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ANALYSIS OF BALLISTIC PROPERTIES OF MULTILAYERED SHIELDS USING APPROXIMATE MODELS

G. Ben-Dor, A. Dubinsky and T. Elperin

The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer-Sheva, 84105, P. O. Box 653, Israel

ABSTRACT

Recent results of analytical investigation of multilayered spaced and non-spaced shields using simplified models describing impactor-shield interaction are discussed. For targets consisting of plates manufactured from ductile materials the influence of the order of the plates and air gaps on ballistic limit velocity is investigated, and some problems of optimal arrangement of the plates in a layered shield are solved. Design of two-component ceramic-faced lightweight armors against ballistic impact is investigated, and approximate analytical formulas are derived for areal density and thicknesses of the plates in the optimal armor as functions of parameters determining the properties of the materials of the armor components, cross-section and mass of an impactor, and of the expected impact velocity.

KEYWORDS

Ballistic limit, impact, perforation, armor, shield, optimization.

INTRODUCTION

Sub-ordnance penetration and perforation of multilayered plates has been a subject of intensive research during recent years since non-monolithic configurations are considered feasible for the designing shields or elements of the shields. Simplified analytical models were derived and used for the analysis and optimization of the shields consisting of the layers manufactured from different materials, e.g., ductile multi-layered shields [1, 2-12], aluminum/Lexan combinations [11], ceramic-faced armors [12-17]. Qualitative laws that are obtained from approximate models can be very useful for further theoretical and experimental investigations. In order to obtain such laws the most appropriate are those models that allow to derive formulas determining the dependence of the ballistic limit velocity (hereafter BLV) on various factors affecting perforation, e.g., a shape of the impactor, simultaneous interaction between the impactor and different layers of the shield during motion of the impactor in a multilayered armor, properties of the materials of the layers, etc. In this respect localized-interaction models [18-19], cavity expansion approximations [20-21], Florence's model [13] appear to be very useful.

In this paper we discuss some our results on the multi-layered shields, and additional information can be found in Ben-Dor et al. [2-10, 12]. All the results for non-ceramic armor described here were derived rigorously using the adopted models for impactor-shield interaction. The obtained results (if not indicated differently) correspond to conical impactors with arbitrary shape of the cross-section. The determined ballistic properties of the shields are valid for any impactor in the considered class. Although validation of the obtained results using the available experimental data in the literature is encouraging, specially designed experiments are required in order to determine the range of the validity of the obtained results.

BALLISTIC PROPETIES OF MULTILAYERED SHIELDS DETERMONED WITH THE AID OF THE LOCALIZED INTERACTION MODELS

Impactor-shield localized interaction model

Consider a high speed normal penetration of a 3-D rigid sharp impactor into an armor with a finite thickness and assume that the localized interaction model is valid, i.e., the impactor-armor interaction at a given location at the surface of the impactor which is in a contact with the armor can be described by the following equation:

$$d\vec{F} = \left[\rho\Omega(u)v^2 + \sigma\right]\vec{n}\,dS, \quad u = -\vec{v}\cdot\vec{n} \tag{1}$$

where $d\vec{F}$ is the force acting at the surface element dS of the impactor along the inner normal unit vector $\vec{n}$ at a given location at the surface of the impactor, $\vec{v}$ is the unit local velocity vector, Ω is function determining the particular model for the impactor-shield interaction. Equation (1) with constant parameters ρ,σ comprises the most widely used phenomenological models for homogenous targets (see, e.g., [18,19]). Usually parameters ρ,σ are density and distortion pressure of the armor, respectively, and $\Omega(u) = u^2$. We consider the armor consisting of N plates, the material of i-th plate is characterized by values ρ_i,σ_i. Often these parameters appear in our results as a combination $\chi_i = \sigma_i/\rho_i$. Thus the parameters ρ,σ in eqn (1) depend on the distance of the surface element from the front plane of the target. We assume that the adjacent plates are in contact and do not interact. If the "plate" with the number i is an air gap then $\rho_i = \sigma_i = 0$.

The total force $\vec{F}$ is determined by integrating the local force given by eqn (1) over the impactor-armor contact surface that depends on the position of the impactor inside the shield. This allows us to write equation of motion of the impactor in the normal direction and to determine the BLV that is defined as the initial velocity of the impactor required for its nose to emerge from the target with a zero velocity. Corresponding cumbersome expressions we do not write here (see, e.g., [2,8]).

Optimum multilayered shield. Plates with the same density and given thicknesses

For the shield consisting of several plates with the same density but having different values of distortion pressure perforated, generally, by a non-conical impactor the following properties are valid. If two adjacent plates in a multilayered armor are such that the value of the distortion pressure for the first plate is larger than that for the second plate, the BLV of the armor can be increased by interchanging these plates. The maximum BLV of the armor is achieved when the plates are arranged in the order of increasing values of the distortion pressure of the material of the plates; the minimum BLV is achieved when the plates are arranged in the armor in an inverse order.

Optimum two-layered shield. Plates of different densities and with given thicknesses

The maximum BLV for two-layered armor is attained when the plates are arranged according to the increase of the magnitude of the parameter $\chi = \sigma/\rho$.

Optimum multilayered shield. Plates manufactured from one of the two possible materials

Ballistic properties of multilayered shields are studied when the shield consists of the adjacent plates made from one of two possible materials and the total thickness of the plates manufactured from every material is fixed. The following ballistic properties of the shield are proved. The displacement of any plate inside the target in direction of penetration yields monotone change of the BLV of the shield and the criterion of increasing or decreasing of the BLV depends of the properties of the materials of the plates, namely, relocation of a plate with a larger (smaller) value of the parameter χ yields an increase (decrease) of the BLV. The maximum BLV is obtained for the two-layered shield without alternating the plates manufactured from different materials; the front plate in the optimum shield must be the plate manufactured from the material with the smaller value of the parameter χ.

Optimum multilayered shield with a given areal density and thickness. Plates manufactured from different materials

The problem is formulated as follows. There are several materials with different properties which can be used for manufacturing the plates in a mshield. The areal density of the shield (its mass per surface unit) and its thickness are given. The goal is to determine the structure of the shield (the order and the thicknesses of the plates from different materials) that provides the maximum BLV of the shield. It is proved that the shield with maximum BLV must consist of one or several adjacent plates (these cases are equivalent is point of view of the model) manufactured from the material with the maximum χ. The shield with minimum BLV consist of one or several adjacent plates manufactured from the material with the minimum χ. The values of BLV of different shields with given areal density and thickness are between these limiting values.

Optimum multilayered shield with large air gaps

It is assumed that the impactor perforates the plates in a multi-layered shield sequentially, i.e., it does not interact with two or several plates simultaneously. One would expect that this assumption is approximately valid if the length of the impactor is much less than the thickness of every plate. In the framework of the adopted penetration model this assumption corresponds to the spaced armor when the widths of the air gaps are greater than the length of the impactor. The set of plates is given. We proved that the maximum BLV is attained when the plates are arranged in the order of increasing values of χ.

Influence of air gap on the ballistic resistance of the two-layered shield

The following property is proved. If $\chi_1 > \chi_2 \left(\chi_1 < \chi_2\right)$ then the BLV decreases (increases) with increasing the air gap thickness from zero to the length of the impactor (BLV becomes constant with the further increase of the air gap thickness). If $\chi_1 = \chi_2$, i. e., the properties of the material of both plates are the same, the ballistic limit velocity does not depend on the thickness of the air gap. Numerical calculations performed for armors consisting of plates manufactured from different materials show that the developed model predicts a very negligible effect of an air gap upon the ballistic resistance.

Influence of the order of the plates on the ballistic resistance of a two-layered spaced shield

The maximum BLV of the armor with a fixed width of an air gap is attained when the plates are arranged in the order of the increasing values of parameter χ.

Influence of air gap on a ballistic resistance of a multilayered shield consisting of the plates are manufactured from the same material.

The BLV of the spaced shield is determined by the total thickness of the plates, i.e., it is independent of the air gap sizes between the layers, of the sequence of the plates in the shield and of the distribution of the total thickness among the plates. Monolithic and spaced shields are equivalent in the framework of the considered model.

Optimal shapes of 3D impactors.

We studied optimization of 3D impactors with a given longitudinal contour, length and volume. We determined the existence of the "universal" optimal impactor among the 3D conical and non conical slender impactors penetrating normally into non-homogeneous (layered) semi-infinite shield or into a shield with a finite thickness. The impactor having the minimum drag moving inside a homogeneous medium with a constant velocity penetrates to the maximum depth into a semi-infinite shield and has the maximum BLV when it penetrates into a shield with a finite thickness, regardless of the distribution of the properties of the material in the shield along the penetration path. Using the analogy with the hypersonic flow over the flying projectiles ($\Omega = u^2$ in eqn (1)) it is predicted that the optimal impactors have a star-shaped cross-section.

INFLUENCE OF AIR GAPS ON THE BALLISTIC PROPERTIES OF MULTILAYERED SHIELDS DETERMINED USING CAVITY EXPANSION MODEL

Cylindrical cavity expansion model (CCEM)

The model is based on the assumptions that the impactor (a body of revolution) moving in a shield causes hole expansion in every plane which is normal to the direction of its motion when it reaches this plane and these layers do not interact. Expression for hole expansion vs. the time ($t=0$ is the beginning of the hole expansion) at every plane reads (for details see, e.g., [20-21]):

$$p=\alpha\dot{R}^2+\beta R\ddot{R}+\gamma \tag{2}$$

where R is radius of the hole, p is a pressure applied in the normal direction at the part of the impactor's surface, coefficients α,β,γ depend on the properties of the material of the corresponding plate in the multilayered shield. Taking into account kinematic relation between the location of the impactor in the shield, its shape and the radius of the hole at every plane, the equation of motion of the impactor allows us to determine the BLV. Corresponding formulas can be found in [4]. It is important to emphasize that, even for conical impactor, in eqn (2) $\ddot{R}\neq 0$ and cavity expansion model does not reduce to the localized interaction model. Such special models for homogeneous metal shields can be found, e.g., in [21]. Thus, BLV depends, generally, of the parameters $\alpha_i,\beta_i,\gamma_i$ where the subscript i denotes the number of the plate in the shield.

Optimum multilayered shield consists of given plates. Large air gaps.

The following properties are proved. If two adjacent plates in a shield with large air gaps are such that the value of the parameter $\tilde{\chi}=\gamma/\alpha$ for the first plate is larger than that for the second plate, the BLV of the shield can be increased by interchanging these plates. The maximum (minimum) BLV of the shield is attained when the plates are arranged in the order of increasing (decreasing) values of the parameter $\tilde{\chi}$. The values β_i do not effect the optimal order of the plates.

Comparison of ballistic properties of monolithic and spaced shields

The simplified models that we use imply that monolithic target and the target consisting of several adjacent plates are equivalent if the total thickness of the plates and their material are the same. However, in contrast to localized interaction model, CCEM predicts the difference in BLV for monolithic and spaced shields. It was shown analytically (for large air gaps) that the BLV of a spaced shield is larger than that of a monolithic shield, and the BLV of the shield increases with the increase of the number of the plates with the same thickness while the total thickness of the plates is kept constant. Numerical simulation using the model [21] showed that the influence of air gaps on BLV of the shield is weak for slender conical impactors and can be more pronounced with the increase of the apex half angle of its nose and the density of the material of the shield.

OPTIMUM TWO COMPONENT CERAMIC ARMOR

Model description

Consider a normal impact by a rigid projectile on a two-layer composite armor consisting of a ceramic front plate and a ductile back plate. We employ the following model $v_*^2=\alpha\varepsilon_2\sigma_2h_2z(Az+m)/\left(0.91m^2\right)$ where v_* is the BLV, m is a projectile's mass, R is a projectile's radius, h_1,h_2 are the plate's thicknesses, σ is the ultimate tensile strength, ε is the breaking strain, ρ is density, $A=\rho_1h_1+\rho_2h_2$ is the areal density, $z=\pi\left(R+2h_1\right)^2$; subscripts 1 and 2 refer to a ceramic plate and a back plate, respectively. For $\alpha=1$ this model was suggested by Florence [13] and re-worked by Hetherington [14]. We generalized slightly this model introducing a coefficient α which can be determined using the available experimental data in order to increase the accuracy of the predictions.

The objective of our study is to find the thicknesses of the plates h_1,h_2 which provide the minimum areal density of the armor for a given BLV v_*.

Optimum two component armor

We found that using the dimensionless variables

$$\overline{h}_i=\frac{h_i}{R},\ \overline{\rho}_i=\frac{\pi R^3\rho_i}{m},\ i=1,2,\ \overline{w}=v_*\sqrt{\frac{0.91\rho_2}{\alpha\varepsilon_2\sigma_2}},\ \overline{A}=\frac{\pi R^2A}{m}$$

the problem is reduced to finding a positive $\bar{h}_1$ that provides the minimum $\bar{A} = \bar{A}(\bar{h}_1, \bar{\rho}_1, \bar{w})$. The dimensionless areal density $\bar{A}$ is a function of one variable $\bar{h}_1$ and depends on only two parameters, $\bar{\rho}_1$ and $\bar{w}$. Therefore, although the exact analytical solution of the problem does not exist, the latter property allows us to find the simple approximations for characteristics of optimum shield in a general case, namely, for arbitrary combination of materials of the plates. Such approximations (with the average accuracy of 3% in the range $0.04 \le \bar{\rho}_1 \le 0.1, 1 \le \bar{w} \le 10$) for the thickness of the ceramic plate and the areal density of the optimum armor are given by the the following expressions:

$$\bar{h}_1^{opt} = \frac{(0.04 + 1.12\bar{\rho}_1)\bar{w}^{1.895}}{\bar{\rho}_1(\bar{\rho}_1 + 1.29\bar{w}^{1.47} + 0.1)}, \quad \bar{A}^{opt} = (0.04 + 1.12\bar{\rho}_1)\bar{w}^{0.425}$$

The optimal thickness of metallic plate is $\bar{h}_2^{opt} = (\bar{A}^{opt} - \bar{\rho}_1\bar{h}_1^{opt})/\bar{\rho}_2$.

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