ASYMPTOTIC FIELDS NEAR A CRACK TIP LOCATED AT INTERFACES OF SEVERAL ANISOTROPIC MATERIALS

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ABSTRACT

In this paper, we studied the stress singularities near tip of a two-dimension notch, which could be a crack tip, formed from several elastic materials, each of them may be generally anisotropic. By introducing the dual variables in the state space, the basic equations governing the posed problem were established. We also proposed a numerical method to solve the governing equations. It was shown that the mathematical formulations advanced are quite simple and the numerical method proposed is easy and highly accurate.

KEYWORDS
Notch, crack, interface, multi-material, bimaterial, anisotropy, plane stress, generalized plane strain

1. INTRODUCTION

Knowledge about the stress concentration near the tip of a notch in anisotropic materials, or a crack as a special case, has a particular importance. The fracture behaviors of such a structure may be interesting for many engineering applications such as composites, crystals, welded structures or reinforced polymers etc. In this topic, one can note the pioneering works of Stroh (1958), Sih *et al*. (1965) or Hoenig (1982) concerning the asymptotic fields near a crack tip in homogenous orthotropic or general anisotropic materials. The next studies were carried out in determining the near-tip fields when the crack lying at or touching an interface between two anisotropic materials. Several basic crack problems have been solved (Gotoh, 1967, Clements, 1971, Willis, 1971, Delale and Erdogan, 1979, Ting and Hoang, 1984, Ting, 1986, Qu & Bassani, 1989, Suo, 1990, Gupta *et al*., 1992, Ting, 1996, Sung and Liou, 1996, Lin and Sung, 1997, Matntic *et al*., 1997 among others).

In all the studies mentioned above, the linear elastic anisotropy theory developed by Lekhnitskii (1953) and Eshelby *et al* (1953) were essentially followed. This theory provides explicit results for some problems such as cracks in homogenous materials or cracks lying at an interface etc. However, for more complex problems, the methods provided by this theory leads to long and difficult mathematical formulations.

In this paper, we propose to study the stress singularities near the tip of a notch formed from several generally anisotropic elastic materials. We will use, in this work, another methodology than that of Lekhnitskii and Eshelby. This new methodology consists in introducing the Hamiltonian system and the state space method into the continuum mechanics and has been successfully used in the reform of the elasticity theory (Zhong, 1995). In this work, we deduced the governing equations allowing the determination of the stress singularities and the asymptotic fields near the notch tip. The mathematical formulation is quite simple comparing with those currently appeared in the literature. We also proposed a numerical method to solve the governing equations. It has been shown that this numerical method is simple and highly accurate.

2. GOVERNING EQUATIONS OF THE PROBLEM

Let consider a notch formed from several elastic anisotropic materials. We establish a Cartesian coordinate system and a cylindrical coordinate system with their origins at the notch tip and the z-axis representing the notch front. The material 1 occupies the sectorial domain $[\theta_0,\theta_1]$, named zone 1; the material 2 occupies the zone 2, bounded by $[\theta_1,\theta_2]$, and so on. Under remote loading, the stress concentration at the notch tip will take a mixed mode nature due to the anisotropy of the materials.

First, we write the stress components in the Cartesian system and in the cylindrical system as $\sigma_{xyz} = \{\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{xz} \quad \tau_{yz}\}^T$ and $\sigma_{r\theta z} = \{\sigma_r \quad \sigma_\theta \quad \sigma_z \quad \tau_{r\theta} \quad \tau_{rz} \quad \tau_{\theta z}\}^T$ respectively. The corresponding strain components are $\varepsilon_{xyz} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{xz} \quad \gamma_{yz}\}^T$ and $\varepsilon_{r\theta z} = \{\varepsilon_r \quad \varepsilon_\theta \quad \varepsilon_z \quad \gamma_{r\theta} \quad \gamma_{rz} \quad \gamma_{\theta z}\}^T$ respectively. In the Cartesian system, each material has a homogeneous and anisotropic elasticity:

$$\boldsymbol{\sigma}_{xyz} = \boldsymbol{C}_{xyz}\boldsymbol{\varepsilon}_{xyz} \qquad (1)$$

$\boldsymbol{C}_{xyz}$ is the stiffness matrix of the material. All its components c_{ij} $(i, j = 1,6)$ are constant. In the cylindrical system, the stress and the strain components can be obtained from their corresponding quantities in the Cartesian system with a coordinate rotation, namely,

$$\boldsymbol{\sigma}_{r\theta z} = \boldsymbol{T}_\sigma \boldsymbol{\sigma}_{xyz} \qquad \varepsilon_{r\theta z} = \boldsymbol{T}_\varepsilon \boldsymbol{\varepsilon}_{xyz} \qquad (2)$$

where $\boldsymbol{T}_\sigma$ and $\boldsymbol{T}_\varepsilon$ are the coordinate rotation matrices about the stresses and strains respectively. Therefore, the stress-strain relationship in the cylindrical system is:

$$\begin{aligned} &\boldsymbol{\sigma}_{r\theta z} = \boldsymbol{C}_{r\theta z}\boldsymbol{\varepsilon}_{r\theta z} \\ &\boldsymbol{C}_{r\theta z}(\theta) = \boldsymbol{T}_\sigma \boldsymbol{C}_{xyz}\boldsymbol{T}_\varepsilon^{-1}. \end{aligned} \qquad (4)$$

This shows that in the cylindrical system, the stiffness matrix is not a constant matrix but a function of θ. Hereafter we work exclusively in the cylindrical system, therefore the subscript $_{r\theta z}$ will be omitted in order to simplify the notations. At the present, we do not distinguish the different materials in the formulation.

We write now the fundamental equations of the anisotropic elasticity in the cylindrical system:

(a): *The equilibrium equations:* In the case when the stress components are independent of the z-axis, the equilibrium equations are:

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad \frac{\partial\tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{2\tau_{r\theta}}{r} = 0 \quad \frac{\partial\tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta} + \frac{\tau_{rz}}{r} = 0 \qquad (5)$$

We perform the following variable changes:

$$\xi = \ln r \qquad r = \exp(\xi); \qquad (6)$$

and

$$\begin{aligned} &S_r = r\sigma_r \qquad \sigma_r = S_r / r; \\ &S_{r\theta} = r\tau_{r\theta} \qquad \tau_{r\theta} = S_{r\theta}/r; \quad \text{...etc} \end{aligned} \qquad (7)$$

Then by using the notation $\dot{(\cdot)} = \frac{\partial}{\partial\theta}$, the equilibrium equations (5) can be rewritten as:

$$\dot{S}_{r\theta} = S_\theta - \frac{\partial S_r}{\partial\xi} \quad \dot{S}_\theta = -\frac{\partial S_{r\theta}}{\partial\xi} - S_{r\theta} \quad \dot{S}_{\theta z} = -\frac{\partial S_{rz}}{\partial\xi} \qquad (8)$$

We define the following variable vectors:

$$\boldsymbol{p} = \{S_\theta \quad S_{r\theta} \quad S_{\theta z}\}^T \quad \boldsymbol{p}_t = \{S_r \quad S_z \quad S_{rz}\}^T \qquad (9)$$

Hence, the equilibrium equations (8) can be rewritten as:

$$\dot{\boldsymbol{p}} = \boldsymbol{E}_1\boldsymbol{p} + \boldsymbol{E}_2\frac{\partial\boldsymbol{p}}{\partial\xi} + \boldsymbol{E}_3\frac{\partial\boldsymbol{p}_t}{\partial\xi} \qquad (10)$$

where

$$\boldsymbol{E}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{E}_2 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \boldsymbol{E}_3 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b): *The displacement-stress relationship:* If the displacement components are independent of the *z*-axis, the relations between the strain and displacement components are:

$$\begin{aligned} &\varepsilon_r = \frac{\partial u_r}{\partial r} && \varepsilon_\theta = \frac{1}{r}\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right) && \varepsilon_z = 0 \\ &\gamma_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} && \gamma_{rz} = \frac{\partial w}{\partial r} && \gamma_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial \theta} \end{aligned} \qquad (11)$$

By substituting (11) into (4) and by using the variable changes (6) and (7), one obtains:

$$\begin{Bmatrix} S_r \\ S_\theta \\ S_z \\ S_{r\theta} \\ S_{rz} \\ S_{\theta z} \end{Bmatrix} = \begin{bmatrix} c_{12} & c_{14} & c_{16} \\ c_{22} & c_{24} & c_{26} \\ c_{32} & c_{34} & c_{36} \\ c_{42} & c_{44} & c_{46} \\ c_{52} & c_{54} & c_{56} \\ c_{62} & c_{64} & c_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_\theta}{\partial \theta} \\ \dfrac{\partial u_r}{\partial \theta} \\ \dfrac{\partial w}{\partial \theta} \end{Bmatrix} + \begin{bmatrix} -c_{14} & c_{12} & 0 \\ -c_{24} & c_{22} & 0 \\ -c_{34} & c_{32} & 0 \\ -c_{44} & c_{42} & 0 \\ -c_{54} & c_{52} & 0 \\ -c_{64} & c_{62} & 0 \end{bmatrix} \begin{Bmatrix} u_\theta \\ u_r \\ w \end{Bmatrix} + \begin{bmatrix} c_{14} & c_{11} & c_{15} \\ c_{24} & c_{21} & c_{25} \\ c_{34} & c_{31} & c_{35} \\ c_{44} & c_{41} & c_{45} \\ c_{54} & c_{51} & c_{55} \\ c_{64} & c_{61} & c_{65} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_\theta}{\partial \xi} \\ \dfrac{\partial u_r}{\partial \xi} \\ \dfrac{\partial w}{\partial \xi} \end{Bmatrix} \qquad (12)$$

Similarly, we define a displacement vector

$$\{\boldsymbol{q}\} = \{u_\theta \quad u_r \quad w\}^T \qquad (13)$$

By using the definitions (9) and (13), the relationship (12) can be rewritten as:

$$\boldsymbol{p} = \boldsymbol{C}_d \dot{\boldsymbol{q}} + \boldsymbol{C}_e \boldsymbol{q} + \boldsymbol{C}_f \frac{\partial \boldsymbol{q}}{\partial \xi} \quad \boldsymbol{p}_t = \boldsymbol{C}_{d1} \dot{\boldsymbol{q}} + \boldsymbol{C}_{e1} \boldsymbol{q} + \boldsymbol{C}_{f1} \frac{\partial \boldsymbol{q}}{\partial \xi} \qquad (14)$$

Or :

$$\dot{\boldsymbol{q}} = \boldsymbol{C}_d^{-1}\left(\boldsymbol{p} - \boldsymbol{C}_e \boldsymbol{q} - \boldsymbol{C}_f \frac{\partial \boldsymbol{q}}{\partial \xi}\right) \quad \boldsymbol{p}_t = \boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\boldsymbol{p} + \left(\boldsymbol{C}_{f1} - \boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\boldsymbol{C}_f\right)\frac{\partial \boldsymbol{q}}{\partial \xi} \qquad (16)$$

with:

$$\begin{aligned} &\boldsymbol{C}_d = \begin{bmatrix} c_{22} & c_{24} & c_{26} \\ c_{42} & c_{44} & c_{46} \\ c_{62} & c_{64} & c_{66} \end{bmatrix} \quad \boldsymbol{C}_e = \begin{bmatrix} -c_{24} & c_{22} & 0 \\ -c_{44} & c_{42} & 0 \\ -c_{64} & c_{62} & 0 \end{bmatrix} \quad \boldsymbol{C}_f = \begin{bmatrix} c_{24} & c_{21} & c_{25} \\ c_{44} & c_{41} & c_{45} \\ c_{64} & c_{61} & c_{65} \end{bmatrix} \\ &\boldsymbol{C}_{d1} = \begin{bmatrix} c_{12} & c_{14} & c_{16} \\ c_{32} & c_{34} & c_{36} \\ c_{52} & c_{54} & c_{56} \end{bmatrix} \quad \boldsymbol{C}_{e1} = \begin{bmatrix} -c_{14} & c_{12} & 0 \\ -c_{34} & c_{32} & 0 \\ -c_{54} & c_{52} & 0 \end{bmatrix} \quad \boldsymbol{C}_{f1} = \begin{bmatrix} c_{14} & c_{11} & c_{15} \\ c_{34} & c_{31} & c_{35} \\ c_{54} & c_{51} & c_{55} \end{bmatrix} \end{aligned} \qquad (18)$$

In (17) the relationship $\boldsymbol{C}_{e1} - \boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\boldsymbol{C}_e = \boldsymbol{0}$ is used. Since the strain energy in solids is always positive, consequently, $\boldsymbol{C}_d$ is a positively definite matrix. Therefore, the inversion of the matrix $\boldsymbol{C}_d$ is permitted

(c): *The governing equations:* By substituting equation (17) into the equilibrium equation (10), the variable vector $\boldsymbol{p}_t$ is eliminated. Then we obtain, from (10) and (16), the following dual equations that govern the posed problem:

$$\dot{\boldsymbol{q}} = \boldsymbol{H}_{11}\boldsymbol{q} + \boldsymbol{H}_{12}\boldsymbol{p} \quad \dot{\boldsymbol{p}} = \boldsymbol{H}_{21}\boldsymbol{q} + \boldsymbol{H}_{22}\boldsymbol{p} \qquad (19)$$

with:

$$\begin{aligned} &\boldsymbol{H}_{11} = \boldsymbol{E}_1 - \boldsymbol{C}_d^{-1}\boldsymbol{C}_f \frac{\partial}{\partial \xi} && \boldsymbol{H}_{12} = \boldsymbol{C}_d^{-1} \\ &\boldsymbol{H}_{21} = \boldsymbol{E}_3\left(\boldsymbol{C}_{f1} - \boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\boldsymbol{C}_f\right)\frac{\partial^2}{\partial \xi^2} && \boldsymbol{H}_{22} = \boldsymbol{E}_1 + \left(\boldsymbol{E}_2 + \boldsymbol{E}_3\boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\right)\frac{\partial}{\partial \xi} \end{aligned} \qquad (20)$$

In fact, it is more convenient to define a total vector $\boldsymbol{v}$ as variables in the state space:

$$\boldsymbol{v} = \left\{\boldsymbol{q}^T \quad \boldsymbol{p}^T\right\}^T \qquad (21)$$

such that the governing equations (19) become:

$$\dot{\boldsymbol{v}} = \boldsymbol{H}\boldsymbol{v} \qquad (22)$$

with:

$$\boldsymbol{H} = \begin{vmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{vmatrix} \qquad (23)$$

(d): *The boundary conditions and the continuity conditions:* Referring to Fig.1, we adopt the superscript $^{(i)}$ to indicate the quantities in the zone i , for example, $\boldsymbol{v}^{(i)}$, $\boldsymbol{H}^{(i)}$, etc.. The boundary conditions at the two free surfaces of the notch are:

$$\boldsymbol{p}^{(1)}(\theta = \theta_0) = \boldsymbol{0} \quad \boldsymbol{p}^{(n)}(\theta = \theta_n) = \boldsymbol{0} \qquad (24)$$

The continuity conditions across the interfaces are:

$$\boldsymbol{v}^{(1)}(\theta = \theta_1) = \boldsymbol{v}^{(2)}(\theta = \theta_1) \quad \cdots\cdots \quad \boldsymbol{v}^{(n-1)}(\theta = \theta_{n-1}) = \boldsymbol{v}^{(n)}(\theta = \theta_{n-1}) \qquad (25)$$

These relations show the advantage of the choice of the dual variables in the present study: the multi-material problem can be dealt with as a single material problem since the variable vector $\boldsymbol{v}$ is continuous across all the interfaces. This makes much easier the resolution of the governing equation (22).

If we suppose the stress component σ_z=0, from the third equation of (4), one deduces the strain component ε_z:

$$\varepsilon_z = -\frac{c_{31}}{c_{33}}\varepsilon_r - \frac{c_{32}}{c_{33}}\varepsilon_\theta - \frac{c_{34}}{c_{33}}\gamma_{r\theta} - \frac{c_{35}}{c_{33}}\gamma_{rz} - \frac{c_{36}}{c_{33}}\gamma_{\theta z} \qquad (26)$$

Introducing (26) into (4) eliminates all components in the third colon and the third row of the stiffness matrix $\boldsymbol{C}$. The other components become:

$$c_{ij\ (plane\ stress)} = c_{ij} - \frac{c_{i3}c_{3j}}{c_{33}} \qquad (27)$$

By adapting this new stiffness matrix, all formulations deduced for the generalized plane strain can directly be used for plane stress problems.

3. SOLUTION METHOD

By examining the governing equation (22), it is self-evident to try to solve it by using the variable separation method. We suppose that the variable vector $\boldsymbol{v}(\xi, \theta)$ can be written under separable form:

$$\boldsymbol{v}(\xi,\theta) = \exp(\lambda\xi)\boldsymbol{\psi}(\theta) \qquad (31)$$

where λ is an undetermined eigenvalue, $\boldsymbol{\psi}(\theta)$ is a variable vector depending exclusively on θ. Then equation (22) becomes:

$$\dot{\boldsymbol{\psi}}(\theta) = \boldsymbol{H}(\theta)\boldsymbol{\psi}(\theta) \qquad (32)$$

In (32), $\boldsymbol{H}$ is function of θ only,

$$\boldsymbol{H}(\theta) = \begin{vmatrix} \boldsymbol{E}_1 - \boldsymbol{C}_d^{-1}\boldsymbol{C}_f\lambda & \boldsymbol{C}_d^{-1} \\ \boldsymbol{E}_3\left(\boldsymbol{C}_{f1} - \boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\boldsymbol{C}_f\right)\lambda^2 & \boldsymbol{E}_1 + \left(\boldsymbol{E}_2 + \boldsymbol{E}_3\boldsymbol{C}_{d1}\boldsymbol{C}_d^{-1}\right)\lambda \end{vmatrix} \qquad (33)$$

The continuity conditions across the interfaces become:

$$\boldsymbol{\psi}^{(1)}(\theta = \theta_1) = \boldsymbol{\psi}^{(2)}(\theta = \theta_1) \quad \cdots\cdots \quad \boldsymbol{\psi}^{(n-1)}(\theta = \theta_{n-1}) = \boldsymbol{\psi}^{(n)}(\theta = \theta_{n-1}) \qquad (34)$$

We believe that equation (32) may be solved by different ways. In this work, we propose a numerical method allowing the determination of the eigenvalue λ and the corresponding eigenvector $\boldsymbol{\psi}(\theta)$. First, we divide a zone, the zone i bounded by the interfaces $\theta=\theta_{i-1}$and $\theta=\theta_i$ for example, into N_i intervals of equal angle size by inserting N_i-1 points. In each interval, we integrate (32) by using the trapezoidal approximation:

$$\boldsymbol{\psi}_1^{(i)} - \boldsymbol{\psi}_0^{(i)} = \left(\boldsymbol{H}_0^{(i)}\boldsymbol{\psi}_0^{(i)} + \boldsymbol{H}_1^{(i)}\boldsymbol{\psi}_1^{(i)}\right)\frac{d}{2} \quad \cdots\cdots \quad \boldsymbol{\psi}_{N_i}^{(i)} - \boldsymbol{\psi}_{N_i-1}^{(i)} = \left(\boldsymbol{H}_{N_i-1}^{(i)}\boldsymbol{\psi}_{N_i-1}^{(i)} + \boldsymbol{H}_{N_i}^{(i)}\boldsymbol{\psi}_{N_i}^{(i)}\right)\frac{d}{2} \qquad (35)$$

where d is the interval size. From (35), we have:

$$\psi_1^{(i)} = \left(\boldsymbol{I}_6 - \boldsymbol{H}_1^{(i)}\frac{d}{2}\right)^{-1}\left(\boldsymbol{I}_6 + \boldsymbol{H}_0^{(i)}\frac{d}{2}\right)\psi_0^{(i)} \quad \cdots\cdots \quad \psi_{N_i}^{(i)} = \left(\boldsymbol{I}_6 - \boldsymbol{H}_{N_i}^{(i)}\frac{d}{2}\right)^{-1}\left(\boldsymbol{I}_6 + \boldsymbol{H}_{N_i-1}^{(i)}\frac{d}{2}\right)\psi_{N_i-1}^{(i)} \qquad (36)$$

where $\boldsymbol{I}_6$ is a 6×6 unite matrix. Hence, we immediately obtain the relation between $\psi_{N_i}^{(i)}$ and $\psi_0^{(i)}$, namely:

$$\psi_{N_i}^{(i)} = \boldsymbol{G}^{(i)}\psi_0^{(i)} \qquad (37)$$

with:

$$\boldsymbol{G}^{(i)} = \left(\boldsymbol{I}_6 - \boldsymbol{H}_{N_i}^{(i)}\frac{d}{2}\right)^{-1}\left[\prod_{k=N_i-1}^{1}\left(\boldsymbol{I}_6 + \boldsymbol{H}_k^{(i)}\frac{d}{2}\right)\left(\boldsymbol{I}_6 - \boldsymbol{H}_k^{(i)}\frac{d}{2}\right)^{-1}\right]\left(\boldsymbol{I}_6 + \boldsymbol{H}_0^{(i)}\frac{d}{2}\right) \qquad (38)$$

According to the continuity conditions (34), one has:

$$\psi_0^{(i)} = \psi_{N\,i}^{(i-1)} \qquad (39)$$

Hence, we obtain the relation between $\psi^{(1)}(\theta = \theta_0)$ and $\psi^{(n)}(\theta = \theta_n)$, namely,

$$\psi^{(n)}(\theta = \theta_n) = \boldsymbol{G}\psi^{(1)}(\theta = \theta_0) \qquad (40)$$

with

$$\boldsymbol{G} = \prod_{i=n}^{1}\boldsymbol{G}^{(i)} \qquad (41)$$

In practice, the trapeze method provides quite a poor accuracy in calculation of $\boldsymbol{G}$. The accuracy can considerably be improved by using the Richardson extrapolation technique.

Now we write (40) in the form of the dual vectors $\boldsymbol{q}$ and $\boldsymbol{p}$:

$$\begin{Bmatrix}\boldsymbol{q}\\ \boldsymbol{p}\end{Bmatrix}(\theta = \theta_n) = \begin{bmatrix}\boldsymbol{G}_{11} & \boldsymbol{G}_{12}\\ \boldsymbol{G}_{21} & \boldsymbol{G}_{22}\end{bmatrix}\begin{Bmatrix}\boldsymbol{q}\\ \boldsymbol{p}\end{Bmatrix}(\theta = \theta_0) \qquad (42)$$

Since $\boldsymbol{p}(\theta = \theta_0) = \boldsymbol{p}(\theta = \theta_n) = \{\boldsymbol{0}\}$, from the second equation of (42), one has:

$$\boldsymbol{G}_{21}\boldsymbol{q}(\theta = \theta_{\boldsymbol{0}}) = \boldsymbol{0} \qquad (43)$$

This leads to:

$$\det(\boldsymbol{G}_{21}) = 0 \qquad (44)$$

Equation (44) is the condition required to determine the eigenvalues λ. Iteration techniques for roots finding can be used for the determination of λ. In this work, the Muler method is used because it can generate complex roots even if a real initial value of λ is chosen, and vice-versa. Once the eigenvalues determined, the vector $\boldsymbol{q}(\theta = \theta_0)$ is obtained from (43). Therefore, the boundary value problem posed becomes an initial value problem. Any numerical method providing a good accuracy can be used for solving equation (32). Otherwise the eigenvectors ψ can straightforwardly be given from (36), and all stress and displacement components can easily be obtained from (31) and (17).

5. CONCLUSIONS

In this work, we have established the general equations governing the asymptotic fields near a notch tip formed from several general anisotropic materials. These equations are expressed under the form of a system of first-order differential equations, instead of a high-order differential equation of a single variable as in the traditional methods. The dual variables chosen present important advantages in the resolution of the problems because of their continuity across all the interfaces. A numerical method has been proposed to solve the egeinvalue problem. This numerical method is simple and highly accurate comparing with results obtained in other existing analytical solutions. Consequently, the present method enables us to deal with a large range of problems in this topic with rather simple mathematical formulations and small numerical effort. Since the new materials developed recently present a large field in which the modeling of the anisotropy is important, we believe that the present work provides a new tool to study problems in this domain.

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