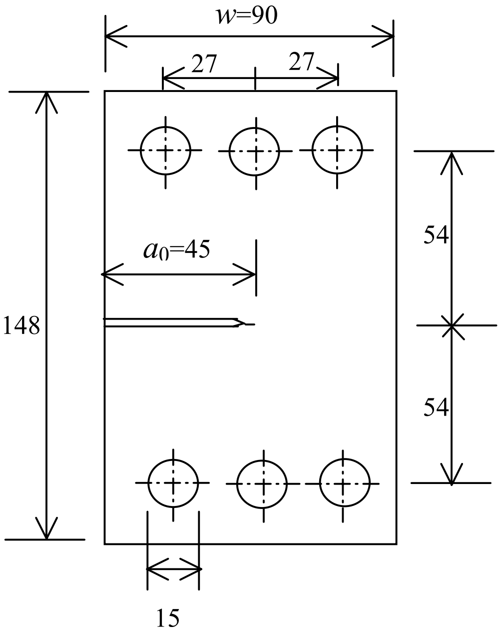

Fig.1.(a) Configuration of the specimen

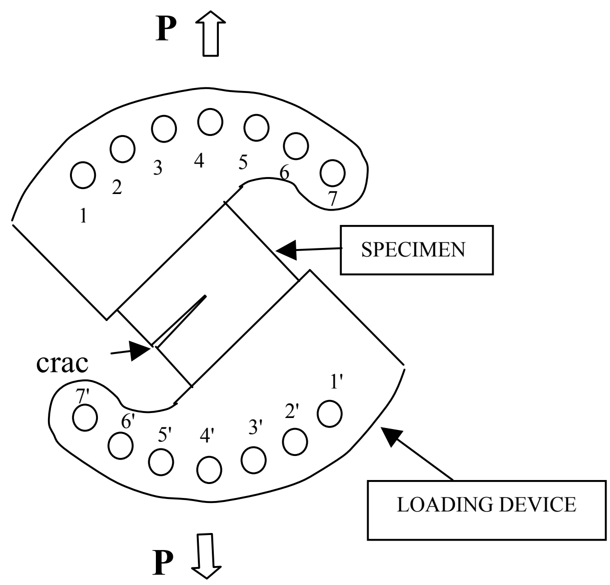

Fig.1.(b) Device for mixed mode loading

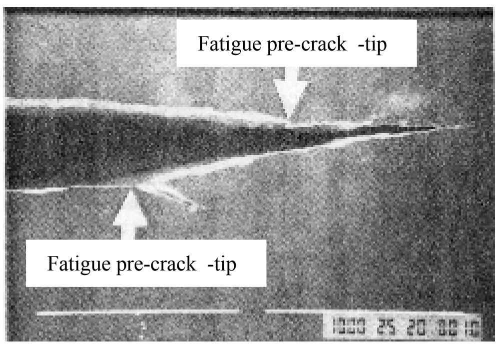

Fig. 2 Crack initiates in two directions (loading hole No. 6)

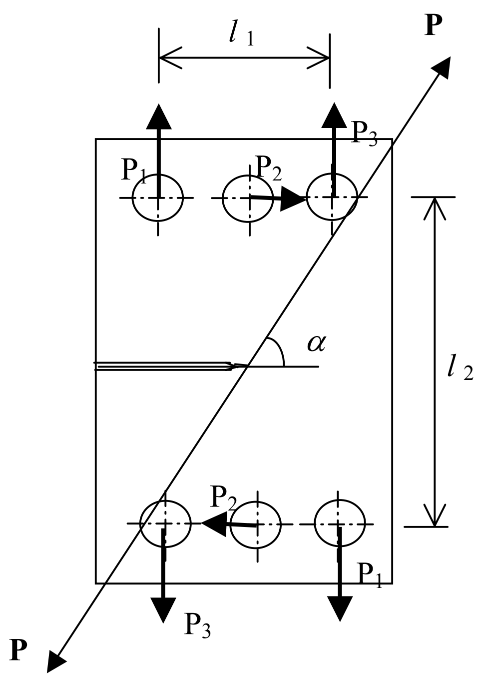

Fig. 3 Loads applied to a specimen

It was found that for the Mode I specimen (loading hole No. 1), the crack initiated from the centre of the blunted tip of the fatigue pre-crack and extended to the direction perpendicular to the loading axis. And for the Mode II specimen (loading hole No. 7), a crack due to shear type fracture is observed to extend in a direction almost parallel to the fatigue pre-crack surface. Under mixed mode loading (loading hole No. 3-6), cracks due to shear type fracture (in slip manner) initiate at the sharpened corner of the pre-crack tip near the surfaces of a specimen, and then another crack due to tensile-type fracture (in cleavage manner) occurs at the midthickness. It develops more rapidly than the shear cracks and causes final fracture of the specimen. The cracks due to tensile-type fracture extend in the direction perpendicular to the loading axis (Fig.2).

4. NUMERICAL ANALYSES

Elastic-plastic finite element analyses are carried out by using a general-purpose finite element program, named CASTEM 2000 developed by CEA (Commessariat à l'énergie atomique – France). The analysis is based on small strain assumption and employs the flow theory of plasticity. Eight-nod and six-nod elements were used in the calculations. The finite element mesh is shown in Fig. 4. The loading conditions were approximated in such a way that the loads P_1, P_2 and P_3 were applied to the holes in the specimen as shown in Fig. 3. The magnitude of load was determined from following equilibrium equations:

$$P\cos\alpha = P_2, \qquad P\sin\alpha = P_1 + P_3, \qquad P_1 l_1 + P_2 l_2 = P_3 l_3 \qquad (7)$$

Where P is the load applied to the loading device shown in Fig.1 (b), and the lengths l_1, l_2 and the loading angle α are defined as shown in Fig. 3

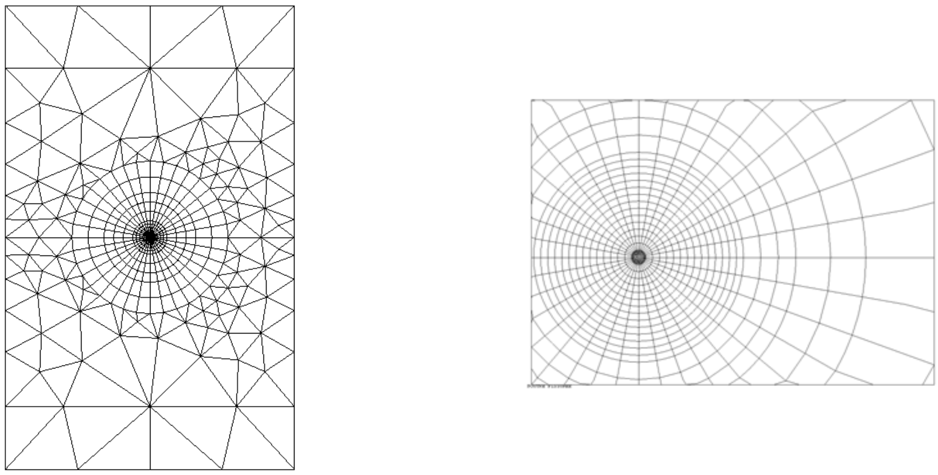

Fig. 4 Finite element mesh

5. NUMERICAL RESULTS AND DISCUSSIONS

After calculation of the two associated J-integrals J^{*I}, J^{*II} and the plastic mixity parameter M^p, the theoretical HRR near-tip stress fields are obtained for each specimen. The distributions of the stress components $\sigma_{\theta\theta}$, $\tau_{r\theta}$ near the crack tip and the J-integral value allow us to analyse the bifurcation angle and the propagation of the cracks.

According to bifurcation criteria, the tensile-type crack will propagate in the direction of maximum $\sigma_{\theta\theta}$, noted θ_σ (angle of bifurcation due to tensile-type fracture) and the shear type crack will propagate in the direction of maximum $\tau_{r\theta}$, noted θ_τ (angle of bifurcation due to shear type fracture). The Tab. 2 shows the numerical results of the specimens under different loading: loading angle $\alpha = 0°$, 15°, 45° and 90°. $\sigma_{\theta\theta}$ and $\tau_{r\theta}$ are the maximum stress components at a distance $r = 2J/\sigma_0$ from the crack tip, J being the J-integral and σ_0 the yield stress.

Hole No.	7	6	4	1
Load P (N)	4850	4900	3500	2900
α (°)	0°	15°	45°	90°
$\sigma_{\theta\theta}$ (MPa)	395	467	426	497
$\tau_{r\theta}$ (MPa)	292	266	177	153
$\sigma_{\theta\theta}/\tau_{r\theta}$	1.35	1.76	2.41	3.24
J^{*I} (N/mm)	0	14.2	14.5	14.6
J^{*II} (N/mm)	46	44.6	3.1	0
J (N/mm)	46	58.7	17.6	14.6
M^p	0	0.38	0.76	1
θ_τ (°)	0	11	40	81
θ_σ (°)	-71	-61	-35	0
Experimental observation	Shear type fracture	Shear type fracture at the beginning and tensile-type fracture after		Tensile-type fracture

Tab. 2 Crack bifurcation angle versus loading angle

From experimental results, the critical values of J_{IC} and J_{IIC} are known. J_{Ic} is equal to 14.6 N/mm (loading angle $\alpha = 90°$). In order to obtain the value of J_{Ic}, the calculation shows that the load P has to be equal to 2900N. Under this load, the stress field is determined and the ratio $\sigma_{\theta\theta}/\tau_{r\theta}$ is than obtained ($\sigma_{\theta\theta}/\tau_{r\theta} = 3.24$). The critical value $\sigma_c = \sigma_{\theta\theta max}$ in this case. Similarly, from $J_{IIc} = 46$ N/mm, the ratio $\sigma_{\theta\theta}/\tau_{r\theta} = 1.35$ is obtained. The critical value $\tau_c = \tau_{r\theta max}$ in this case. The ratio σ_c/τ_c is then equal to 2.16 for this material. The criterion will be:

$$\frac{\sigma_{\theta\theta}}{\tau_{r\theta}} > \frac{\sigma_c}{\tau_c} \Rightarrow \text{tensile-type fracture , and} \qquad \frac{\sigma_{\theta\theta}}{\tau_{r\theta}} < \frac{\sigma_c}{\tau_c} \Rightarrow \text{shear-type fracture}$$

The table 2 shows that the ratio $\sigma_{\theta\theta}/\tau_{r\theta}$ is less important for $\alpha = 0°$ than for 15°, 45° and 90°. This can explain the shear type fracture observed in this case in which the crack bifurcation angle is equal to zero. In the case of $\alpha = 90°$, the ratio $\sigma_{\theta\theta}/\tau_{r\theta}$ becomes very high, so the crack will propagate in the direction of $\theta_\sigma = 0$ in a cleavage manner.

Now, in order to understand the growth behaviour of the crack when the two types of fracture are in competition, we assume that the crack extends simultaneously in two directions (θ_τ and θ_σ)(Fig. 5). This assumption is based on the observation of the experimental results (Fig.2). Let suppose that a_σ and a_τ lengths of small cracks due to tensile-type fracture and to shear type fracture respectively. These cracks are supposed to follow the bifurcation angles determined in Tab. 2.

The J-integrals, the mixity parameter M^p and the maximum stresses $\sigma_{\theta\theta}$ and $\tau_{r\theta}$ for each type of crack are shown in Tab.3 for different extended lengths of crack under the loading corresponding to $\alpha = 45°$ and $\alpha = 0°$. $\sigma_{\theta\theta_\tau}$, $\tau_{r\theta_\tau}$, are the maximum values of $\sigma_{\theta\theta}$ and $\tau_{r\theta}$ near the a_τ crack tip, $\sigma_{\theta\theta_\sigma}$, $\tau_{r\theta_\sigma}$, are maximum values of $\sigma_{\theta\theta}$ and $\tau_{r\theta}$ near the a_σ crack tip. J_τ, J_σ are the J-integral values for the a_τ crack and a_σ crack respectively.

Let suppose that the shear crack a_τ extends (experimental observation) and the extended length is 1mm. At the same time, the tensile crack a_σ initiates and then extends. Giving a_σ crack a extended length of 0.3, 0.4 and 1mm, we analyse the competition of a_τ crack growth and a_σ crack growth.

From Tab.3, one can observe that, in the mixed mode case (α = 45°, loading hole No.4), the value of $\tau_{r\theta_\tau}$ is important when a_σ is small (a_σ=0.3mm) and it decreases as a_σ grows. This can explain the fact that the shear type crack extends at first and stop after (experimental observation). In the other hand, $\sigma_{\theta\theta_\sigma}$ increases when a_σ grows. So the tensile-type crack grows until the final fracture of the specimen. In the Mode II case (α = 0°, loading hole No.7), $\tau_{r\theta_\tau}$ is more important for all length of a_σ. It is way the crack extends always in shear manner until the final fracture of the specimen.

		α =45°			α = 0°	
crack length (mm)	a_τ	1	1	1	1	1
	a_σ	0,3	0,4	1	0,3	0,4
stresses near a_τ (MPa)	$\sigma_{\theta\theta_\tau}$	430	420	310	340	300
	$\tau_{r\theta_\tau}$	170	160	140	270	260
stresses near a_σ (MPa)	$\sigma_{\theta\theta_\sigma}$	550	580	700	370	420
	$\tau_{r\theta_\sigma}$	165	170	180	160	180
J at a_τ (N/mm)	J_τ	14.1	12.4	5	18.5	17
J at a_σ (N/mm)	J_σ	21.3	25	40.7	7.5	10.9

Tab. 3 Maximum stresses near the tips of two cracks in competition and J values

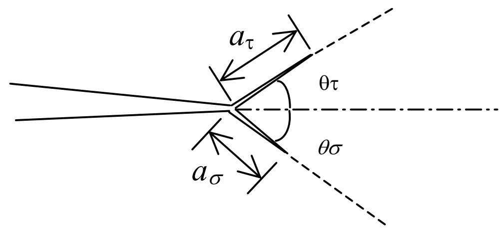

Fig. 5 Two cracks in competition

5. CONCLUSIONS

The numerical method developed recently allows us to obtain the theoretical HRR stress fields near the tip of a crack in elastic-plastic material under mixed mode loading. The numerical results obtained by using this method can explain the experimental results. The crack growth depends on the competition between the ratio $\sigma_{\theta\theta}/\tau_{r\theta}$ compared to the ratio σ_c/τ_c . this ratio could lead to a critical mixity parameter M^p.

Based on experimental observations, this competition was evaluated by numerical studies. More experimental and numerical studies will be necessary to establish suitable bifurcation criteria for a mixed mode crack in elastic-plastic materials.

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