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A SIMPLE METHOD FOR TOPOLOGY OPTIMIZATION USING ORTHOTROPIC MATERIAL PROPERTIES

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ABSTRACT

In this paper, a simple method for topology optimization of linearly elastic continuum structures is presented. For prescribed loading and boundary conditions, and subject to a specified amount of structural material, the optimum structural topology is determined from the condition of maximum integral stiffness, which is equivalent to minimum elastic compliance. The SIMP (Simple Isotropic Material with Penalization) is improved in order to save the computation time. Instead of using isotropic material with SIMP method, the material is assumed to be pseudo orthotropic continuum by setting the principal axis of the material to principal stress directions and introducing a new penalty function to the young's modulus at the minor principal stress direction. Numerical examples illustrate that the present method is more efficient than the SIMP method.

KEYWORDS

Topology optimization, Optimal design, Maximum stiffness structure, CAD

INTRODUCTION

The research in the area of topology optimization is extremely active recent years. Several topology optimization methods have been proposed, and used for the design of practical problem. However, there still exist a number of problems such as checkerboard, mesh-dependence, and local minima being investigated currently.

The topology optimization of continuum structures corresponds to finding the connectedness, shape and number of holes such that the objective function is extremized. Using a density function $\boldsymbol{r}$ defined on design domain $\boldsymbol{W}$ to describe the material distribution, it can only takes the value 0 (void) or. 1 (solid), i.e.

$$\boldsymbol{r}(x) = 0 \;\; or \;\; 1, \forall x \in \boldsymbol{W} \tag{1}$$

It is well known that the 0-1 topology optimization problem lacks solutions in general. The reason is that given one design the introduction of more holes will generally increase the efficiency measure. A general approach to avoid this problem is that relax the 0-1 density constraint to a continuous variable as

$$0 < \boldsymbol{r}(x) \leq 1, \forall x \in \boldsymbol{W} \tag{2}$$

to achieve an approximate solution, and then use other techniques to approach a black/white design. Bendsøe and Kikuchi [1] introduced a periodic microstructure to the material through the use of so-called homogenization approach to topology optimization that allows the volume density of material to cover the complete range of values from 0 to 1 by changing the size of microstructure. To use this method, it is necessary to determine the effective material characteristic by homogenization, and results are obtained with large regions of perforated microstructure or composite materials ($0 < \boldsymbol{r} < 1$). Another approach that is called density function method [2] disregards the details of the microstructure and defines the elasticity tensor as a function of density of material directly. The SIMP (Simple Isotropic Material with Penalization) approach [3] is kind of density function method, in which the stiffness tensor of the intermediate density material is penalized with an exponential function of density to somehow approach a 0-1 design. Using the SIMP approach the stiffness tensor of an intermediate density material is

$$C_{ijkl}(\boldsymbol{r}) = C^0_{ijkl}\, \boldsymbol{r}^p \tag{3}$$

where C^0_{ijkl} is the stiffness tensor of material and p is the penalization factor which ensures that the continuous design variables are forced towards a black/white solution. To control the value of p can control the speed of convergence and the rate of intermediate density material in the result design. It is a popular method and has also been widely used because of its simplicity.

In this study, the SIMP approach is improved in order to be more efficient in optimization process. Using the concept of Michell truss, we assume material to be a pseudo orthotropic continuum and introduce new penalties to the Yang's modulus. An example is attached at the end to show the validity of this approach.

FORMULATION OF OPTIMIZATION PROBLEM

In this paper, we treat the problem of maximum stiffness of structures with the given amount of material. Design for maximum stiffness of statically loaded linearly elastic structures is equivalent to design for minimum compliance defined as the work done bye the set of given loads against the displacements at equilibrium. Consider an initial domain $\boldsymbol{W}$ with a boundary $\boldsymbol{G}$ loaded with a static force $\boldsymbol{P}$. The optimization problem can be formulated as

$$\begin{aligned} &\textit{Minimize} && \int_G P_i u_i \, dG \\ &\textit{Subject to} && \int_W C_{ijkl} u_{k,j} v_{i,j} \, dx = \int_G P_i v_i \, dG \\ & && \boldsymbol{u} \in H^1(\boldsymbol{W}),\ \forall \boldsymbol{v} \in H^1(\boldsymbol{W}) \\ & && \int_W \boldsymbol{r}(x)\, dx \leq M_0 \\ & && 0 < \boldsymbol{r}(x) \leq 1 \end{aligned} \tag{4}$$

where $\boldsymbol{u}$ is the displacement vector, $\boldsymbol{v}$ is the variation of $\boldsymbol{u}$, M_0 is the given amount of material. Using Lagrange multiplier method, this optimization problem can be rewritten to a stationary problem of a Lagrange functional as

$$
\begin{aligned}
L(\boldsymbol{u},\boldsymbol{v},\boldsymbol{r},\boldsymbol{L}) = \int_G P_i\, u_i\, dG - \int_W C_{ijkl} u_{k,j} v_{i,j}\, dx + \int_G P_i v_i\, dG \\
- \boldsymbol{L}\left(\int_W \boldsymbol{r}(x)\, dx - M_0\right)
\end{aligned}
\tag{5}
$$

Taking the variation of the Lagrange functional, the optimality criterion of this problem can derived as

$$
\int_W \left(\boldsymbol{L} - \frac{\partial C_{ijkl}}{\partial \boldsymbol{r}} u_{k,l} v_{i,j} \right) \dot{\boldsymbol{r}}\, dx = 0 \tag{6}
$$

$$
\begin{aligned}
\int_W C_{ijkl} \dot{u}_{k,j} v_{i,j}\, dx = \int_G P_i\, \dot{u}_i\, dG \\
\boldsymbol{u} \in H^1(W), \forall \boldsymbol{v} \in H^1(W)
\end{aligned}
\tag{7}
$$

Eqn.7 is an adjoint equation from which the adjoint variable $\boldsymbol{v}$ can be solved.

INTRODUCING ORTHOTROPIC MATERIAL PROPERTIES

As mentioned above, checkerboard problem is one of the problems occurring frequently in the topology optimization process. As shown in Figure 1, the result design consists of alternating solid and void elements so that it is not useful for practical purpose. To avoid the checkerboard pattern, the use of higher-order finite elements has been suggested [4]. However, this approach is the substantial increase in cpu-time because of not only the increasing of degrees but also the low convergence speed. A large penalty parameter p is used in general to reduce the cpu-time, but it has the possibility to decrease the performance of the structure. In this research, we try to find a new kind of penalty to the intermediate density material so that the optimization process is more efficient in finding the solution and converging to 0-1 material distributions with less performance loss of the result design. The hint is obtained from the so-called Michell truss [5] that is derived by Michell for a minimum weight truss of a plane structure. As shown in

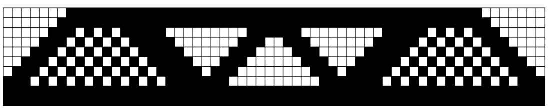


Figure 1: Checkerboard pattern

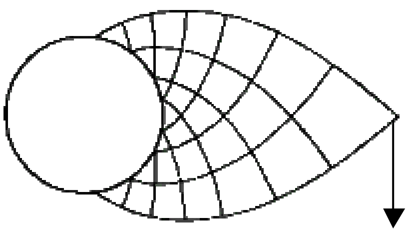


Figure 2: Michell truss

Figure 2, the Michell truss is an orthogonal net structure, in which each component extends in the direction of principal stress and crosses mutually with a right angle. Although this solution is impractical because it derived without the constraint on geometric shape and number of holes, the concept is applied in this paper. Instead of using isotropic material, the material is assumed to be pseudo orthotropic continuum and the principal axis of the material is set to principal stress directions. It is reasonable to consider that this approach will be efficient in generating the topology. In order to create pseudo orthotropic material, we introduce different penalty functions to the young's modulus at the major principal stress direction and that at the minor principal stress direction as

$$E_1 = E^0 \rho^p \tag{8}$$

$$E_2 = E_1 \left| \frac{\sigma_1}{\sigma_2} \right| \tag{9}$$

Where E^0 is the true Young's modulus, σ_1 and σ_2 are the major principal stress and the minor principal stress respectively ($|\sigma_1| \geq |\sigma_2|$), E_1 and E_2 are the Young's modulus at the directions of σ_1 and σ_2 respectively. Eqn.7 is the same penalty with SIMP method and Eqn.8 is a new penalty. The reason why give E_2 a harder penalty is that E_2 has less effect on the performance of the structure than E_1.

NUMERICAL RESULTS

In this section, a square plate example is performed in order to investigate the effect of the new penalties presented in this paper. As shown in Figure 3, the plate is computed for maximizing the stiffness, in which the left side is fixed and the right side is applied with a load. It is modeled by 8-node isoperimetric elements with the material properties Young's modulus E=2.10x10^{11} N/m^2 and Poisson's ratio =0.3. The volume constraint is set to 40% of the entire design domain. The penalty parameter p is raised from 1.0 to 2.0 with the step of 0.1 during the optimization process. Figure 4 shows an optimal result in the case of using isotropic material properties and Figure 5 shows an optimal result in the case of using orthotropic material properties. Comparing these two results, the obvious difference can be found. From Figure4, it is found that the clear topology didn't appear after 150 iterations. The clear topology appeared after 330 iterations. On the other hand, from results of Figure 5, it is found that the clear topology appeared after 150 iterations with p=1.4. It is similar to the result after 330 iterations and it is clear enough to use as a last result. Another fact can be confirmed that results of the two cases are almost the same; there is even no

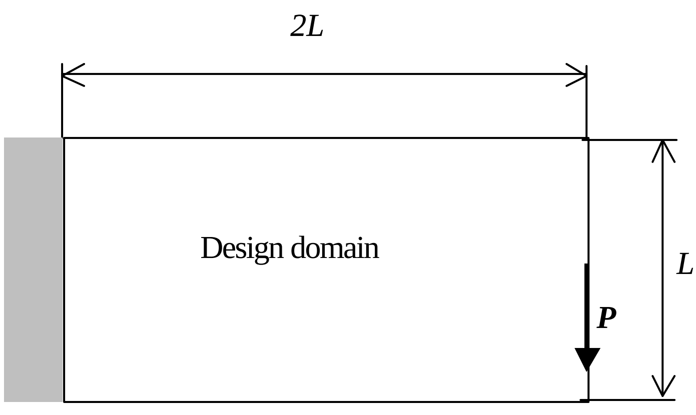
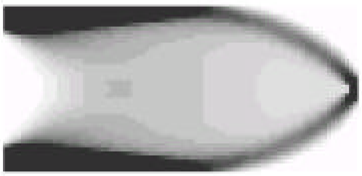


Figure 3: Design problem and boundary conditions

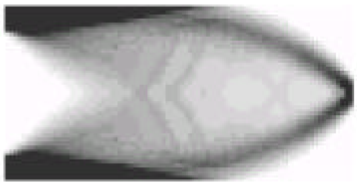


(a) After 30 iterations, p=1.0

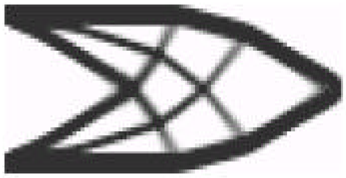
(b) After 150 iterations, p=1.4

(c) After 330 iterations, p=2.0

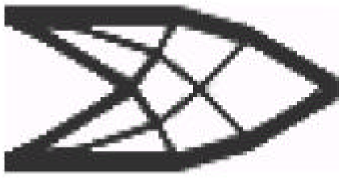
Figure 4: Results for the use of isotropic material properties



(a) After 30 iterations, p=1.0



(b) After 150 iterations, p=1.4



(c) After 330 iterations, p=2.0

Figure 5: Results for the use of orthotropic material properties

difference of the values of two objective functions. This fact means that there is almost no extra loss of the efficiency of last design to use with new penalty functions.

CONCLUSION

In this paper, a simple method for topology optimization of linearly elastic continuum structures is presented. Instead of using isotropic material with SIMP method, the material is assumed to be pseudo orthotropic continuum by setting the principal axis of the material to principal stress directions. We introduce a new penalty function to the young's modulus at the minor principal stress direction. Numerical examples illustrate that the present method is more efficient than the SIMP method.

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