ORAL REFERENCE ICF100308OR

AN INTERFACE ELEMENT FOR THE NUMERICAL ANALYSIS OF FIBRE REINFORCED CONCRETE

Massimo Cuomo

Sezione di Ingegneria Strutturale
Dipartimento di Ingegneria Civile ed Ambientale
University of Catania – ITALY

ABSTRACT

A constitutive model for a joint element is developed based on a generalisation of the Cohesive Crack Model. The model uses a constitutive law based on dual external (tractions and crack opening vector), and internal variables, the latter of damage nature, responsible for the evolution of the softening cohesive. A potential energy of unilateral type couples the crack opening vector with the damage variables, whose evolution is ruled by two yielding modes, one accounting for the slippage of the fibres, the other for the deterioration of the material due to crack opening.

KEYWORDS
Damage Mechanics, Interface elements, Cohesive crack model, Brittle matrices, Fibre reinforced concrete

1. MOTIVATIONS, BACKGROUND AND OBJECTIVES OF THE MODEL PROPOSED

The occurrence of fractures causes two main phenomena that affects the mechanical modelisation: first, energy is dissipated in a domain of measure zero, since its physical dimension is smaller than the dimension of the structure (actually, its dimension is fractal); secondly, the displacement field ceases to be continuous, and finite jumps appear, so that the usual compatibility equations lose their validity, that is the vectorial space of the displacements changes from H_1 to BV. From a physical point of view, these phenomena give rise to an unstable behaviour at the material and structural level.

The numerical counterpart is that a simulation of the process with a continuum model suffers of numerical problems of mesh dependency, so that either a non-local media has to be used, or some form of enhancement of the displacement field has to be introduced. Enhanced elements, with embedded discontinuities, like the X-FEM recently developed by Belitschko are a promising example of the latter approach. An alternate methodology consists in introducing discontinuous interfaces in some predefined locations in the continuum. In the interface model the width of the process zone is assumed to reduce to zero, but the amount of dissipation is controlled, allowing a numerical treatment of the material instability. Tractions are directly related to the displacements jumps, so that there is no need to introduce generalised derivatives.

The latter approach is followed in this paper. Specifically, attention is focused on the constitutive behaviour of the interface model, disregarding the problem of refining the discretisation for better localising the fracture surfaces. Main objective of the paper is to modify an interface model previously proposed in the literature by Carol [1,2], that accounts for mode I and mode II fracture, based on the definition of an intrinsic curve for the interface in the traction space, so that a plastic-like behaviour is assumed for the dual relative displacements. Cohesive forces are supposed to act after crack opening, and softening is introduced assuming a phenomenologically defined degradation of some material parameters. The model, thus, appears as a (non associated) elastic-plastic-softening model. Elasticity is introduced for numerical purposes. Still retaining the idea of an intrinsic curve and of its degradation as consequence of fracture evolution, the model proposed differs substantially from the original one in several aspects that will be now briefly introduced

1. The cohesive traction-displacement laws, as well as the softening behaviour of the interface, are defined on the basis of thermodynamic potentials, so that they can be easily implemented in a variational framework for numerical analysis.
2. The softening law is introduced through the dependency of the limit surface on a damage parameter, dual to the internal variable that rules the reversible loading-unloading. In this way the limit condition of the interface (yield surface) is defined in the extended space of the tractions and of the conjugated forces. The development follow closely a recently proposed model of continuum damage [3].
3. The first consequence of points 1,2 is that it is possible to obtain crack opening and reclosing, without permanent residual relative displacements, as in standard damage models. Furthermore, the thermodynamic framework allows to easily account for additional effects, like fibre bridging. It is sufficient to add an additional term in the internal energy, and additional dissipation mechanisms, that account for fibre slippage or yielding, in the dissipation potential.

2. PRESENTATION OF THE INTERFACE MODEL

The interface model is local, and is ruled by the following fields of dual variables:

$\mathbf{w} = (w_n, w_t) \in U$	*Relative displacements*	$\mathbf{t} = (\sigma, \tau) \in U'$	*Cohesive forces*
$\omega \in \Re$	*Internal damage variable*	$\zeta \in \Re$	*Conjugated damage energy*
$\alpha \in I$	*Hardening internal variable*	$\chi \in I'$	*Conjugated force*

$$h = (\mathbf{w}, \omega, \alpha) = (\mathbf{w}_e, \omega_e, \alpha_e) + (\mathbf{w}_p, \omega_p, \alpha_p) = h_e + h_p \tag{1}$$

The indices n,t refer to normal and tangential components respectively. In the remaining of the paper only the 2-dimensional case will be addressed. It is underlined that a scalar damage mechanism is assumed, while the hardening variables can be in general vectors, so to account for anisotropic friction mechanisms. However, in this paper, they will not be explicitly considered. Following the Standard Generalised Material Model, the kinematic variables are partitioned in a reversible and an irreversible component, identified in (1) by the indices e,p, as done in [3] (the additive decomposition implies linear kinematic). The model is characterised by the functional of the internal energy u, that rules the reversible behaviour, and of the dissipation d, that accounts for irreversible phenomena. Denoting by $(\cdot)^c$ the conjugated potential, the constitutive equations are then obtained as

$$\begin{aligned} s &= (\mathbf{t}, \zeta, \chi) = \partial_{h_e} u(h_e) & \qquad s &\in \partial_{\dot{h}_p} d(\dot{h}) \\ h_e &= \partial_s u^c(s) & \qquad \dot{h}_p &\in \partial_s d^c(s) \end{aligned} \tag{2}$$

The last of (2) are the flow rules for the irreversible kinematic variables.

2.1 The internal energy of the interface

The following form is assumed

$$u = \frac{1}{2}\mathbf{K}_{\omega}\mathbf{w}\cdot\mathbf{w} + \frac{1}{2}\mathbf{K}_{f}\mathbf{w}\cdot\mathbf{w} + \text{ind } W_n + \text{ind } \Omega$$
$$\mathbf{K}_{\omega} = \begin{bmatrix} K_n(\omega) & 0 \\ 0 & K_t(\omega) \end{bmatrix} \qquad \mathbf{K}_{f} = \begin{bmatrix} K_f & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{K}_{\omega} = \mathbf{K}_{m}\,(1+\omega_e)^n \tag{3}$$

In (3) K_m denotes the stiffness of the concrete matrix, while K_f is the stiffness of the fibre phase, assumed to act only in the normal direction. The damage mechanism is assigned only to the concrete matrix, but a further degradation, with the same or with another internal variable, can be introduced in the same way for the fibre properties. Introducing the set $W_n = \{w_n | w_n \geq 0\}$, the presence of the indicator function of W_n ensures the no compenetration condition. The set $\Omega = \{\omega^e | \omega^e \geq -1\}$ has been introduced in order to preserve the positivity of the damaged stiffness, so that the damage variable range from 0 to –1, as usually assumed. However, for the model (3) this term is not strictly necessary, since tends to –1 asymptotically, as will be shown later. Note finally that normal and tangential reactions are uncoupled.

The potential used, while preserves the unilateral character of the interface, does not fulfil the condition that no relative displacement develops until fracture occurs. Although the introduction of a fictitious elastic stiffness is usual in interface models [1], in the author's opinion it introduces serious drawbacks, that, however, will not be commented in this paper.

2.2 The dissipation potential

Following the developments in [3,4], in the time independent case considered in this model, the dissipation functional turns out to be conjugated to the complementary dissipation functional (plastic potential), that is given by the indicator function of the elastic domain S. Multiple dissipation mechanisms can then be included considering S as the convex hull of a finite number of domains S_i. The elastic domain is specified by means of a yield function. For the sake of clearness, the development of the model is followed step by step starting from the form assumed by Carol:

$$g_C = \tau^2 + (c - \mu\sigma_0)^2 - (c - \mu\sigma)^2 \tag{4}$$

with c, , $_0$ material constants. In the plane expression (5) represents an hyperbola having the Coulomb bilateral as asymptotes. The intersection of g_C with the co-ordinate axes are given by $\sigma = \sigma_0\,, 2c/\mu - \sigma_0\,,\ \ \tau = \pm\mu\sqrt{\sigma_0(2c/\mu - \sigma_0)}$. Clearly, any fracture process occurs with irreversible displacements. In order to limit the phenomenon of dilatancy, the authors introduce a non-associative flow potential that becomes flat for compressive normal tension beyond a certain limit.

The first improvement consists in introducing the conjugate damage variable for definitely separating the irreversible plastic effects (due either to void development in the concrete or to fibres yielding or slipping) from the fracture phenomena which is associated mainly to damage. In the original model the parameters c, $_0$ where affected by the evolution of the fracture process, while was kept constant. A possible straightforward generalisation could then be to assume the yield function

$$g_C = \tau^2 + (c - \overline{\zeta} - \mu(\sigma_0 - \overline{\zeta})^2 - (c - \overline{\zeta} - \mu\sigma)^2 \qquad \overline{\zeta} = \zeta / w_0 \qquad w_0 = \sigma_0 / \left(K_n + K_f\right) \tag{5}$$

where the definition of the new damage variable is required for dimensionality reasons (note that has dimensions of a force per unit of length). The choice of (5) is motivated by the assumption that the damage affects equally the cohesion and the uniaxial limit stress. Expression (5) represents a lined surface, whose intersection with the plane =0 are the two straight lines

$$\sigma + \zeta = \sigma_0 \qquad (2-\mu)\,\zeta + \mu\sigma = 2c - \mu\sigma_0 \tag{6}$$

No physical damage mechanism is clearly associated t any of them. In an uniaxial process the initiation of the fracture can be found using the elastic law (2), as will be described soon. It is found that the limit values for the normal traction and the conjugated damage force are

$$\sigma_{\lim} = \frac{K_f + K_m}{K_f + K_m(1+n/2)}\,\sigma_0 \qquad \zeta_{\lim} = \frac{n}{2}\frac{K_m}{K_f + K_m(1+n/2)}\,\sigma_0 \tag{7}$$

so that neither $_0$ has a clear physical meaning, nor the value of the energy per unit area at the initiation of the fracture process matches the value that one would expect, i.e. ½ $_0$ w_0.

A further modification is then proposed, inspirited by the form (6) of the lined surface, that is it is proposed that the intersection of the limit surface with the plane =0 reduce to the two lines

$$\zeta - \zeta_0 = 0 \;\;,\;\; \sigma + \zeta = \sigma_k \quad ; \quad \sigma_k = \sigma_0 + \zeta_0 - 2c/\mu \tag{8}$$

where the value $_0$ is the energy necessary for mode I fracture initiation if in the process only damage occurs. The expression for the limit surface takes the form

$$g = \tau^2 + \left(2c - \mu(\sigma_0 - \sigma)\right)^2 - \left(2c - 2\mu(\overline{\zeta}_0 - \overline{\zeta}) - \mu(\sigma_0 - \sigma)\right)^2 \tag{9}$$

The function (9) presents several differences with respect to (5). The intersection with the - plane is now a parabola, with $\tau \to \infty$ for $\sigma \to -\infty$, as before, but the tangent to the curve tends to 0, so that the problem of dilatancy is substantially reduced. The surface (9) is still a lined one, whose intersection with the plane =0 is given by the two straight lines (8), that intersect for the value $\sigma = 2\sigma_0 - c/\mu$, positive for the common values of the material parameters. The activation value of the conjugate damage energy will be discussed in the next paragraph in connection with the analysis of an uniaxial process. The surface (9) forms in the plane an hyperbola, whose sides are asymptotically tangent to a Coulomb bilateral with slope 1/|2|. Note that, although negative values of the damage conjugate variable are not called out by the admissibility condition (9), they are not attainable on the basis of the elastic relations. A sketch of criterion (9) and its section with the co-ordinate plane are given in figs. 1,2. The flow rules are given by the usual consistency rule, $\dot{h}_p = \lambda \partial_s g(s), \quad \lambda \in \partial\Re^-(g(s))$, and they take the forms

$$\dot{w}_{n_p} = \lambda^* 4\mu(\zeta_0 - \zeta) \qquad \dot{w}_{t_p} = \lambda^* 2\tau \qquad \lambda^* = \lambda / 2\sqrt{g}$$
$$\dot{\omega}_p = \frac{\lambda^*}{w_0} 4\mu^2 \left(2(\zeta_0 - \zeta) + (\sigma_0 - \sigma) - 2c/\mu\right) = \frac{\lambda^*}{w_0} 4\mu^2 (\sigma_k - \zeta - \sigma) + \mu \frac{\dot{w}_{n_p}}{w_0} \tag{10}$$

It is stressed that the derivatives of the dissipation potential are continuous functions. Furthermore the permanent normal opening depends only on the difference between the current value of the damage energy and its limit value $_0$. When the latter is attained, the opening displacement becomes fully reversible, but damage still increases thanks to the other mechanism (first term in the latter of (10)). Additional dissipation mechanisms can be added in order to account for fibre yielding. The simplest choice could be $h_2 = \sigma + d\,\overline{\zeta} - \sigma_{0f} \le 0 \quad d = K_f / K_m$, with $_{0f}$ the limit tensile stress in the fibres. A sketch of the resulting domain in the uniaxial case is presented in fig. 3 First the matrix fails, then fibres plasticise, until pure separation of the interface is reached.

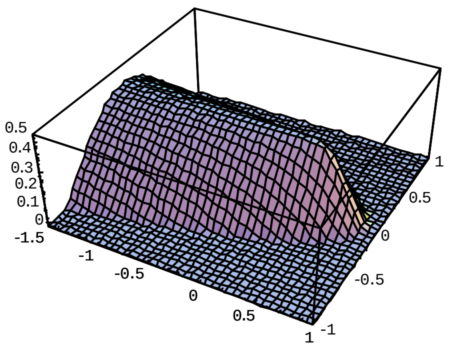

Figure 1 : Elastic domain

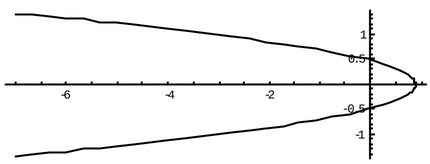

Figure 2 : section of the elastic domain

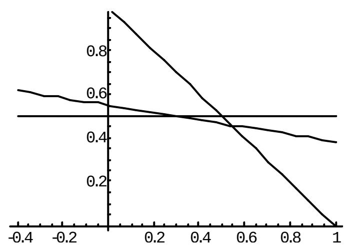

Figure 3 : uniaxial domain

3. EXEMPLIFICATION : UNIAXIAL RESPONSE TO MODE I FRACTURE

The characteristics of the model are investigated with reference to the special case of pure normal traction acting on the interface. Starting from a virgin state ($_e$=0), the elastic equations (2) furnish

$$\sigma=\left(K_m+K_f\right)w\ ;\quad \overline{\zeta}=\frac{n}{2}K_m\frac{w^2}{w_0}=\frac{n}{2}\frac{K_m}{\left(K_m+K_f\right)^2}\frac{\sigma^2}{w_0}\quad ;\ \tau=0 \tag{12}$$

Eqn.(12) is the parametric expression of a curve in the plane, that can intersect the limit surface in one of the 2 points that satisfy eqns. (8), according to the relative values of the material parameters ζ_0,σ_k. First is considered the case that the parametric curve (12) hits the limit surface on the line $=_0$, corresponding to a pure damage (reversible) process. Then one has

$$w_0=\frac{2\overline{\zeta}_0}{K_m n}\quad ;\quad \zeta_{\lim}=\overline{\zeta}_0 w_0=\frac{n}{2}K_m w_0^2\qquad \sigma_{\lim}=\frac{2}{n}\frac{K_m+K_f}{K_m}\overline{\zeta}_0 \tag{13}$$

Therefore, in order to obtain the desired value for the fracture activation energy, it must be $\overline{\zeta}_0=n/2\,\sigma_{0m}$, with $_{0m}$ the limit stress in the matrix. Indeed, substituting, it is found

$$w_0=\frac{\sigma_{0m}}{K_m}\quad ;\quad \zeta_{\lim}=\overline{\zeta}_0 w_0=\frac{n}{2}\sigma_{0m}w_0\qquad \sigma_{\lim}=\frac{K_m+K_f}{K_m}\sigma_{0m} \tag{14}$$

where the stress in the last expression is relevant to the whole composite. Proceeding with the extension, the stress progressively decreases on the fracture surfaces, and tends to 0 asymptotically, as can be easily proved. This is in contrast with the cohesive model, that is based on the existence of a limit critical value of the crack opening. However, the energy for the entire process is finite, and it can be shown that it is equal to [4]

$$G_f=\frac{1}{2}\sigma_{0m}\,w_0+\frac{n^2}{4}\zeta_0=\left(\frac{1}{n}+\frac{n^2}{4}\right)\zeta_0=\left(1+\frac{n^3}{4}\right)\frac{\sigma_0 w_0}{2} \tag{15}$$

The previous equation can be used for estimating the value of n. For whatever value of n, the descending branch of the -w curve is always sublinear.

In the case the path (12) intersects first the second line (8) of the boundary, the following is found (note that in the case of absence of fibres, the limit stress coincides with the uniaxial limit tension of the matrix, and for the initiation energy one finds $\zeta_{\lim 1}=\overline{\zeta}_{\lim 1}w_0=\left(\sigma_{0m}/K_m\right)\left(\sigma_k-\sigma_{0m}\right)$.

$$\begin{aligned}
w_{o1}&=\frac{2\overline{\zeta}_0}{K_m^2\,n^2}\left(\sqrt{K_f^2-2nK_m^2+n^2K_m^2\frac{\sigma_k}{\overline{\zeta}_0}}-K_f\right)\\
\sigma_{\lim 1}&=\sigma_{0m}\left(1+\frac{K_f}{K_m^2\,n}\left[\sqrt{K_f^2+2\,nK_m^2\left(\frac{\sigma_k}{\sigma_{0m}}-1\right)}-K_f\right]\right)\\
\overline{\zeta}_{\lim 1}&=\frac{\overline{\zeta}_0}{K_m^2\,n^2}\left[\sqrt{K_f^2-nK_m^2\left(2-n\frac{\sigma_k}{\overline{\zeta}_0}\right)}-K_f\right]^2\qquad \overline{\zeta}_0=\frac{n}{2}\sigma_{0m}
\end{aligned} \tag{16}$$

Loading-unloading uniaxial processes are shown in fig. 4,5, comparing the cases of pure damage activation, and of mixed mechanisms. Fig.4 refers to plain concrete and fig.5 to a small addition of fibres. Note that no residual displacement is found after unloading in the pure damage mechanism, and the value of the damage parameter tends asymptotically to 1(full damaged state).

A different case is encountered if the limit surface is hit on the line $\sigma + \zeta = \sigma_0$. In this case some irreversible displacement is present, as it happens when fibres are present, and yielding occurs. At the same time the stiffness decreases, as damage develops. Increasing the relative displacement the stress decreases, and the state point moves on the limit curve until it eventually reaches the condition $_{0}$. At this stage the fibres start to slip, and no more permanent displacement is added, while damage in the matrix increases further. Note, however, that thanks to the hypothesis (2) the rigidity of the fibres remains constant, so that a residual plateau is finally reached with a residual stiffness.

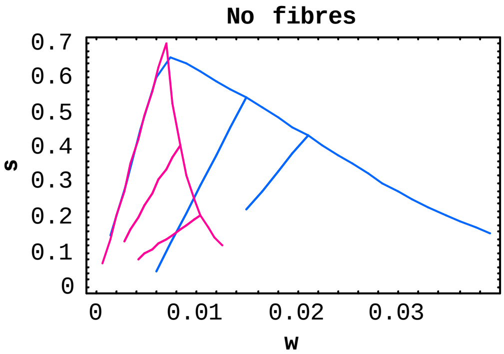

Figure 4 : Uniaxial fracture process for plain concrete

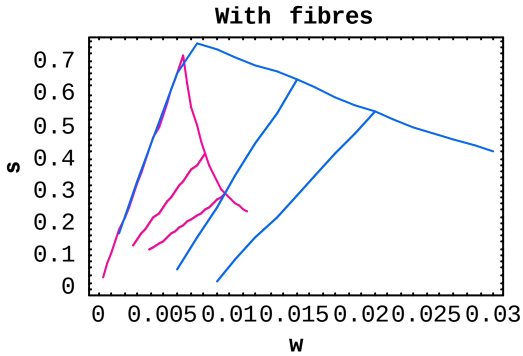

Figure 5 : Uniaxial fracture process for fibre reinforced concrete

4 FINAL COMMENTS

A constitutive model for a joint element has been developed based on a generalisation of the Cohesive Crack Model. The element is intended to be used for the microstructural analysis of fibre reinforced high strength concrete. The model is thermodynamically based, and differentiate both in the elastic energy and in the dissipation the contribution of the matrix and of the reinforcement. The unilaterality of the interface is guaranteed in compression, but some elastic opening is still admitted before fracture occurs. The choice of the elastic stiffness Kn is based on the energetic equivalence ½ $_0/K_m$ = $_0$, fracture activation energy. The author is conscious of the fact that the parameter is somewhat arbitrary, and that it introduces an internal length (the limit elastic opening w_0), that could affect the response of the model. A better model, where unilaterality is exactly fulfilled, can be implemented using a logarithmic damage law, and will be presented in a future paper.

In the paper only the simplest dissipation potential has been presented, but extra terms can be introduced, in the form of additional dissipation modes, for accounting explicitly for fibres yielding and other dissipative phenomena. However, the calibration of the parameters requires careful comparison with experimental data.

REFERENCES

[1] Carol, I., Prat, P.C., Lopez, C.M., Normal/shear cracking model: appliation to discrete crack analysis, *J. Engng. Mech.*, 123, 765-773, 1997

[2] Garcia-Alvarez, V.O., Gettu, R., Carol,I., "Numerical analysis of mixed model fracture in concrete using interface elements", Proceedings ECCOMAS 2000, Barcelona.

[3] Contrafatto, L., Cuomo, M., A new thermodynamically consistent continuum model for hardening plasticity coupled with damage, submitted to *Int. J. of Solids and Structures*.

[4] Cuomo, M., Forms of the dissipation function for some classes of viscoplastic models, submitted to *ZAMM*