Ref. ICF100489OR

ACCOUNT OF PLASTICITY IN MACRO-MICROCRACK INTERACTION

V. Tamuzs[1], V. Petrova[2], S. Tarasovs[1]

[1] Institute of Polymer Mechanics, Riga, LV1006, Latvia
[2] Voronezh State University, Voronezh, Russia

ABSTRACT

Several papers during the last years were devoted to the problem of plastic zone growth for collinear cracks. In this work more general case was studied: plastic zone creation in isotropic, homogeneous elastic-perfectly plastic infinite plate containing crack and several microcracks near its tip. The macrocrack is subjected to a normal loading, acting at the infinity. The Dugdale model was used for plastic zones growing from the tips of macrocrack, microcracks are assumed to be elastic. The plastic zone length and crack opening displacement are found from asymptotic solution and compared with finite element solution.

KEYWORDS

Microcracks interaction, Dugdale model, crack opening displacement.

INTRODUCTION

The model of plastic zone ahead of the crack tip, introduced by Dugdale [1], is widely used in fracture mechanics. Many works were devoted to the problem of plastic zone creation in cracks interaction. Leonov and Onishko [2] considered the problem for two collinear equal cracks. The crack tips were supposed to close smoothly due to cohesive forces distributed along zones near the crack tips. The interior relaxed zones were coalesced. The review of Karihaloo [3] is devoted to the author's results on the fracture characteristics of elastic solids containing inhomogenities in the form of slitlike cracks with plastic zones. A perturbation solution was presented for widely spaced cracks. The numerical solution based on the Chebishev polynomials of the first kind was also obtained. Harrop [4] extended the Dugdale model for the case when plastic zones were being closed by cohesive parabolic stress distribution. Theocaris [5] studied the Dugdale model for two unequal collinear cracks. The theoretical and experimental works devoted to the modified Dugdale model were reviewed in the paper.

ASYMPTOTIC SOLUTION

The solution of Romalis and Tamuzs [6] for the problem of micro-macrocrack interaction was adopted in this work for macrocrack with plastic zones. Microcracks are assumed to be elastic.

Let an isotropic elastic-perfectly plastic plane contain a macrocrack and N microcracks of length $2a_k$. It will be assumed that all microcracks have the same length, i.e. $a_k = a$. Cartesian coordinates x and y are centered at the midpoint of the main crack with the crack along the x-axis. The local coordinate systems x_k , y_k are attached to each microcrack. The microcrack position is determined by its midpoint coordinate z^0_k and the inclination angle α_k to the x-axis.

The main crack consists from an open zone [- c_1 , c_2] and two plastic zones created at the vicinities of the crack tips - c_1 and c_2 such that the full length of the main crack is considered to be $2a_0$. In such case the lengths of the plastic zones are $|a_0 - c_1|$ and $|a_0 - c_2|$. The tensile stress T is applied at infinity while the plastic zones are closed by stress q which is normally identified with the yield stress of the material. The problem can be reduced to solving the problem with boundary conditions on the crack lines. For non-dimensional parameters $b = c_1/a_0$, $d = c_2/a_0$ and $\tau = x_k/a_0$ (k=0,1, …, N) the boundary conditions are written as

$$
\begin{aligned}
&p_0(\tau) = \begin{cases} -T & -b < \tau < d \\ -T + q & -1 < \tau < -b \ \ and \ \ d < \tau < 1 \end{cases} \\
&p_k(\tau) = -\frac{T}{2}\left(1 + e^{-2i\alpha_k}\right), \ |\tau| < a_k / a_0 \qquad (k = 1,2,...,N)
\end{aligned}
\tag{1}
$$

Figure 1: Coordinates of crack tips and plastic zones.

Singular integral equations for the system of cracks with self-equilibrium stresses on their faces have been derived by Panasyuk et al [7]

$$
\int_{-a_n}^{a_n} \frac{g'_n(\tau)}{t-x} dt + \sum_{\substack{k=0 \\ k \neq n}}^{N} \int_{-a_k}^{a_k} \left[g'_k(t) K_{nk}(t,x) + \overline{g'_k(t)} L_{nk}(t,x) \right] dt = \pi p_n(x) \quad (n = 0,1,...,N)
\tag{2}
$$

where K_{nk} and L_{nk} are the regular kernels containing the geometric parameters of the problem [7]. The $g'_k(x)$ represents the derivatives of displacement jumps on the crack lines:

$$
g'_k(x) = \frac{2\mu}{i(\kappa+1)} \frac{\partial}{\partial x}([u_k] + i[v_k]) = v'_k - iu'_k ,
\tag{3}
$$

where $\mu = E/2(1+v)$ is the shear modulus, E Young's modulus, v Poisson's ratio, $\kappa = 3-4v$ for the plane strain state and $\kappa = (3-v)/(1+v)$ for the plane stress state.

By using variables $t = a_k \tau$, $x = a_n \chi$ the system (2) can be reduced to a dimensionless form and the solution of it is sought as a power series with regard to the small parameter $\lambda = a/a_0$

$$
g'_n = \sum_{p=0}^{\infty} g'_{np} \lambda^p , \ \ K_{nk} = \sum_{p=0}^{\infty} m_{nkp} \lambda^p , \quad L_{nk} = \sum_{p=0}^{\infty} n_{nkp} \lambda^p
\tag{4}
$$

Inserting series (4) into Eqn. 2. and equating the expressions of like powers of λ, the recurrent relations are obtained for the subsequent determination of coefficients g'_{np}. We derived them by retaining terms up to λ^2 :

$$g'_{00}(\chi) = -\frac{1}{\pi\sqrt{1-\chi^2}}\int_{-1}^{1}\frac{\sqrt{1-\tau^2}}{\tau-\chi}p_0(\tau)d\tau$$

$$g'_{02}(\chi) = \frac{1}{\pi\sqrt{1-\chi^2}}\sum_{k=1}^{N}\int_{-1}^{1}\left[g'_{k0}(\tau)m_{0k1}(\tau,\chi)+\overline{g'_{k0}(\tau)}n_{0k1}(\tau,\chi)\right]d\tau \tag{5}$$

$$g'_{n0}(\chi) = \frac{1}{\pi\sqrt{1-\chi^2}}\left\{-\int_{-1}^{1}\frac{\sqrt{1-\tau^2}}{\tau-\chi}p_n(\tau)d\tau+\int_{-1}^{1}[g'_{00}(\tau)m_{n01}(\tau,\chi)+\overline{g'_{00}(\tau)}n_{n01}(\tau,\chi)]d\tau\right\}$$

Taking into account the expression for $p_0(\tau)$ (1) the derivative of the vertical displacement on the isolated crack line is obtained by first two approximations of $v'_0 = \mathrm{Re}\{g'_0\}$.

$$v'_{00}(\chi) = \mathrm{Re}\,g'_{00} = \frac{\chi(q-T)}{\sqrt{1-\chi^2}}+\frac{q}{\pi\sqrt{1-\chi^2}}\int_{-b}^{d}\frac{\sqrt{1-\tau^2}}{\tau-\chi}d\tau \qquad |\chi|\le 1,\ [-b,d]\subset[-1,1] \tag{6}$$

$$(b,d>0)$$

$$v'_{02}(\chi) = \mathrm{Re}\,g'_{02} = \frac{1}{2\sqrt{1-\chi^2}}\sum_{k=1}^{N}\mathrm{Re}\left[(p_k+I_{k0})m'_{0k1}(\chi)+(\overline{p}_k+\overline{I}_{k0})n'_{0k1}(\chi)\right] \tag{7}$$

$$\begin{aligned}I_{k0} = \frac{T-q}{2}&\left[2-\frac{\overline{u}_k}{\sqrt{\overline{u}_k^2-1}}-\frac{u_k}{\sqrt{u_k^2-1}}+e^{-2i\alpha_k}\frac{u_k-\overline{u}_k}{(\overline{u}_k^2-1)^{3/2}}\right]\\ &+\frac{q}{2\pi}\left[\left(\frac{1}{\sqrt{\overline{u}_k^2-1}}+e^{-2i\alpha_k}\frac{u_k(\overline{u}_k-u_k)}{(\overline{u}_k^2-1)^{3/2}}\right)\int_{-b}^{d}\frac{\sqrt{1-t^2}}{t-\overline{u}_k}dt\right.\\ &\left.+\frac{1}{\sqrt{u_k^2-1}}\int_{-b}^{d}\frac{\sqrt{1-t^2}}{t-u_k}dt+e^{-2i\alpha_k}\frac{u_k-\overline{u}_k}{\sqrt{\overline{u}_k^2-1}}\int_{-b}^{d}\frac{\sqrt{1-t^2}}{(t-\overline{u}_k)^2}dt\right]\end{aligned} \tag{8}$$

and

$$m'_{0k1}(\chi) = \frac{e^{i\alpha_k}}{2}\left[\frac{e^{-i\alpha_k}(\overline{u}_k\chi-1)}{(\chi-\overline{u}_k)^2\sqrt{\overline{u}_k^2-1}}+\frac{e^{i\alpha_k}(u_k\chi-1)}{(\chi-u_k)^2\sqrt{u_k^2-1}}\right];$$

$$n'_{0k1}(\chi) = \frac{e^{-i\alpha_k}}{2}\left[(u_k-\overline{u}_k)e^{-i\alpha_k}\frac{-\chi^2+2\overline{u}_k^3\chi-3\overline{u}_k^2+2}{(\chi-\overline{u}_k)^3(\overline{u}_k^2-1)^{3/2}}+(e^{i\alpha_k}-e^{-i\alpha_k})\frac{\overline{u}_k\chi-1}{(\chi-\overline{u}_k)^2\sqrt{\overline{u}_k^2-1}}\right] \tag{9}$$

The derivatives of the vertical displacement discontinuities on the macrocrack line are written as

$$v_0'(\chi) = v_{00}'(\chi) + \lambda^2 v_{02}'(\chi), \qquad \lambda = a / a_0 \tag{10}$$

Then the stress intensity factors at the macrocrack tips -a_0 and a_0 are defined

$$k_I^{\pm} - ik_{II}^{\pm} = \mp \lim_{\chi \to \pm 1} \sqrt{1-\chi^2}\, a_0^{1/2} [v_0'(\chi) - iu_0'(\chi)] \tag{11}$$

The length of the unknown plastic zones will be calculated from the condition

$$k_I(\pm a_0) = 0, \tag{12}$$

which means that the crack faces are closed smoothly.

Solution for an Isolated Crack
Using the zero-th approximation of derivative of the vertical displacement for the symmetric case, Eqn. 6., the SIF for isolated crack is

$$k_{I00} = -\lim_{\chi \to 1} \sqrt{1-\chi^2}\, a_0^{1/2} v_{00}'(\chi) = a_0^{1/2} \left[T - q + \frac{2q}{\pi} \arctan \frac{b}{\sqrt{1-b^2}} \right] \tag{13}$$

The SIFs at right and left tips are equal for an isolated crack as well as the lengths of the plastic zones. The size (1-b) of the plastic zone is obtained by equating the expression of k_{I00} to zero

$$b = \cos \frac{\pi}{2} \frac{T}{q} \tag{14}$$

We obtain well known expression for an isolated relaxed crack with plastic zones of size (1-b) in an infinite solid (Dugdale, [1]). The value of CTOD can be determined by integrating $v_{00}'(\chi)$ and then calculating the obtained expression at $\chi = b$

$$v_{00}(b) = -q \frac{2}{\pi} b \ln\left(\cos\left(\frac{\pi}{2} \frac{T}{q} \right) \right) \tag{15}$$

Solution for the Macrocrack Interacting with Microcracks
For the macro-microcrack system the derivatives of the normal displacements on the macrocrack line are determined by Eqn. 6-10. Consider the symmetrical case when the plastic zones have same length, i.e. b=d. We should determine the second term k_{I02} in the SIF.

$$k_{I02} = \mp \lim_{\chi \to \pm 1} \sqrt{1-\chi^2}\, a_0^{1/2} v_{02}'(\chi) \tag{16}$$

The equation for determination the plastic zones is

$$\left[\frac{T}{q} - \frac{2}{\pi} \arccos b \right] + \frac{\lambda^2}{2q} \sum_{k=1}^{N} \operatorname{Re}\left[\left(p_k + I_{k0}(T,q,u_k,b)\right) m_k + \left(\overline{p}_k + \overline{I_{k0}(T,q,u_k,b)}\right) n_k \right] = 0 \tag{17}$$

where

$$m_k = \frac{e^{i\alpha_k}}{2}\left[\frac{e^{-i\alpha_k}}{(\bar{u}_k \mp 1)\sqrt{\bar{u}_k^2 - 1}} + \frac{e^{i\alpha_k}}{(u_k \mp 1)\sqrt{u_k^2 - 1}}\right]$$

$$n_k = \frac{e^{-i\alpha_k}}{2}\left[(\bar{u}_k - u_k)e^{-i\alpha_k}\,\frac{2\bar{u}_k \pm 1}{(\bar{u}_k \mp 1)(\bar{u}_k^2 - 1)^{3/2}} + \frac{e^{i\alpha_k} - e^{-i\alpha_k}}{(\bar{u}_k \mp 1)\sqrt{\bar{u}_k^2 - 1}}\right] \quad (18)$$

The second term in the value of CTOD can be determined by integrating $v'_{02}(\chi)$ and then calculating the obtained expression at χ=b

$$v_{02}(\chi) = \int v'_{02}(\chi)d\chi = \frac{1}{2}\sqrt{1-\chi^2}\sum_{k=1}^{N} \mathrm{Re}\left[(p_k + I_{k0})\overset{*}{m}_{0k1}(\chi) + (\bar{p}_k + \bar{I}_{k0})\overset{*}{n}_{0k1}(\chi)\right] \quad (19)$$

where

$$\overset{*}{m}_{0k1}(\chi) = e^{i\alpha_k}\ \mathrm{Re}\left[\frac{e^{i\alpha_k}}{(u_k - \chi)\sqrt{u_k^2 - 1}}\right]$$

$$\overset{*}{n}_{0k1}(\chi) = \frac{e^{-i\alpha_k}}{2}\frac{1}{(\bar{u}_k - \chi)\sqrt{\bar{u}_k^2 - 1}}\left[(u_k - \bar{u}_k)e^{-i\alpha_k}\,\frac{2\bar{u}_k^2 - \bar{u}_k\chi - 1}{(\bar{u}_k - \chi)(\bar{u}_k^2 - 1)} + e^{i\alpha_k} - e^{-i\alpha_k}\right] \quad (20)$$

The full value of CTOD is $v_0(b) = v_{00}(b) + \lambda^2 v_{02}(b)$.

NUMERICAL SOLUTION

Finite element method (FEM) was used to solve the problem. Plastic zones are modelled by introducing non-linear interface elements along the line of supposed plastic zone growth. The interface elements have user defined traction-opening relationship (constant traction equal to the yield stress of the material in the case of perfect plasticity).

RESULTS

As an example the macrocrack with two horizontal microcracks ahead of its tip was considered. Each microcrack is ten times smaller then macrocrack and geometry of the problem is presented in Fig.2. (distance between macrocrack and microcracks, d, and between microcrack and center line, h, is equal to the size of microcrack).

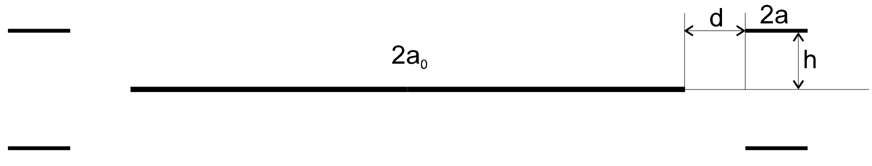
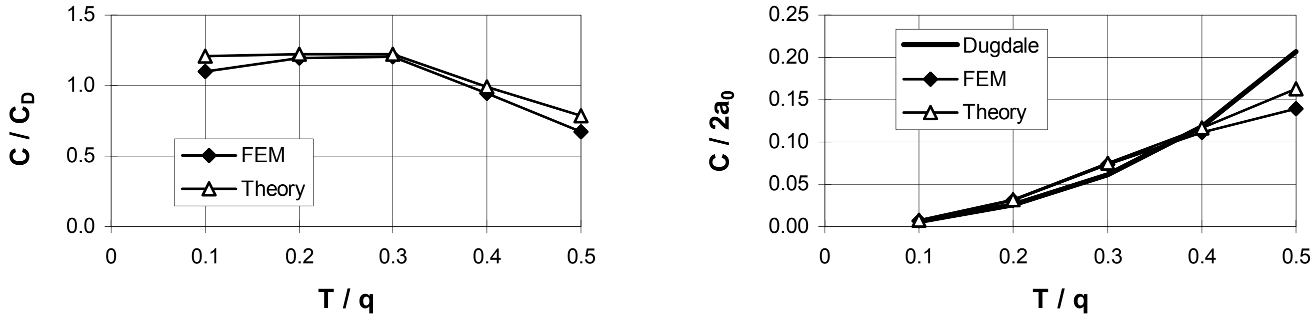


Figure 2: Geometry of the problem.

In Fig. 3-4 plastic zone length, C, and crack opening displacement, COD, are presented. The results are normalised with respect to plastic zone length and COD of single crack. The analytical results are compared with finite element solution.



a) b)

Figure 3: Plastic zone length: a) results are normalised with respect to Dugdale solution, b) results are normalised with respect to the length of macrocrack.

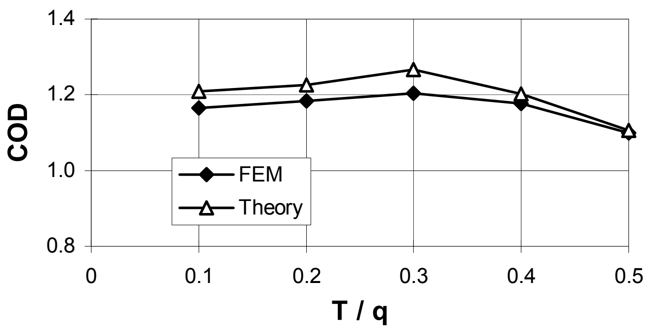


Figure 4: Crack opening displacement.

CONCLUSION

The pair of microcracks ahead of the crack tip increases the COD comparably with COD of a single crack whereas the plastic zone diminishes when it approaches to microcracks. So the defects ahead of the crack tip can enlarge the brittleness of material.

REFERENCES

1. Dugdale, D.S. (1960) *J. Mech. Phys. Solids.* 8, 100.
2. Leonov, M.Ya. and Onishko, L.V. (1962) In: *Teorija Plastin i Obolochek* (in Russian). Izd. AN Ukr.SSR, Kiev, pp. 200-203.
3. Karihaloo, B.L. (1979) *Eng. Frac. Mech.* 12, 49.
4. Harrop, L.P. (1978) *Eng. Frac. Mech.* 10, 807.
5. Theocaris, P.S. (1983) *Eng. Frac. Mech.* 18, 545.
6. Romalis, N. and Tamuzh, V. (1984) *Mechanics of Composite Materials.* 20, 35.
7. Panasyuk, V.V., Savruk, M.P., Dacishin, A.P. (1976) *Raspredelenie naprjazhenij okolo treschin v plastinah* (in Russian). Kiev.