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A NEW MATERIAL MODEL AND ITS APPLICATION TO FRACTURE ANALYSIS IN FIBER PULL-OUT TESTS

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ABSTRACT

A new material model based on the assumption that the material consists of hard blocks and soft layers is proposed. In such materials, extensive plastic deformation develops within the soft layers whereas the blocks are rigid (elastic). The plastic deformation may lead to failure within layers. The dominated mode of deformation is shear. Assuming that the only non-zero deviatoric stress is a shear stress it is possible to obtain an analytical (semi-analytical) solution within the layer for quite an arbitrary constitutive law of the layer material. Using this solution the behavior of block/layer continuum may be described. To demonstrate the main features of the model the axisymmetric problem of the fiber pull-out is solved semi-analytically under simplified assumptions. In particular, the hard blocks and soft layers are circular cylinders of different thickness. The end effects are neglected. Based on these assumptions, the solution for arbitrary number of layers and blocks is given by induction. The material of the layers is assumed to be elastic/plastic, hardening. The evolution of damage is described by the Lemaitre law. The fracture condition is defined by a critical value of the damage variable. It is shown that the plastic zone develops in a jump-like manner and fracture initiates in the layer adjacent to the fiber. The displacement of the fiber at the initiation of fracture is determined. The effects of geometric and material parameters on the size of the plastic zone and fracture initiation are discussed.

KEYWORDS

pull-out test, elastic/plastic solids, composites, damage mechanics

INTRODUCTION

Studies of surfaces (including fracture surfaces) of samples of elastic/plastic materials demonstrated that at the micro- and meso scales zones of plastic deformations are alternated with zones of elastic deformation. In particular, it was shown in [1] that the process of deformation near interfaces consists of the formation of rigid blocks and thin zones where intensive straining occurs. This feature of the deformation process needs to be taken into account at evaluation of the limit deformation characteristics as well as fracture and damage parameters of structural elements. Note that such a deformation scheme is inherent to filled composites with compliant matrix and rigid inclusions at a high degree of filling.
The present paper is devoted to modeling of the aforementioned effect on the basis of a model proposed in [2] and its application to the analysis of fiber pull-out test.

MATERIAL MODEL

A general model for a continuum consisting of rigid blocks and elastic/plastic layers between the blocks has been proposed in [2]. Here the model is specialized to the case of anti-plane axisymmetric deformation using the assumption that all blocks are hollow cylinders with the same axis of symmetry. Then, all elastic/plastic layers are also hollow cylinders. Strains are small and are localized within the layers whereas the blocks can only move as rigid bodies along the axis of symmetry. A representative element of the continuum consists of an elastic/plastic layer and a rigid block adjacent to this layer as shown in Figure 1. Since another rigid block is adjacent to the layer at $r = a$, it is possible to assume,

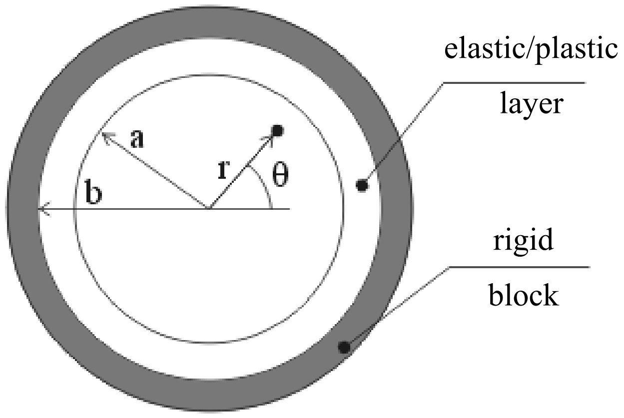

Figure 1: A representative element of the continuum

without loss of generality, that $u = -u_0 \ (u_0 > 0)$ at $r = a$ and $u = 0$ at $r = b$ where u is the axial velocity in the elastic/plastic layer. We will search for a solution with the only non-zero component of the stress tensor τ_{rz} in a cylindrical coordinate system whose z – axis coincides with the axis of symmetry of the blocks and layers. Then, for isotropic materials the only non-zero component of the strain tensor will be ε_{rz}. Using the definition for the equivalent strain and for the shear strain one can find

$$\varepsilon_{eq} = \left(2/\sqrt{3}\right)\varepsilon_{rz} = \left(1/\sqrt{3}\right)\partial u/\partial r \tag{1}$$

The only non-trivial equilibrium equation is $\partial \tau_{rz}/\partial r + \tau_{rz}/r = 0$ and its general solution is given by

$$\tau_{rz} = k_0 \rho / r \tag{2}$$

where k_0 is the initial shear yield stress and ρ is an arbitrary function of u_0.

If the entire layer is elastic then $\rho \leq a$ and combining Eqn. 1, Eqn. 2 and Hooke's law leads to

$$\varepsilon_{eq} = \frac{1}{\sqrt{3}} \frac{k_0}{G} \frac{\rho}{r} \tag{3}$$

If the entire layer is plastic then $\tau_{rz} = k$ at each point of the layer. Here k is the current shear yield stress which is supposed to depend on the equivalent plastic strain ε_{eq}^{p} and a damage parameter D. A possible representation of this dependence is [3]

$$k = k_0\left[1 + f\left(\varepsilon_{eq}^{p}\right)\right]\left(1 - D\right) \tag{4}$$

where $f\left(\varepsilon_{eq}^{p}\right)$ is an arbitrary function of ε_{eq}^{p} satisfying the following conditions, $df/d\varepsilon_{eq}^{p} > 0$ at any ε_{eq}^{p} and $f(0) = 0$. Since the hydrostatic stress vanishes, the damage evolution equation may be written in the form [3]

$$\dot{D} = \alpha \dot{\varepsilon}_{eq}^{p} \tag{5}$$

where the superposed dot stands for derivatives with respect to time and α is a material constant. For many materials $D = 0$ at $\varepsilon_{eq}^{p} = 0$ [3]. Using this condition, Eqn. 5 can be immediately integrated to give

$$D = \alpha \varepsilon_{eq}^{p} \tag{6}$$

Substituting Eqn. 6 into Eqn. 4 gives

$$k = k_0 \left[1 + f\left(\varepsilon_{eq}^{p}\right)\right]\left(1 - \alpha \varepsilon_{eq}^{p}\right) \tag{7}$$

Since $\tau_{rz} = k$, combining Eqn. 2 and Eqn. 7 results in

$$\rho/r = \left[1 + f\left(\varepsilon_{eq}^{p}\right)\right]\left(1 - \alpha \varepsilon_{eq}^{p}\right) \tag{8}$$

This equation determines ε_{eq}^{p} as a function of $\rho/r \equiv s$ in implicit form

$$\varepsilon_{eq}^{p} = \varphi(s) \tag{9}$$

Assuming that $\varepsilon_{eq} = \varepsilon_{eq}^{e} + \varepsilon_{pl}^{p}$, we arrive, with the use of Eqn. 3 and Eqn. 9, at

$$\varepsilon_{eq} = \varphi(s) + \frac{1}{\sqrt{3}} \frac{k_0}{G} s \tag{10}$$

If the elastic/plastic boundary is within the layer then its position is given by the equation $s = 1$, as follows from Eqn. 2. Therefore, the total equivalent strain is

$$\varepsilon_{eq} = m\varphi(s) + \frac{1}{\sqrt{3}} \frac{k_0}{G} s \tag{11}$$

where $m = 0$ at $s \le 1$ and $m = 1$ at $s > 1$. Thus Eqn. 2, Eqn. 3, Eqn. 10, and Eqn. 11 determine the dependence of τ_{rz} on ε_{eq} in the representative element in parametric form.

FIBER PULL OUT TEST

Neglecting end effects, fiber pull out test can be considered as an anti-plane problem. The fiber of radius r_f is assumed to be rigid. Its velocity is w_f ($u = -w_f$ at $r = r_f$). The material around the fiber is modeled by the continuum described in the previous section. It is assumed that $u = 0$ at $r = R_{max}$. The region $r_f \le r \le R_{\max}$ consists of *N* representative elements. The geometry of the element *i* is defined by $a = r_i$ and $b = R_i$ (Figure 1). Also, we will use the nomenclature $u = -w_i$ at $r = r_i$. Hence $r_1 = r_f$,

$R_N = R_{\max}$ and $w_1 = w_f$. The equivalent strain in each representative element is given by Eqn. 11. Since $\partial u/\partial r = -(s^2/\rho)\partial u/\partial s$, substituting Eqn. 11 into Eqn. 1 gives

$$u^{(i)} = -\rho \int_{\rho/r_i}^{\rho/r} \frac{\left\lfloor \sqrt{3} m \varphi(s) + k_0 s/G \right\rfloor}{s^2} ds - w_i \quad \text{at} \quad r_i \le r \le R_i \tag{12}$$

where $u^{(i)}$ is the velocity in the layer of the element *i*. Due to the continuity of velocity, Eqn. 12 leads to

$$w_{i+1} = \rho \int_{\rho/r_i}^{\rho/R_i} \frac{\left\lfloor \sqrt{3} m \varphi(s) + k_0 s/G \right\rfloor}{s^2} ds + w_i \tag{13}$$

Using Eqn. 13 it is possible to show by induction that

$$w_f = \rho \sum_{i=1}^{N} \int_{\rho/R_i}^{\rho/r_i} \frac{\left\lfloor \sqrt{3} m \varphi(s) + k_0 s/G \right\rfloor}{s^2} ds \tag{14}$$

where we have taken into account that $u = 0$ at $r = R_N$. The solution to Eqn. 14 determines the variation of ρ with w_f. Let r_* be the radius of the elastic/plastic boundary. It follows from Eqn. 2 that $r_* = \rho$ if ρ is within one of the intervals $r_i \le \rho \le R_i$. However, if $R_i \le \rho \le r_{i+1}$ then $r_* = R_i$. Therefore, the plastic zone develops in a jump-like manner. The damage parameter attains its maximum value at $r = r_f$. Using Eqn. 6 and Eqn. 11 at $s = \rho/r_f$ it is possible to find *D* at $r = r_f$ as a function of ρ in the following form

$$D = \alpha \left[\varphi\left(\frac{\rho}{r_f}\right) + \frac{1}{\sqrt{3}} \frac{k_0}{G} \frac{\rho}{r_f} \right] \tag{15}$$

Eqn. 14 and Eqn. 15 give the variation of *D* at $r = r_f$ with w_f in parametric form. In particular, assuming that $D = D_c$ the displacement of the fiber at which fracture starts, $w_f = w_f^{(f)}$, can be found.

NUMERICAL RESULTS AND CONCLUSION

The solution to Eqn. 14 has been found assuming that $r_f = 1$ (without loss of generality), $R_{\max} = 1.46$, $r_i = 1 + (i-1)(h+\delta)$, $R_i = 1 + h + (i-1)(h+\delta)$, $f(\varepsilon_{eq}^p) = 1.072\varepsilon_{eq}^p$, $\alpha = .3$, and $k_0/G = 3.3 \cdot 10^{-3}$. The mechanical properties are typical for a structural steel [4]. Figure 2 illustrates the development of the plastic zone with the displacement of the fiber. It is important to mention that the solution breaks down at $w_f = w_{\max}$. The same result has been obtained in [5, 6]. It is possible to show that this feature is independent of the hardening law and other material and geometric parameters involved in the calculation performed. Differentiating Eqn. 8 with respect to time at $r = r_f$ gives

$$\dot{\rho}/r_f = \left\{ \left(df/d\varepsilon_{eq}^p\right)\left(1 - \alpha\varepsilon_{eq}^p\right) - \alpha\left[1 + f\left(\varepsilon_{eq}^p\right)\right] \right\} \dot{\varepsilon}_{eq}^p \tag{16}$$

For the plastic zone to develop at the initial instant, $\varepsilon_{eq}^p = 0$, it is necessary that $\dot{\rho} > 0$ and $\dot{\varepsilon}_{eq}^p > 0$. Using Eqn. 16 this necessary condition can be written in the form

$$df \big/ d\varepsilon_{eq}^{p} > \alpha \quad \text{at} \quad \varepsilon_{eq}^{p} = 0 \tag{17}$$

If this inequality is not satisfied then no solution exists (the elastic solution does not exist because the

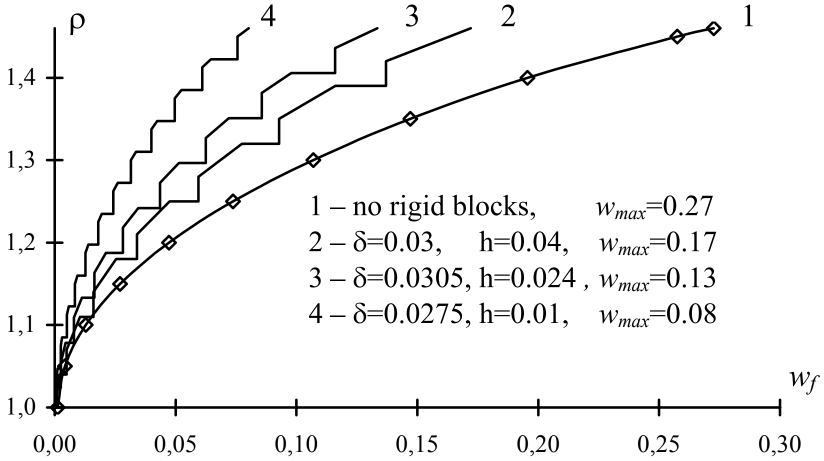

Figure 2: Variation of the elastic/plastic radius, ρ,
with the displacement of the fiber, w_f

initial yield stress is attained, and the elastic/plastic solution does not exist because the plastic zone cannot develop). Assume that Eqn. 17 is satisfied. Then,

$$\left(df \big/ d\varepsilon_{eq}^{p}\right)\left(1 - \alpha\varepsilon_{eq}^{p}\right) - \alpha\left[1 + f\left(\varepsilon_{eq}^{p}\right)\right] > 0 \quad \text{at} \quad \varepsilon_{eq}^{p} = 0 \tag{18}$$

On the other hand, it is clear that

$$\left(df \big/ d\varepsilon_{eq}^{p}\right)\left(1 - \alpha\varepsilon_{eq}^{p}\right) - \alpha\left[1 + f\left(\varepsilon_{eq}^{p}\right)\right] < 0 \quad \text{at} \quad \varepsilon_{eq}^{p} = 1/\alpha \tag{19}$$

Therefore,

$$\left(df \big/ d\varepsilon_{eq}^{p}\right)\left(1 - \alpha\varepsilon_{eq}^{p}\right) - \alpha\left[1 + f\left(\varepsilon_{eq}^{p}\right)\right] = 0 \quad \text{at} \quad \varepsilon_{eq}^{p} = \varepsilon_{\max} \tag{20}$$

where $0 < \varepsilon_{\max} < 1/\alpha$. It follows from Eqn. 7 that k reaches its maximum value, $k_{\max}$, at $\varepsilon_{eq}^{p} = \varepsilon_{\max}$. Consider a possibility to obtain a solution if $\varepsilon_{eq}^{p} = \varepsilon_{\max}$ at a point $r = r_m$ of the interval $r_f < r < R_{\max}$. The distribution of k is shown schematically in Figure 3 (solid line). The plastic solution in the interval $r_f \le r \le r_m$ cannot exist because k is an increasing function of r (Figure 3) whereas τ_{rz} is a decreasing function of r, as follows from Eqn. 2, but $\tau_{rz} = k$ in the plastic zone. The elastic solution in the interval $r_f \le r \le r_m$ (dot line in Figure 3) cannot exist because τ_{rz} is a decreasing function of r and, therefore, $\tau_{rz} > k$ at $r < r_m$ that violates the yield criterion. Therefore, the solution breaks down if ε_{eq}^{p} attains the value of $\varepsilon_{\max}$ at $r = r_f$. Of course, it is possible that fracture occurs at $\varepsilon_{eq}^{p} < \varepsilon_{\max}$. However, since the critical value of damage, D_c, is an independent parameter, it is important to account for the possibility that the solution breaks down in numerical calculations. In particular, in the case of the material under consideration $w_{\max} = w_{\max 2} = 0.13$ at $h = 0.024$ and $\delta = 0.0305$. The corresponding dependence D of ρ at $r = r_f$ found from Eqn. 14 and Eqn. 15 is shown in Figure 4 (solid line), and $D = 0.33$ at $w = w_{\max}$. Since $D_c = 0.22$ [4] for this material, fracture occurs at $w_f < w_{\max}$. However, for a material with a lower strain-hardening modulus, say $a = 0.5$, $D = 0.18$ at $w_f = w_{\max 1} = 0.016$ (dot line in Figure 4, $h = 0.006$, $\delta = 0.004$). In this case the fracture condition is nowhere satisfied at $w_f = w_{\max}$ and the solution cannot be extended for $w_f > w_{\max}$.

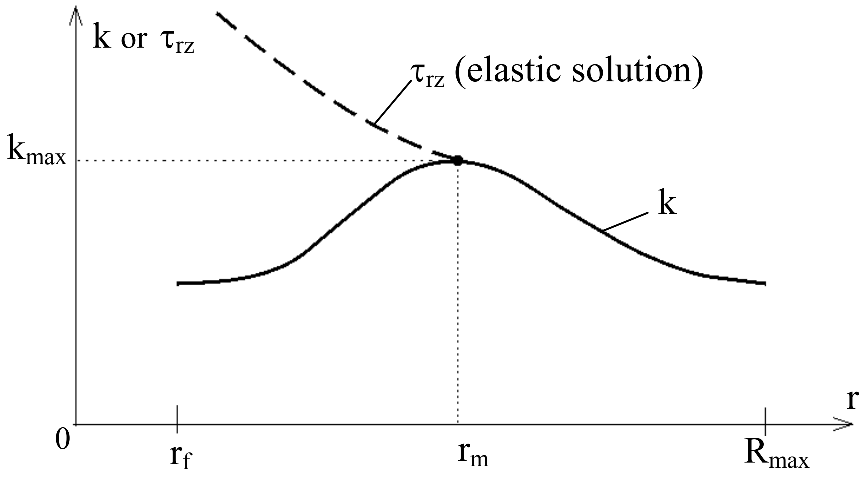

Figure 3: Schematic diagram illustrating non-existence of solution

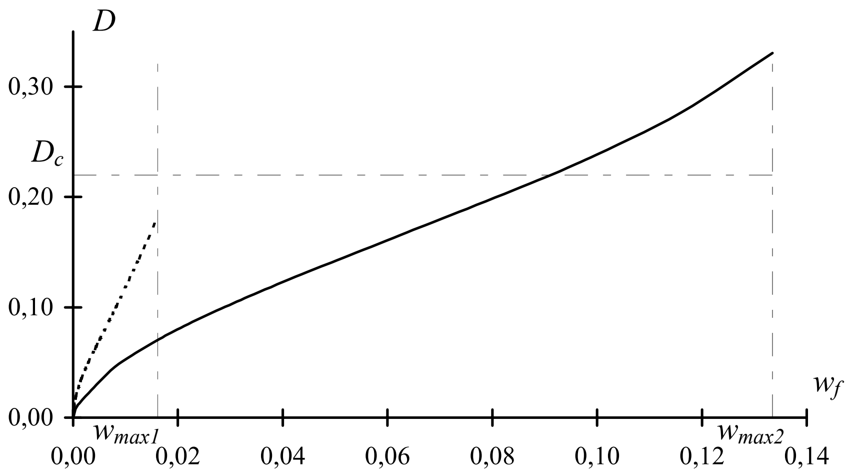

Figure 4: Evolution of the damage variable at $r = r_f$

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References

1. Panin, V.E. (1998). *Physical mesomechanics,* 5,22.
2. Alexandrov, S. and Goldstein, R.V. (1999). *Sov. Phys. Dok*. 368(6), 768 [in Russian].
3. Lemaitre, J. (1987). In: *Continuum Damage Mechanics: Theory and Applications*, pp. 37-89, Krajcinovic, D. and Lemaitre, J. (Eds). Springer, Wien - New York.
4. Prahl, U., Aboutayeb, M., Achenbach, U. and Wang, X. (1999). *Comp. Mater. Sci.* 16, 206.
5. Alexandrov, S. and Goldstein, R.V. (1999). *C. R. Acad. Sci*. Ser. IIb 327, 193.
6. Alexandrov, S. and Goldstein, R.V. (2000). *J. Appl. Math. Mech*. 64, 155.