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An elasto-plastic damage model for cementitious materials

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ABSTRACT
In the present paper, an elasto-plastic nonlocal damage model is proposed for studying the mechanical response of structural elements made of cementitious materials. An isotropic damage model, able to describe the different behavior in tension and in compression of the material is presented. To overcome the analytical and computational problems induced by the softening constitutive law, a regularization technique, based on the introduction of the damage Laplacian in the damage limit function, is adopted. A Drucker-Prager type of plastic limit function is proposed considering an isotropic hardening. A numerical procedure, based on an implicit 'backward-Euler' technique for the time integration of the plastic and damage evolutive equations, is developed and implemented in a finite element code. Some numerical examples are carried out in order to study the structural behavior of elements made of concrete and of fiber reinforced concrete.

KEYWORDS: Damage, Plasticity, Softening response, Nonlocal theory.

INTRODUCTION

Cementitious materials, such as concrete and masonry, are widely used in structural civil engineering. These materials are characterized by softening response coupled with plastic effects, due to the development of microcracks and of anelastic deformations.
The continuum damage mechanics represents an effective framework to model the softening behavior of cementitious materials [1], while the plasticity theory allows to take into account the anelastic material behavior [2]. Various macromechanical models have been proposed in literature to describe the mechanical response of structural elements made of cementitious materials. These models are mainly based on damage mechanics [3,4,5] and on plasticity theories [6,7].
In this paper, an elasto-plastic nonlocal damage model is proposed with the aim of developing an effective model able to predict the main features of concrete or masonry elements response. The stress-strain law accounts for damage and plastic effects.
The damage evolution process is controlled by a variable, which represents an equivalent deformation. The damage limit function considers the different response in tension and compression of the material.
In order to circumvent the pathological drawback due to strain and damage localization, a first gradient-enhanced theory is proposed. The nonlocal damage model is obtained by introducing the Laplacian of the damage variable in the loading function. The presence of the gradient term has a regularizing effect and avoids mesh-dependence when finite element analyses are performed.
The plasticity evolution law is governed by a plastic yield function with different threshold in tension and compression and with an isotropic hardening. The yield function and the plastic deformation evolution law depend on the effective stress.

The proposed model is implemented in the finite element code FEAP [8]. Some applications are developed to study the behavior of structural elements made of concrete and of fiber reinforced concrete.

AN ELASTO-PLASTIC NONLOCAL DAMAGE MODEL

The following stress-strain law is adopted for cementitiuos materials:

$$\boldsymbol{\sigma} = (1-D)^2 \mathrm{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) \tag{1}$$

where **C** is the second order elastic isotropic constitutive matrix, $\boldsymbol{\varepsilon}^P$ is the plastic deformation, D is the damage variable that can vary in the range [0,1]; $D=0$ corresponds to the virgin material state and $D=1$ to the total damaged state.
The rate constitutive equation is obtained by differentiating equation (1) with respect to the time:

$$\dot{\boldsymbol{\sigma}} = (1-D)^2 \mathbf{C}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^P) - 2(1-D)\dot{D}\mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P) \tag{2}$$

The elasto-plastic nonlocal damage model is based on the following assumption:

- the damage evolution is governed by the elastic strain $\boldsymbol{\varepsilon}^e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P$ in tension, and by the total strain $\boldsymbol{\varepsilon}$ in compression.
- the plastic deformation evolution is controlled by the effective stress $\tilde{\boldsymbol{\sigma}}$ defined as:

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{(1-D)^2} \tag{3}$$

Nonlocal damage model
An isotropic nonlocal damage model is proposed. The damage evolution is controlled by the consistency condition with the classical Kuhn-Tucker conditions:

$$\dot{F}\,\dot{D} = 0 \tag{4}$$

$$\dot{D} \geq 0 \qquad F \leq 0 \qquad \dot{D}\,F = 0 \tag{5}$$

where $F(Y,D)$ defines the damage limit function and Y is the variable associated to the parameter D, which controls the damage evolution. In particular, the variable Y is defined as follows:

$$Y = \frac{Y_t}{Y_{0t}} + \frac{Y_c}{Y_{0c}} \tag{6}$$

where Y_{0t} and Y_{0c} are the initial damage thresholds in tension and in compression, respectively. The quantities Y_t and Y_c represents the equivalent tensile and compressive deformations and they are function of the elastic deformation and of the total deformation, respectively [7].
The following damage limit function is proposed:

$$F = (Y-1) - (aY + K)D + h\nabla^2 D \tag{7}$$

In formula (7) the nonlocal effect is due to the presence of the Laplacian of the variable D, i.e. $\nabla^2\mathrm{D}$, in the damage limit function $F(Y,D)$. The parameter h is linked to the characteristic length of the material and controls the size of the localization region. The material constants K and a control the damage rate growth and the softening branch slope, respectively [7].

Plastic model

A plastic model with isotropic hardening, which takes into account the different strength in tension and in compression, is proposed. A plastic limit function $F_P = F_P(\tilde{\boldsymbol{\sigma}}, q)$, which depends on the effective stress $\tilde{\boldsymbol{\sigma}}$ (3) and on the thermodynamic force q, is introduced. The force q is associated to the internal hardening variable α by the rational relation:

$$q = -\frac{\alpha}{\alpha + \chi} \tag{8}$$

where χ is the hardening parameter.

The plastic deformation evolution is governed by the following equations:

$$\dot{\boldsymbol{\varepsilon}}_P = \dot{\lambda}_P \frac{\partial F_P}{\partial \tilde{\boldsymbol{\sigma}}} \tag{9}$$

$$\dot{\alpha} = \dot{\lambda}_P \frac{\partial F_P}{\partial q} \tag{10}$$

$$F_P \leq 0 \qquad \dot{\lambda}_P \geq 0 \qquad F_P\, \dot{\lambda}_P = 0 \tag{11}$$

where $\dot{\lambda}_P$ is the plastic multiplier that can be evaluated from the classical consistency equation $\dot{F}_P\, \dot{\lambda}_P = 0$. In the present model the following yield function is considered:

$$F_P(\tilde{\boldsymbol{\sigma}}, q) = 3J_2 + (\sigma_c - \sigma_t)I_1 - \sigma_c\sigma_t \tag{12}$$

where σ_c and σ_t are the compressive and tensile yield stresses, respectively, I_1 is the first invariant and J_2 the second deviatoric invariant of the effective stress tensor $\tilde{\boldsymbol{\sigma}}$.

SOLUTION PROCEDURE

A numerical procedure, based on an implicit 'backward-Euler' technique for the time integration of the plastic and damage evolutive equations of the model, is developed. Each non-linear step is solved using a predictor-corrector iterative technique within the splitting method.

In the predictor phase, the elasto-plastic problem (8)-(12) is solved with the damage field frozen. In this phase the plastic evolution is computed through a further nested predictor-corrector phase based on a return-mapping algorithm.

In the corrector phase the strain field is taken frozen and the damage evolution is evaluated solving the problem defined by equations (4)-(7).

Hence, the solution algorithm consists in the following two steps:

- an elasto-plastic predictor phase;
- a damage corrector phase.

The equation governing the two phases are reported in the following scheme:

Elastic-plastic predictor	Damage corrector
$\dot{\boldsymbol{\varepsilon}}^P = \begin{cases} \dot{\lambda}_P \dfrac{\partial F}{\partial \tilde{\boldsymbol{\sigma}}} & if\ F_P \geq 0 \\ 0 & if\ F_P < 0 \end{cases}$ $\dot{\alpha} = \begin{cases} \dot{\lambda}_P \dfrac{\partial F}{\partial q} & if\ F_P \geq 0 \\ 0 & if\ F_P < 0 \end{cases}$ $F_P \leq 0 \qquad \dot{\lambda}_P \geq 0 \qquad F_P\, \dot{\lambda}_P = 0$	$\dot{\boldsymbol{\varepsilon}}^P = 0$ $\dot{\alpha} = 0$
$\dot{D} = 0$	$\dot{F}\, \dot{D} = 0$ $\dot{D} \geq 0 \qquad F \leq 0 \qquad \dot{D}\, F = 0$
$\dot{\boldsymbol{\sigma}} = (1-D)^2 \mathbf{C}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^P)$	$\dot{\boldsymbol{\sigma}} = -2(1-D)\dot{D}\mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^P)$

NUMERICAL APPLICATIONS

The plastic nonlocal damage model is implemented in plane-stress 3 and 4 node finite elements in the code FEAP [8].
Some numerical examples are developed in order to study the structural behavior of elements made of concrete and of fiber reinforced concrete (FRC).
In order to reproduce the concrete behavior the nonlocal damage model without plasticity is adopted; on the contrary to simulate the FRC response, characterized by the matrix softening and the fiber debonding and pull-out, the nonlocal damage model with plasticity is used. In fact, the adoption of a model characterized by a plasticity with rational hardening, reproduces the fact that when the matrix is completely damaged, the FRC response tends to a limit value corresponding to the fiber bridging action.
The material parameters used for the concrete nonlocal damage model and for the FRC plastic nonlocal damage model are:

- Concrete:

$$\mathrm{E_m} = 3.0 \cdot 10^4\ \mathrm{N/mm^2} \qquad \nu_\mathrm{m} = 0.2$$
$$\mathrm{Y_{0t}} = 0.8 \cdot 10^{-4} \qquad \mathrm{Y}_{0c} = 0.8 \cdot 10^{-3}$$
$$K_t = 1.0 \cdot 10^{-4} \qquad K_c = 2.5 \cdot 10^{-3}$$
$$a_t = 0.97 \qquad a_c = 0.85$$
$$h = 1.0\ \mathrm{mm^2}$$

- FRC:

$$\mathrm{E_m} = 3.0 \cdot 10^4\ \mathrm{N/mm^2} \qquad \nu_\mathrm{m} = 0.2$$
$$\mathrm{Y_{0t}} = 0.9 \cdot 10^{-4} \qquad \mathrm{Y_{0c}} = 0.9 \cdot 10^{-3}$$
$$\mathrm{K_t} = 1.5 \cdot 10^{-4} \qquad \mathrm{K_c} = 2.5 \cdot 10^{-3}$$
$$\mathrm{a_t} = 0.99 \qquad \mathrm{a_c} = 0.90$$
$$\mathrm{h} = 1.0\ \mathrm{mm^2}$$
$$\sigma_\mathrm{t} = 100\ \mathrm{N/mm^2} \qquad \sigma_\mathrm{c} = 200\ \mathrm{N/mm^2}$$
$$\chi = 1.0 \cdot 10^{-3}\ \mathrm{N/mm^2}$$

where K_t, a_t and K_c, a_c are damage materials parameter in tension and in compression, respectively. In order to take into account the beneficial effects of the fibers in improving the material mechanics response the values of the parameters K_t, K_c, a_t and a_c adopted for in the FRC model are higher than the ones used for the concrete.

Initially, some analyses are performed to set the values of the material parameters in order to reproduce the concrete and FRC behavior in the pure tensile and compressive states.

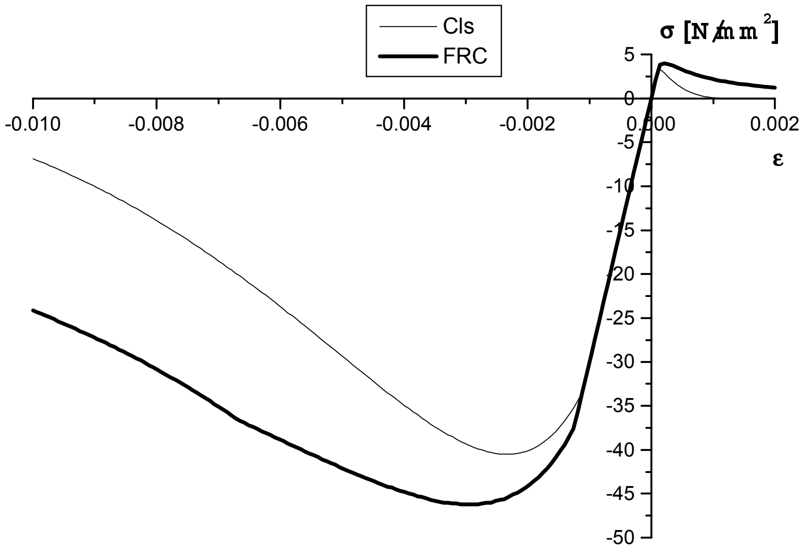

Figure 1: Tensile and compressive behavior of concrete and of FRC

In Figure 1 the stress-strain behavior in tension and in compression for the concrete and the FRC material is represented. It can be pointed out the beneficial effects of fibers in improving the mechanical response of concrete. In fact, in the post peak phase, when the fibers debonding and pull-out occurs the softening branch for FRC composite materials is less steep than for concrete.
The bending behavior of a concrete and of a FRC beam is investigated.
The geometrical parameters characterizing the analyzed beam are:

$$L = 800mm \qquad\qquad w = 250mm$$

where L is the length of the beam and w is the height of the cross section.
In Figure 2, the damage distribution in the FRC beam for different values of the prescribed displacement v is plotted.

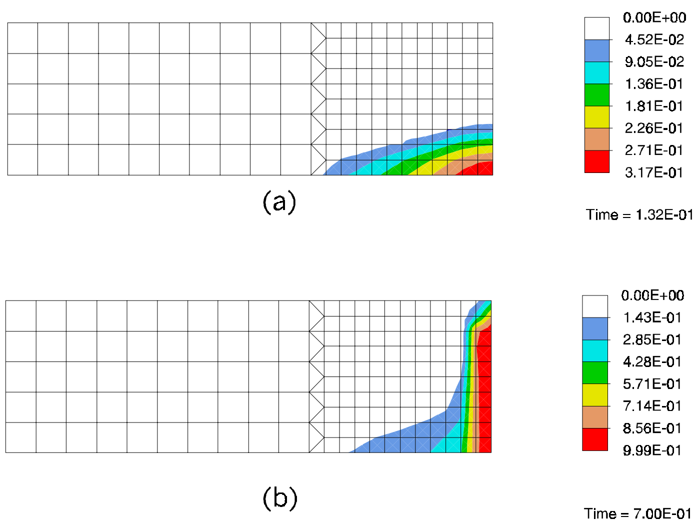

Figure 2: Damage evolution: a) v=*0.26 mm*, b) v=*1.4 mm*

It can be noted that the introduction of the damage Laplacian in the limit function *F* prevents the damage localization in the weakest point of the beam. The damage process starts at the bottom of the middle section (see Figure 2(a)), where the maximum tensile strains are concentrated. Then it propagates towards the topside of the beam when the compressive strain becomes significative (see Figure 2(b)).

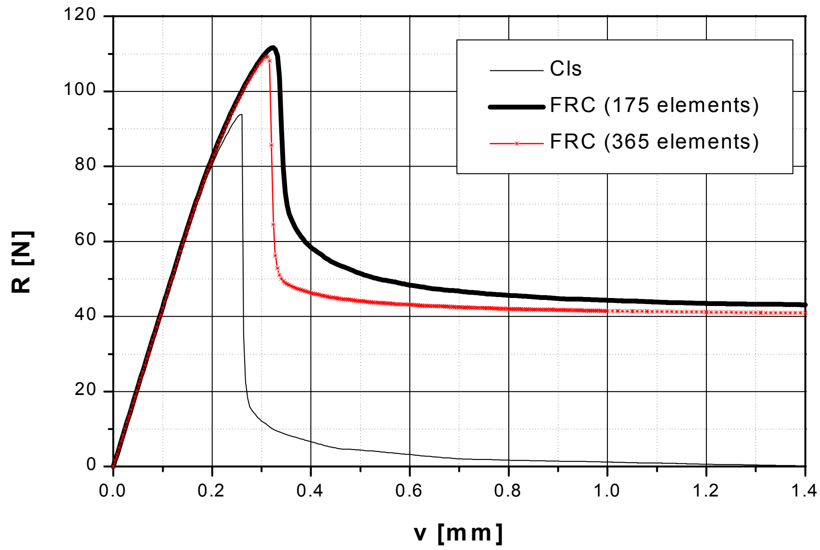

Figure 3: Load displacement curves for concrete and FRC

In Figure 3 the bending behavior of the concrete and the FRC beam is plotted. It can be pointed out that the plastic nonlocal damage model is able to reproduce the post-peak behavior of the FRC and the results are mesh independent.

CONCLUSIONS

A plastic nonlocal damage model for cementitious material is presented. The model is able to take into account the different behavior in tension and in compression of the material. To avoid the mathematical and numerical problems, due to the localization phenomenon, a gradient nonlocal model is adopted. The numerical results show the capability of the model in describing the mechanical behavior and the damage processes in concrete and FRC structural elements.

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