ANALYSIS OF THE PROCESS ZONE AHEAD OF A CRACK TIP

J. Lemaitre
Prof. Em. Universite Paris 6
Laboratoire de Mécanique et Technologie
61 av. du président Wilson F-94235 CACHAN Cedex, FRANCE
and
R. Desmorat
Laboratoire de Modélisation et Mécanique des Structures
8, rue du Capitaine Scott, F-75015 PARIS, FRANCE

ABSTRACT

At the head of a crack tip of plane stress problem, the near far stress field is taken as the Westergaard equations for elasticity. Closer, it is given by the HRR plastic field. More closer, where damage occurs, it is given by a linear approximation of stress from the ultimate stress σ_u at the border of the damage zone to $\sigma_u(1 - D_c)$ at the real crack tip where the damage reaches its critical value D_c. It is shown that the damaged zone is very small and homothetic of the plastic zone and that the ductile crack growth rate may be deduced from the plasticity and damage parameters. [1]

STRESS ANALYSIS AHEAD OF A CRACK TIP

Consider the simple reference case of fracture mechanics : a crack of length a loaded in mode I by a state of quasi-static monotic plane stress σ_∞ at infinity. Due to the properties of a material subjected to elasticity, plasticity and damage, the domain close to the crack tip is divided into three regions as shown in figure 1.

- A region E far from the crack tip where the behavior of the material is purely elastic. The state of stress not too far from the crack tip is given by the Westergaard analysis [1, 2].
- A region P closer to the crack tip where the behavior of the material is purely plastic. The state of stress is given by the H.R.R. field [3, 4].
- A region D surrounding the crack tip where the behavior of the material is elasto plastic softened by damage up to the crack tip where the damage reaches its value at fracture.

Elastic zone E

Using Westergaard equations the purely elastic domain is limited by a line along which the von Mises equivalent stress $\sigma_{eq} = \sqrt{\frac{3}{2}\sigma^D_{ij}\sigma^D_{ij}}$ reaches the yield stress σ_y of the material. Its distance from the crack tip along the x-axis is (G: strain energy release rate),

1. ICF 10 - March 2001

$$x - a = r_y = \frac{EG}{2\pi\sigma_y^2} = \frac{\sigma_\infty^2 a}{2\sigma_y^2} \qquad \text{for plane stress} \tag{1}$$

Figure 1: Elasticity, plasticity and damage ahead of a crack

Plastic zone P

The plastic property of the material is represented by the Ramberg-Osgood constitutive equation,

$$p = \left(\frac{\sigma_{eq}}{K}\right)^M \tag{2}$$

where $p = \sqrt{\frac{2}{3}\epsilon_{ij}^p \epsilon_{ij}^p}$ is the accumulated plastic strain and K and M are material parameters. Eq. (2) models plasticity as a nonlinear elasticity problem. It is valid as long as no unloading occurs and neglects the elastic part of the strain. Ramberg Osgood law is the constitutive equation used in the HRR analysis in order to determine the von Mises equivalent stress σ_{eq} and the equivalent plastic strain p along the x axis,

$$\sigma_{eq} = Const \cdot \sigma_\infty{}^{\frac{2}{M+1}} \left(\frac{a}{x-a}\right)^{\frac{1}{M+1}} \tag{3}$$

$$p = \left(\frac{Const}{K}\right)^M \sigma_\infty{}^{\frac{2M}{M+1}} \left(\frac{a}{x-a}\right)^{\frac{M}{M+1}} \tag{4}$$

$x = a + r_y$ is the abscisse corresponding to the yield limit where the yield stress σ_y is reached, or in an equivalent manner where p takes a conventional value, let us say $p_y = \sigma_y/E$, of the order of magnitude of $0.2 \cdot 10^{-2}$, and

$$p_y = \left(\frac{Const}{K}\right)^M \sigma_\infty{}^{\frac{2M}{M+1}} \left(\frac{a}{r_y}\right)^{\frac{M}{M+1}} \tag{5}$$

$x = a + r_D$ is the abscisse corresponding to the damage limit, i.e. $D = 0$ if $x > a + r_D$, $D \neq 0$ if $x < a + r_D$ considering the continuous damage variable D (surface density of microcracks or microcavities) as a scalar for isotropic damage. The damage threshold p_D, below which $D = 0$, is generally loading dependent and related to the damage threshold in pure tension ϵ_{pD} characteristic of each material [5]. For simplicity we take here $p_D = \epsilon_{pD}$,

$$p_D = \left(\frac{Const}{K}\right)^M {\sigma_\infty}^{\frac{2M}{M+1}} \left(\frac{a}{r_D}\right)^{\frac{M}{M+1}} \tag{6}$$

This shows that the damage zone is homothetic to the plastic zone

$$\frac{r_D}{r_y} = \left(\frac{p_y}{p_D}\right)^{\frac{M+1}{M}} \tag{7}$$

In fact, this zone is very small. If p_y is of the order of 0.2 10^{-2}, ϵ_{pD} for metals is often of the order of 5 to 20 10^{-2} and M of 3 to 8,

$$r_D \approx \frac{r_y}{10 \text{ to } 100} \tag{8}$$

Damage zone D

The abscisse of the damage zone r_D is defined by $p = p_D$ which corresponds for most metals to an equivalent stress close to the ultimate stress σ_u [5]. Then $\sigma_{eq} = \sigma_u$ for $x = a + r_D$.

At the crack tip $x = a$, the damage reaches its critical value at mesocrack initiation $D = D_c$ ($D_c \approx 0.2$ to 0.5 depending upon the material). Considering the hardening saturated for $p > p_D$, the plasticity criterion coupled to damage by the effective stress concept is written as

$$\frac{\sigma_{eq}}{1 - D} - \sigma_u = 0 \tag{9}$$

which gives the value of the stress at the crack tip $x = a$

$$\sigma_{eq} = \sigma_u (1 - D_c) \tag{10}$$

Figure 2: Behavior of the material by zone

The damage zone being small, a linear variation of the stress σ_{eq} is assumed for $a < x < a + r_D$,

$$\sigma_{eq} = \sigma_u \left[1 - D_c \left(1 - \frac{x - a}{r_D} \right) \right] \tag{11}$$

The behavior of the material, elastic for $\sigma_{eq} < \sigma_y$, plastic by the Ramberg-Osgood law for $\sigma_y \leq \sigma_{eq} < \sigma_u$ is softened by damage for larger plastic strain p_u corresponding to σ_u. The hardening being saturated, the accumulated plastic strain, is in this range a linear function of the von Mises stress (figure 2) and

$$dp = -\frac{p_R - p_u}{\sigma_u D_c} d\sigma_{eq} \tag{12}$$

This allows to calculate the plastic evolution and the damage from its law of evolution taken as a function of the total elastic energy and of the accumulated plastic strain [6], two variables governing also the crack propagation [7]

$$dD = \left(\frac{\sigma_u^2 R_\nu}{2ES} \right)^s dp \qquad \text{if} \quad p > p_D \tag{13}$$

also from the plasticity criterion: $dD = -d\sigma_{eq}/\sigma_u$ E is the Young's modulus, S and s two material damage parameters. R_ν is the triaxiality function. The linearity of the plastic strain induces the linearity of the damage,

$$\frac{dD}{dx} = -\frac{D_c}{r_D} \qquad \text{and} \qquad D(x, a) = D_c \left(1 - \frac{x - a}{r_D(a)} \right) \tag{14}$$

CRACK GROWTH

Consider the loading σ_∞ at the level which has just created the crack of length a. The loading increment is noted $\delta\sigma_\infty$. It induces a virtual increment of the stress $\delta\sigma_{eq}$ but as the stress σ_{eq} is bounded by σ_u at $x = a + r_D$, this corresponds to a decrease $-\delta\sigma_{eq}$ which induces first an increment of the plastic strain, then an increment of the damage, and finally a real increment of the crack δa (a discontinuity of crack mechanics from continuous damage mechanics) as the Damage is bounded by D_c (Figure 3).

Figure 3: Crack increment

It is necessary to indroduce a link which can be an energetic equivalence $\mathcal{D}_a = \mathcal{D}_d$ between the crack growth dissipation $\mathcal{D}_a$ and the damage dissipation $\mathcal{D}_d$ [5, 8]. If $a(t_0) = a_0$ and $a(t) = a$,

$$\mathcal{D}_a = \int_{a_0}^{a} Ghda \qquad \mathcal{D}_d = \int_{a_0}^{\infty}\int_{t_0}^{t} Y\dot{D}dth^2dx \tag{15}$$

where $G = \sigma_\infty^2 \pi a/E$ is the strain energy release rate, Y is the damage energy release rate density (a constant here $Y = Y_c = \sigma_u^2 R_\nu/2E$), h is the size of the Representative Volume Element. The second integral is in fact an integral between a_0 and $a + r_D(a)$ as outside of the damage zone $\dot{D}$ remains equal to zero,

$$\mathcal{D}_d = h^2 Y_c \left(\int_{a_0}^{a+r_D(a)} D(x,a)dx - \int_{a_0}^{a_0+r_D(a_0)} D(x,a_0)dx \right) \tag{16}$$

Equations (1) and (8) may be combined in order to give r_D as:

$$r_D = \alpha G \qquad \alpha = \frac{E}{2\pi\sigma_y^2}\left(\frac{p_y}{p_D}\right)^{\frac{M+1}{M}} \tag{17}$$

Then

$$\mathcal{D}_d = \frac{1}{2}h^2\alpha Y_c D_c \left[\left(1 + \frac{a-a_0}{\alpha G}\right)^2 G - G_0 \right] \tag{18}$$

Using the approximation $\mathcal{D}_a \approx h(a-a_0)(G+G_0)/2$ and anticipating the fact that the initial strain energy release rate does not have a quantitative effect, the energetic equivalence $\mathcal{D}_a = \mathcal{D}_d$ allows for the determination of G as

$$G \approx \sqrt{\frac{hY_cD_c}{\alpha}(a-a_0)} \tag{19}$$

which is the equation equivalent to the one of the R-curve in Fracture Mechanics. Furthermore it gives the possibility to evaluate the danger of a damage crack initiation regarding to an increase of the loading.

Références

[1] H. M. Westergaard. *J. Appl. Mech.*, **A 49**, 1939.

[2] Irwin. Analysis of stress, strains near the end of a crack transversing a plate. *J. Appl. Mech.*, **24**:361–364, 1957.

[3] J. W. Hutchinson. Singular behavior at the end of a tensile crack in a hardening material. *J. Mech. Phys. Solids*, **16**:13–31, 1968.

[4] J. R. Rice and G. F. Rosengreen. Plane strain deformation near a crack tip in a power-law hardening material. *J. Mech. Phys. Solids*, **16**:1–12, 1968.

[5] J. Lemaitre. *A Course on Damage Mechanics*. Springer Verlag, 1992-96.

[6] J. Lemaitre, R. Desmorat, and M. Sauzay. Anisotropic damage law of evolution. *Eur. J. Mech., A/ Solids*, **19**:187–208, 2000.

[7] G. C. Sih. *Mechanics of fracture initiation and propagation*. Kluwer, 1991.

[8] O. Allix and P. Ladevèze. Damage mechanics of interfacial media: basic aspects, identification and application to delamination. *Damage and Interfacial debonding in Composites, Ed. G.Z. Voyiadjis and D.H. Allen, Elsevier Science B.V.*, pages 167–188, 1996.