DI-2 THE GROWTH OF FATIGUE CRACKS

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ABSTRACT

It is convenient to divide the study of fatigue-crack growth into

(a) the initiation and development of surface microcracks, and

(b) the growth of macrocracks.

The former is discussed briefly; work done at the National Engineering Laboratory over the last eight years on the latter forms the main subject matter of the paper. In this latter work, experimental parameters have been determined which describe

(a) the minimum alternating stress necessary to cause a crack of a given length to grow, and

(b) the subsequent rate of crack growth.

It was found that whether or not an edge crack of length l grew in a plate specimen subjected to a nominal alternating stress $\pm\sigma_a$ (zero mean load) depended on whether the value of the parameter $\sigma_a^3 l$ was greater or less than a material constant C and that the initial rate of growth of a central transverse crack in a sheet specimen, subjected to the loading cycle $T \pm \sigma_a$, where $T > \sigma_a$, was given by

$$\frac{dl}{dN} = A\sigma_a^3 l$$

where l is crack length, N is number of stress cycles from the start of crack growth, and A is a material constant which may or may not depend on the value of the tensile mean stress T. Experimentally determined values of C and A for a wide variety of materials are tabulated.

It is considered that, for a complete understanding of fatigue phenomena, a knowledge of both the plain fatigue strength and the fatigue-crack growth characteristics of a material are required, the parameters discussed in this paper providing a simple means of describing this latter behaviour. Their use is illustrated with reference to the interpretation of notched fatigue data and to whether or not fatigue failure originates at a free surface.

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1. INTRODUCTION

The breakdown of engineering plant and machinery as a result of the initiation and subsequent growth of a fatigue crack in a metal part is still an all too common event, despite the fact that the subject of metal fatigue has been studied seriously since the turn of the century. One reason for this is the large number of variables that must be considered when the question of fatigue strength arises. For example, the fatigue properties of all materials of engineering interest under all possible combinations of stress systems, surface finish, surface condition, heat treatment, temperature, environment, etc. are not, and probably never will be, available[1]. Another reason is that the behaviour, under cyclic loading, of a component containing a notch or change of section, a bolted, riveted or welded joint, or inherent flaws or defects, or which is subjected to fretting, cannot always be predicted from a knowledge of the component geometry and the plain fatigue properties of the material (i.e. those obtained by testing un-notched polished specimens). In some instances, whether or not the component fails catastrophically may depend less on the value of the cyclic stress required to initiate a crack than on that required to cause a crack to grow. Thus, to understand fully the behaviour of a metallic component or structure under a cyclic loading, parameters describing the behaviour of a fatigue crack may be just as necessary as a knowledge of the plain fatigue properties.

Further, as a consequence of the increasing acceptance of 'fail-safe' design principles, it would be useful to know both the conditions governing the rate of growth of a fatigue crack and the relative rates of growth of cracks in different materials. Harper[2] states that a 'fail-safe' design implies a limiting crack length can be established, above which a crack must be detected by inspection if it is not to extend to cause failure before the next inspection. He suggests that designers should use all possible means to achieve the ideals of a low rate of crack propagation and a high residual static strength in the presence of a crack. Probably the best way of assessing the merits of a 'fail-safe' design is by the time between inspection periods which it allows in relation to the weight involved.

It is convenient to divide the study of fatigue-crack growth into

(a) the initiation and development of surface microcracks, and

(b) the growth of macrocracks,

even though the former must precede the latter as a direct cause. The former are visible only under high magnification whereas the latter are often visible to the naked eye.

2. THE INITIATION AND DEVELOPMENT OF SURFACE MICROCRACKS

When a polycrystal is subjected to a cyclic stress of sufficient magnitude, slip will first occur in those surface grains which contain active slip planes favourably oriented with respect to the maximum shear stress. Continuing localized slip will result in slip bands forming on the surface, the slip steps produced in the persistently active slip bands eventually developing into grooves or intrusions which can be considered as the first stage of a microcrack. The ease with which slip can be localized within certain bands is dependent on the ease with which cross slip can occur in the material. Slip-band cracking has been observed experimentally by, for example, Thompson[3], Hempel[4], Forsyth[5] and Kemsley[6] and occurs, at room temperature, in most pure metals and simple alloys.

The fact that slip-band cracking is known to occur at liquid-helium temperature[7] suggests that the basic processes depend only on the mechanical movement of atoms and both Cottrell and Hull,[8] and May[9] have suggested models based on the sliding of material in and out of slip bands whereby deep grooves or intrusions can form. May[9], for example, treated the problem by assuming that dislocations made

return journeys along paths which were shifted in a random fashion with respect to previous paths. Such a random distribution of slip would roughen the surface, leading to a redistribution of stress, so that later slip would tend to be concentrated in valleys already formed on the surface. Some valleys will become deeper, and May showed that the predicted number of cycles required to form valleys deep enough to be considered as microcracks was of the same order as was found experimentally.

It is interesting to note that with single crystals the effectiveness of slip-band cracking will depend on the crystal geometry and applied stress system. For example, surface cracking due to simple slip accumulation will be least effective[10] in a crystal of square cross-section subjected to cyclic torsional stresses whose operative slip vector lies parallel to the observation face for, as the shear stresses are a maximum in the middle of each face and reduce to zero at the corners, at no point in the crystal is there a slip movement with a component normal to the surface.

Although slip-band cracking is common in many metallic alloys subjected to cyclic stresses which at room temperature do not result in gross distortion, grain-boundary cracking may occur if the grain boundaries are weakened relative to the grains, either by raising the temperature[11] or by alloying[12]. At high cyclic stress levels, surface rumpling can cause drastic changes in the surface geometry and grain-boundary cracking may occur even though, at stresses nearer the fatigue limit, the material may exhibit transcrystalline cracking[6]. Grain-boundary cracking will be a consequence of the inability of surrounding grains to deform so as to accommodate a grain which is deforming plastically or at high temperature of void formation due to grain-boundary sliding. When the cyclic stresses are such that the whole of the specimen is deformed plastically the material may either work-harden or work-soften depending on its initial condition[12,14] and these effects can be detected by bulk specimen measurements. Although changes in the bulk properties may cause differences between the results obtained from different types of fatigue tests (for example, between constant stress-amplitude and constant strain-amplitude tests), fatigue failure still arises through the initiation of surface cracks. This is convincingly demonstrated by the fact that repeatedly machining away the specimen surface increases the life irrespective of the stress amplitude[15,16]. With certain metallurgically unstable alloys, such as the high-strength solution-treated and precipitation-hardened aluminium alloys, cyclic stressing can result in local over-ageing and softening, locally softened areas on the surface being the preferential sites of cracking[17]. In highly alloyed metals, such as the high-strength steels, the complex microstructure makes observation of slip difficult; cracking in these materials may often be initiated at surface inclusions.

Because cyclic slip processes are controlled by cyclic shear stresses, a mean stress would not be expected to have a marked effect on the minimum alternating stress necessary to initiate surface microcracks (provided the maximum stress in the loading cycle does not result in gross yielding) and thus they will be able to form under a wholly compressive loading cycle[18]. The mean stress will have a more pronounced effect on the development of surface microcracks, for compressive mean stresses will tend to keep them closed so retarding their growth, whereas tensile mean stresses will tend to keep them open and so enhance their growth.

For a given stress and material, a surface microcrack develops into the body of material in the direction of the active slip planes until it reaches a certain depth from the free surface, after which it will continue growing on a plane normal to the direction of the maximum cyclic principal tensile stress; this changeover length will be discussed later. Forsyth[19] calls the former Stage I growth, and the latter Stage II growth.

3. THE GROWTH OF MACROCRACKS

To study the growth of cracks long enough to be seen by the naked eye, it is usual to employ a sheet specimen containing a small central transverse slit[20-22] as shown in Fig. 1. Buckling is prevented by subjecting the sheet to the uniaxial loading cycle $T \pm \sigma_a$, where $T > \sigma_a$. Fatigue cracks form at the ends of the slit and grow across the sheet, normal to the loading direction. The initial slit should be made small compared to the sheet width, so that a well defined growth curve can be obtained before the crack length becomes an appreciable fraction of the sheet width. Provided the value of σ_a is small (σ_a is usually of the order of E/5000 where E is Young's modulus) cracks grow initially from the ends of the slit on a plane through the sheet thickness at 90° to the plane of the sheet (called 90° growth). Growth will continue in this manner until a certain crack length is reached when, for some materials, subsequent growth will occur on a plane through the sheet thickness inclined at 45° to the plane of the sheet (called 45° growth). This latter type of growth occurs in materials in which the static tensile fracture of a piece of material of the same thickness as the crack-growth specimen occurs on a 45° plane through the thickness. For materials in which static tensile fracture occurs on a 90° plane, fatigue-crack growth occurs on a 90° plane throughout. The length of 90° growth varies with material, for example, the length of 90° growth in an 18/8 austenitic steel sheet may be four times that in a mild steel sheet subjected to the same loading cycle[21]. The length of 90° growth in a given material and constant mean stress T increases as σ_a decreases, while for a given σ_a, the length of 90° growth is independent of T for materials in which the growth rate is independent of T and decreases with increasing T for materials whose growth rate increases with increasing T[22]. It would thus seem that for a given material and sheet thickness, the changeover from 90° to 45° growth is associated with some critical growth rate, or in other words, occurs when the increment of growth per cycle exceeds a certain value. Although most ductile metals in this sheet form exhibit 45° growth (two common exceptions are titanium[23] and zinc[21]), the thicker the sheet, the less tendency is there for the fracture face to tip on to the 45° plane; for example, although a particular loading cycle may result in growth occurring eventually on a 45° plane in a 0.1-inch thick mild steel sheet, the fracture face of a similarly loaded 1.0-inch thick sheet will remain throughout on a 90° plane. However, a small shear lip can usually be detected at either edge of the 90° fracture face of the thicker sheet beyond a crack length corresponding to the onset of 45° growth in the similarly loaded thinner sheet[24].

Although on a microscopic scale the leading edge of a crack is a complex shape through the thickness of the sheet, that of a crack, which is not growing too fast, can on a macro-scale be considered as a straight line joining the ends of the two surface cracks visible on either side of the sheet. Examination under high magnification of the fracture faces created by a fatigue crack often reveals the presence of line markings in certain areas; an example of those found on a titanium fracture face is shown on Fig. 2. These line markings (which are either troughs[25] or ridges[19]) have been called striations[19], arrest lines[26] and ripples[25]. They run parallel to each other, normal to the direction of crack growth, the distance between two adjacent lines corresponding to that calculated from a knowledge of the growth rate at the particular point, assuming that the crack front moves forward each stress cycle. These striations appear not only on metallic materials but also in rubbers[27] and polymers[28]. In these latter materials cracks can only grow from inherent or artificial flaws, thus any mechanism of crack growth must be able to account for growth in a material in which the cyclic slip processes leading to surface cracking in a metal do not occur.

A more pronounced marking appears on the fracture face of materials susceptible to fast fracture (e.g. the high-strength aluminium alloys). Examples of those found on a 5½%Zn aluminium alloy at two different loading cycles are shown on Fig. 3. It is seen that quite distinct light and dark areas exist; a dark area corresponds to an element of fast fracture occurring in a single cycle, a light area corresponds to a period of fatigue-crack growth.

There are several methods of presenting fatigue-crack growth data. One may

simply plot the current crack length (l) against N, the number of stress cycles from the time that cracks first appear at the ends of the initial slit, the resulting growth curve being of the form shown on Fig. 4. However, it is more useful to express the results in terms of some parameter which enables the growth rate in a given material to be estimated for any stress cycle and crack length. Some methods that have been suggested for correlating crack-growth data are:

(a) Frost and Dugdale[20] and later Liu[29] assumed that the strain distribution ahead of a transverse crack, in a sheet of infinite width, remained geometrically similar as the crack progressed in a continuous manner. This leads to a growth law of the form

$$\frac{dl}{dN} = Af(\sigma)l \qquad (1)$$

where A is a material constant and $f(\sigma)$ a function of the stress cycle. For a given test, log l should plot linearly against N. Neither $f(\sigma)$ nor A can be predicted; they must be determined experimentally.

(b) Head[30] assumed that the material ahead of the crack in an infinitely wide sheet work-hardened progressively, until the stress in an element of material immediately ahead of the crack tip reached its 'fracture stress'. By considering the material to be composed of elastic and plastic elements he developed a growth relationship of the form

$$\frac{dl}{dN} = Bg(\sigma)l^{\frac{3}{2}},$$

where B is a material constant and $g(\sigma)$ a function of the stress cycle. $g(\sigma)$ is predicted by the theory but B must be determined experimentally. For a given test $l^{-\frac{1}{2}}$ should plot linearly against N.

(c) The above two relationships apply to tests in which the crack length is small compared to the sheet width and in which the external loading cycle applied to the sheet is maintained constant. Weibull[31], however, conducted tests (0 to σ_{max}) in which the external load applied to a sheet specimen was continually reduced so that the stress (corresponding to σ_{max}) on the uncracked area remained constant. By expressing the crack length l as a fraction of the sheet width b, i.e. $x = l/b$, he found that, for a given test $\frac{dx}{dN} = C$ for values of x up to about 0.8, i.e. the growth rate was independent of x. Thus, for a given test, x should plot linearly against N.

(d) Master curve methods. These involve plotting the measured growth rate at a particular value of l against some parameter such as the stress intensity factor[32], $\alpha\sigma l^{\frac{1}{2}}$, where α is some correction factor appropriate to the specimen geometry and the crack-length/sheet-width ratio, and σ is the gross area stress, or $K_N S_{net}$[33] (where S_{net} is the stress on net uncracked area at the particular value of l, and K_N is the Neuber effective stress-concentration factor, assuming the crack to be an ellipse having an effective root radius equal to the size of the Neuber elemental structural particle). Paris and Erdogan[36] have argued that because the elastic stress field and the size of any plastic zone around the crack tip are proportional to the stress intensity factor K, then the growth rate should be some function of K,

$$\text{i.e. } \frac{dl}{dN} = Mf(K).$$

They indeed found that the growth rate in the high-strength aluminium alloys under a repeated tensile loading cycle $0 - \sigma_{max}$ could be represented by

$$\frac{dl}{dN} = MK^4$$

The relationships mentioned above each predict, for a given test, a different growth-rate dependence on crack length. As mentioned previously, Head considered the material ahead of the crack tip could be divided into elastic and plastic elements, as shown on Fig. 5. He was able to define d, the length of the elastic element by comparing the crack opening predicted by his model for wholly elastic conditions with the known elastic solution, but was unable to make any such comparison to define a_o. He assumed, therefore, that a_o remained constant throughout a test. This constant appears in the coefficient $\bar{B}$ as $B'/a_o^{\frac{1}{2}}$. Thus, Head's equation can be written

$$\frac{dl}{dN} = \frac{B' l^{\frac{3}{2}}}{a_o^{\frac{1}{2}}}.$$

Both theory[34] and experiment[20] have now shown that the extent of the plastic zone ahead of the crack tip is proportional to l, i.e. $a_o = Zl$, thus substitution in the above equation gives

$$\frac{dl}{dN} = \frac{B' l^{\frac{3}{2}}}{Z^{\frac{1}{2}} l^{\frac{1}{2}}} = Cl.$$

In order to compare Weibull's relationship with the relationships suggested for a crack growing in an infinitely wide sheet subjected to a constant load amplitude it is necessary to know

(1) the stress dependence of the growth rate of a crack in an infinitely wide sheet, and

(2) the ratio of the nominal stresses which would cause either a crack of given length in an infinite sheet to open the same amount as a crack of the same length in a finite sheet, or alternatively, which would for an elastic material result in the same elastic stress distributions in the vicinity of the crack.

From photo-elastic tests on sheets containing a narrow ellipse, Dixon[35] found that if σ is the nominal stress based on gross area applied to a sheet of width 2b containing a central slit 2l, and if σ_o is the gross area stress applied to an infinitely wide sheet containing a central slit 2l, similar elastic stress distributions resulted if

$$\sigma = \sigma_o \left\{1 - \left(\frac{l}{b}\right)^2\right\}^{-\frac{1}{2}},$$

while Frost and Dugdale[20] found that from measurements of the opening of a slit in a metal sheet, similar slit openings occurred for $l/b < 0.6$ if

$$\sigma = \sigma_o \left\{1 + w\left(\frac{l}{b}\right)^2\right\}$$

where $w \approx \frac{1}{2}$ to 1. Some of Dixon's photo-elastic results are shown on Fig. 6.

Experimental data discussed later suggest that for the loading cycle $T \pm \sigma_a$ where $T > \sigma_a$, equation (1) can be written as $dl/dN = A\sigma_a^3 l$, where A is a material constant which, for some materials, does not depend markedly on T. Consider a test on a finite sheet of material in which the growth rate does not depend on T and in which the external loading is maintained constant, then using Dixon's correction factor

$$\frac{dl}{dN} = Al\sigma_{oa}^3 \left\{1 - \left(\frac{l}{b}\right)^2\right\}^{-\frac{3}{2}}$$

where $\pm\sigma_{oa}$ is the nominal stress cycle based on gross area. Now consider a Weibull test, in which the external loads are adjusted so that the stress on net area σ_n remains constant, and l is expressed as a fraction of b, the sheet width, say $l/b = x$, then $\sigma_{oa} = \sigma_n(1 - x)$ and $dx/dN = A\sigma_n^3(1 - x)^3 x(1 - x^2)^{-\frac{3}{2}}$. As in a given test, σ_n is constant

$$A' \int dN = \int \frac{1}{x}\left(\frac{1 + x}{1 - x}\right)^{\frac{3}{2}} dx$$

which on writing $Z = \left(\frac{1 + x}{1 - x}\right)^{\frac{1}{2}}$ gives

$$A'N + C = 4Z - \ln\frac{Z + 1}{Z - 1} - 2\tan^{-1} Z.$$

Fig. 7 shows N, in arbitrary units, plotted against x; it is seen that the curve can be represented by a straight line over the range x = 0.06 to 0.7. Thus neither Head's nor Weibull's relationships are inconsistent with equation (1), and for a crack growing in a wide sheet, provided l/b is small, the rate of growth would be expected to be proportional to the current crack length l. Indeed, experimental data[20-23] on 10-inch wide sheets of various materials have shown this to be so. Some examples are shown on Figs. 8(a) and 8(b). It is seen, for small values of l/b, that log l plots linearly against N, i.e. $dl/dN = Cl$. For a given tensile mean stress, C is found to plot linearly against σ_a on logarithmic axes, giving a line of slope about 3 for all metals tested, (typical results for copper are shown in Fig. 9) thus

$$\frac{dl}{dN} = A\sigma_a^3 l. \qquad (2).$$

Typical values[20-23] of A are given in Table 1. For some materials A is seen to be dependent on the tensile mean stress T, whereas for others it is independent of T. Copper is a material in which the growth rate is independent of T; the growth curves on Fig. 10 obtained on copper sheets tested at 4 ±1 and 10 ±1 tons/in² are identical over the period of 90° growth. The greatest dependence on tensile mean stress occurs with the high-strength aluminium alloys, in which pronounced elements of fast fracture occur during the growth process. Even in these materials, however, the growth rate for a given value of σ_a never exceeded a linear dependence on T. When A does depend on T, it can be shown that for values of $T > 2$ tons/in² (2 tons/in² is the lowest value of T applied in any test), A is given by P + QT, where P and Q are material constants. Values of A, of course, apply only to that stage of crack growth during which the growth rate is proportional to l. There appears to be no correlation between any of the static strength or plain fatigue properties and the corresponding value of A for a material. Also, provided the crack front remains sensibly straight through the specimen thickness, A is not significantly dependent on specimen thickness[24] and, although crack-growth tests have not been carried out over a wide frequency range, A does not appear to vary markedly with testing speed over the range 100 to 8000 c/min.

It is easily shown[20] that, if equation (2) is true for a crack growing in an infinitely wide sheet, then it will apply with negligible error to a sheet of finite width for values of $l/b < \frac{1}{8}$. This implies that, for the 10-inch wide sheets used in the experimental work, deviation from linearity on a log l v. N plot will occur at half-crack length of about $\frac{5}{8}$ inch. In general, the experimental results show this to be true and in tests at relatively low mean stresses (i.e. T only slightly greater than σ_a) equation (2) applies up to $l/b = \frac{1}{8}$ irrespective of whether growth is on a 90° or 45° plane. However, at higher mean stresses, there is evidence that, if the fracture face deviates from the 90° plane at $l/b < \frac{1}{8}$, the growth rate at a given σ_a depends on the current crack length only as long as the fracture face is on the 90° plane, becoming faster than given by equation (2) when the fracture face tips onto the 45° plane. There is also some indication that for a given stress cycle the length of 90° growth is less in a material of low ductility than in one of high ductility. For example, although A has about the same value for both normalized and cold-rolled mild steels, the length of 90° growth for a given stress cycle and hence the crack length over which equation (2) is applicable is less for the latter than for the former material. In general, the higher the value of T, the more abrupt is the change from 90° to 45° growth and the greater the increase in growth rate when growth occurs on a 45° plane. This faster growth rate can be represented empirically by $dl/dN = C\sigma_a^n l^m$, where C depends on material and tensile mean stress and n and m vary with test conditions. Frost and Dugdale[20] found that for crack lengths up to $l/b \approx \frac{1}{2}$, n and m had values of 3 and 2. For a crack approaching its critical length (i.e. that at which fast fracture occurs), Harpur[2] found n was about 10 and m varied from 3 to 10. It is over this range of crack lengths that the master curve methods provide a useful means of correlating the data.

Although it has been shown that over a limited range of crack lengths the rate of crack growth is given by equation (2), there is evidence that over the same range of crack lengths, the results will, within the limits of experimental error, also plot linearly on Head's $l^{-\frac{1}{2}}$ v.N axes; Fig. 11 shows examples of the same test results plotted on both log l v. N and $l^{-\frac{1}{2}}$ v. N axes. Moreover, slopes of the $l^{-\frac{1}{2}}$ v.N lines obtained at different σ_a values but constant T values again conform to a cube stress dependence. Thus, for a material in which the growth rate is independent of T,

$$\frac{dl}{dN} = B\sigma_a^3 l^{\frac{3}{2}}.$$

However, as the stress intensity factor K, based on the stress amplitude σ_a, is given by $\sigma_a\sqrt{l}$ it follows

$$\frac{dl}{dN} = BK_a^3.$$

Equation (2) can be written for a mean stress sensitive material as

$$\frac{dl}{dN} = (P + QT)\sigma_a^3 l$$

and, if it is assumed this can be written in stress intensity factor form as was the case for a mean stress insensitive material, then

$$\frac{dl}{dN} = B'K_a^3(1 + Q'K_{max})$$

where $K_{max} = (T + \sigma_a)l^{\frac{1}{2}}$. In the case of repeated tension crack growth tests on the high-strength aluminium alloys, $Q'K_{max} > 1$ and $K_{max} = 2K_a$, therefore

$$\frac{dl}{dN} = B''K_a^4$$

which is the relationship Paris and Erdogan[36] found experimentally. It would therefore seem that the experimental data can be used to support equally well any of the crack growth laws mentioned previously.

Equation (2) predicts that a crack of a given length will grow at a finite rate for any value of σ_a. However, the now well-established fact that small non-propagating cracks can be present in certain sharply-notched specimens, unbroken after testing, implies a value of σ_a exists below which a crack of a given length will not grow. Fig. 12, for example, shows the growth curves of cracks formed at the roots of vee-notches (0.2-inch deep and 0.010-inch root radius) in 2½-inch wide mild-steel plates tested at zero mean load[37]; it is clear that at ±3 and 2½ tons/in², the cracks reach a certain length and then remain dormant.

Experimental data relating to the minimum value of $\pm\sigma_a$, for the particular case of zero mean load, required to just cause an edge crack of length l to grow have been obtained using ¼-inch thick plate specimens 2½ inches wide, containing two opposite edge vee-notches of about 0.004-inch root radius and 0.2-inch deep[38-41]. The plates were subjected to an appropriate alternating stress such that cracks formed at the notch roots, a test being stopped when the cracks had grown to the required length. They were then reprofiled so as to remove the edge notches, leaving a plate specimen, 2.1 inches wide, containing two small edge cracks. After stress-relieving, to minimize any residual stresses introduced during either the machining or cracking processes, a cracked plate was subjected to a given $\pm\sigma_a$ and details of the subsequent crack behaviour noted. For all metals tested to date, it has been found that, whether or not an edge crack grows, depends on the value of the parameter $\sigma_a^3 l$ (where $\pm\sigma_a$ = nominal alternating stress, zero mean load, based on the gross area of the plate and l = length of edge crack, values of l being restricted to about 0.25 inch so that the edge cracks could be considered as being in an infinite body). If $\sigma_a^3 l > C$ a crack will grow, if $\sigma_a^3 l < C$ a crack will remain dormant, where C is a material constant. Values of C (zero mean load) for various metallic materials are given in Table 2. It is seen that C (as for the values of A discussed previously) is not related to either the static or fatigue properties of the material; for example, the three ferritic steels listed have different static and fatigue properties yet all have the same value of C. However, those materials having a high Young's modulus (E) have a higher C value than those having a low E, i.e. nickel and ferrous alloys have the highest C values and these are followed in order of decreasing E by the copper alloys and then the aluminium alloys. This is not unexpected, for whether or not a crack grows must depend on the amount it is opened and closed during the loading cycle. For a given σ_a and l this will depend inversely on E, as the stress levels used are such that the body of the cracked plate deforms elastically. The fact that there is a minimum cyclic crack opening below which a crack will not grow implies that either the change in crack-tip profile during the loading cycle can be wholly accommodated by continuing cyclic deformation or if a certain amount of cracking does occur at the crack tip, the freshly-created surfaces are able to rebond completely when they are pressed together during the compression half of the loading cycle.

It would be expected that, whether or not a crack grows under the wholly-tensile loading cycle $T \pm \sigma_a$, where $T \geqslant \sigma_a$, is again determined by whether $\sigma_a^3 l$ is greater or less than C_T, where C_T is a material constant for a given T. If, at zero mean load, a crack merely remains closed during the compression half of the loading cycle and no rebonding of freshly-created surfaces at the crack-tip occurs, then the crack growth behaviour under the stress cycle 0 ±P will be the same as under the stress cycle 0 to P. i.e. P/2 ±P/2. Thus for $T = \sigma_a$ the maximum value of C_T will be equal to C/8. However, if rebonding does occur at the crack tip under zero mean load conditions, then C_T will be less than C/8, as less rebonding would be expected to occur in air when the crack faces are kept apart throughout the loading cycle. The change in C_T with further increase in T might be expected to be generally similar to the change in the corresponding value of 1/A (Table 1) with increasing T.

For some materials, both A and C would be expected to vary with environment.

A corrosive environment will obviously tend to worsen a material's crack-growth characteristics, while there is some experimental evidence that excluding the atmosphere from the crack tip results in an improvement. For example, Wadsworth[42-43] found that the lives at similar zero mean load loadings of copper, iron and aluminium specimens were longer in vacuum than in air. As slip-band cracking was observed to form after the same number of cycles in either vacuum or air he concluded that the increased life in vacuum was due to the crack-growth rate in vacuum being slower than in air, the improved crack-growth characteristics resulting from the greater amount of crack-tip rebonding possible when the amount of available oxygen was decreased. The iron specimens exhibited a definite fatigue limit in both air and vacuum, that, in a vacuum, being about 25 per cent higher than in air. As slip-band cracks were present in both air and vacuum specimens tested either at or just below their corresponding fatigue limits, it would seem that as well as A decreasing, C was increased by the exclusion of air. Frost[44] also found for mild steel that excluding air from the crack tip, by either coating the specimen with butyl rubber or immersing it in oil, improved the crack propagation characteristics. The greatest improvement was at zero mean load (Fig. 13 shows the S/N curves obtained from sharply-notched specimens tested either in oil or in air, the notches being sufficiently sharp so that all specimens unbroken after testing contained non-propagating cracks) the in-air value of C = 5.5 being roughly doubled as a result of excluding the atmosphere. Under a wholly-tensile loading cycle, the exclusion of air resulted in a decrease in A of about 25 per cent. It would seem, therefore, that rebonding of the freshly-created surfaces at the crack tip is a factor in determining the crack propagation characteristics of a material, the effect being greatest when the atmosphere is kept away from the crack tip and the crack faces are pressed together by compressive loads for part of the loading cycle.

4. MECHANISM OF GROWTH OF A MACROCRACK

Unlike a surface microcrack which is able to form and develop under a wholly compressive loading cycle, a macrocrack cannot grow under this loading cycle, as its rate of growth depends on the amount it is opened and closed. Fig. 14 shows the growth curve for a crack formed at the root of an edge notch in a mild steel plate loaded at -8 ± 6 tons/in^2. It is seen that, although the crack forms in the locally high compressive-stress field at the notch root, its rate of growth decreases quickly to zero and it is unable to grow across the plate. Compared to the not inconsiderable effort devoted to the study of the formation and development of surface microcracks, the study of the possible mechanisms by which a macrocrack grows, as it is continually opened and closed, has received little attention. Recent thin-film electron-microscopy work[10,45,46] has shown that within the individual grains of a cyclically-strained metal, the density of dislocation in those regions of the grain having a dislocation density slightly greater than average is progressively increased, while in the less dense regions the density is further reduced, the dimensions of the regions being a few microns. When formed, these regions give rise to characteristic X-ray effects which allow recognition of this cyclic structure on the fracture face created by a growing fatigue crack[47]. However, although the grains through which a slowly growing fatigue crack has passed exhibit a well developed sub-structure, there is no experimental evidence that the sub-boundaries are the sites of preferential cracking.

This absence of evidence to support internal cracking in the material ahead of the crack tip, together with the fact that crack growth cannot be considered merely as a continuation of the slip processes which lead to the initiation of a surface microcrack (since it is known that cracks can grow in materials in which such slip processes do not operate), make it reasonable to suggest that crack growth is a consequence of the changing crack-tip profile as it is repeatedly blunted and sharpened. Laird and Smith[25] found, by examining the profile of a crack in a specimen at various stages during the loading cycle, that the crack tip first opened as a sharp notch, then blunted, ears finally forming on the blunted profile. On unloading the crack tip re-sharpened and the crack closed. They suggested that growth occurred by a series of miniature tensile tests resulting in a ripple appearing on the fracture face each stress cycle.

A crack under a normal tensile load will have atomic bonds in every stage of elongation and rupture at its tip no matter how low the load, but before a crack of a given length can be self-propagating, it is necessary for the stress to reach such a level that the energy conditions for creating fresh surfaces are satisfied. Under a cyclic loading, however, a fatigue crack can move forward each cycle at a stress level less than that required for self-propagation thus implying that some atomic bonds break at the crack tip although the crack as a whole remains stable. It could be argued that this situation arises because as soon as an increment of load is applied to the crack, high elastic stresses at the tip cause instantaneous fracture of some of the atomic bonds, stability at the increased load following because the freshly-created surfaces, together with subsequent plastic deformation, enable the crack tip to blunt. Unloading will re-sharpen the crack tip and the process will be repeated each cycle. As the loaded crack-tip profile will have a greater peripheral length than that of the unloaded crack one could, for example, assume that the increment of growth per cycle is proportional to the increase in length of that part of the crack-tip profile which is subjected to a tensile stress, for this increased length must be accommodated by unbonding and by elastic and plastic deformation. The environment might also be expected to influence the amount of crack growth per cycle, a corrosive environment increasing the increment of growth whereas an inert environment might result in a decrease as in the absence of any chemically active substances, some proportion of the freshly-created surfaces at the crack tip may be able to rebond when the crack is unloaded. A crack-growth hypothesis must, therefore, be based on a detailed knowledge of the geometry of the changing crack-tip profile and on the corresponding relative amounts of deformation, unbonding, corrosive attack, and rebonding. At the present time there are no means available whereby these quantities can be calculated. However, a rough analysis of the crack-tip geometry, ignoring environmental effects, shows that growth rates of the right order can be derived. For example, it can be shown that the increase in length of that part of the crack-tip profile which is subjected to a tensile stress, assuming the crack opens from a narrow to a wider ellipse as predicted by elastic theory, is to a first approximation proportional to $\sigma^2 l/E^2$ where σ is the stress-range and E is Young's modulus. Thus, over that part of the growth curve in which the growth rate is proportional to the current crack length, its values for different materials should, for the same loading cycle be inversely proportional to E^2. Table 3 shows the values of A, the experimentally determined growth-rate parameter, and values of $20/E^2$ for the three cubic metals, mild steel, copper and aluminium, in which the growth rate is not markedly dependent on the tensile mean stress. The relationship between A and $1/E^2$, however, appears to be different for hexagonal metals; for example, the value of A for titanium (Table 1) corresponds to a value of $50/E^2$. However, as the increment of growth per cycle can be of the order of the lattice spacing, a difference between metals of one general lattice type and another is not perhaps unexpected. The fact that the increment of growth/cycle depends on the stress squared, whereas experimentally the growth rate is proportional to the cube of the alternating stress, is possibly associated both with the fact that the crack-tip radius deviates further from that given by elastic theory as the stress level increases and that some rebonding of the freshly-created surfaces at the crack tip is possible even in air, the amount being greater at small crack lengths and low σ_a values than at longer crack lengths and high σ_a values. The predicted dependence of the growth rate on E is supported by experimental data. For example, both copper and steels respectively have the same crack-growth characteristic irrespective of their static strength properties, whilst the crack-growth characteristics of the titanium alloys are similar to those of the base metal. These facts may be associated with Holden's findings that the details of the cyclic sub-structure are not dependent on the initial state of the material. He also found that material having this cyclic structure flowed readily, as illustrated by the creep found to occur in tests on plain specimens subjected to relatively high cyclic and low mean stresses. The creep rate in these specimens was much greater than in those subjected to a static stress equal to the maximum stress in the fatigue cycle for comparable times[10]. Thus, for slowly growing cracks, it may be the ductility of the material in this cyclic condition which is important in determining how much of the changing crack-tip profile is accommodated by deformation rather than by the static ductility of the material. The aluminium-based group of alloys are, however, exceptions to the argument that the growth-rate characteristics of a particular alloy system do not vary widely from each other or from the base metal for they are the only group of alloys whose growth-rate characteristics differ from alloy to alloy and from base metal. These alloys also exhibit a more pronounced dependence of growth rate on tensile mean stress than any other group of materials tested. As mentioned previously this marked dependence on mean stress is associated with the spasmodic elements of fast fracture known to occur during crack growth. It could be argued that the increased growth rate in the alloys compared to the base material is also due to the presence, in the former, of these spasmodic elements of fast fracture. This is illustrated diagrammatically on Fig. 15, where for a given cyclic stress, growth curves (a) and (b) for aluminium and an aluminium alloy respectively are compared on a log l v.N plot. On curve (b) the basic crack-growth rate is shown by elements xy having the same slope as curve (a) and increments of fast fracture shown by elements yz. The elements of fast fracture could arise from the presence of brittle inclusions which do not permit the crack tip to blunt until it has passed right through them. Thus, for a given stress cycle, it could be argued that the magnitude of yz will depend on inclusion size and that of xy on inclusion spacing.

If a marked mean stress dependence is due to the presence of spasmodic elements of fast fracture, it follows that there will be no marked mean stress dependence in a ductile pure metal. The growth-rate characteristics of any of its alloys which do not contain inherent sources of fast fracture will also be independent of the tensile mean stress. In addition, provided the elastic modulus remains sensibly constant and little plastic deformation occurs at the crack tip, the growth-rate characteristics of the alloys will be similar to those of the base metal, as cracks of similar length will open similar amounts in either alloy or base metal, thus giving rise to similar minimum lengths of freshly-created surfaces at the crack tip necessary for it to assume its stable loaded profile. However, if the alloying does provide fast fracture sources, the growth rate, due to a given loading cycle, of cracks of equal length, will not only be faster in the alloy than in the base metal but will also depend markedly on the tensile mean stress. However, the opening of a small crack, under a particular loading cycle, may not give rise to a sufficiently high maximum tensile strain ahead of the crack tip to operate the fast fracture sources. Thus a marked mean stress dependence may not become apparent until the crack exceeds a certain length. For example, although it has been stated previously that ferritic steels have essentially the same initial crack-growth characteristics, the growth rate, at a given loading cycle in a hard steel may well become faster than that in a softer steel after a certain crack length has been exceeded, for now, the fast fracture sources may become operative. If this does happen, the growth-rate characteristics of the hard steel will become markedly dependent on the tensile mean stress. Thus, in discussing the crack-growth characteristics of different materials, it is essential to define clearly the stress levels and crack lengths under consideration. For example, in a series of crack-growth tests on steel sheets having different mechanical properties and containing initial slits longer than the crack length, for a given stress cycle, at which the fast fracture sources become operative, the initial period of growth in which the growth rate is independent of both the mean stress and the mechanical properties would be missed. Instead it would be concluded that the growth-rate characteristics varied from steel to steel and with the mean stress. The high strength aluminium alloys seem to be materials in which the fast fracture sources seem to operate at crack lengths less than the minimum possible initial slit lengths, for they always exhibit a marked mean stress dependence and growth characteristics faster than those of the base metal.

The variation of A, in equation (2) with the tensile mean stress T, could be represented by

$$A = P + QT$$

where P and Q were stated to be material constants. It could be argued that P defines the true fatigue crack-growth characteristics of a material (i.e. as deter-

mined by the minimum amount of cracking per cycle that is just necessary to allow the crack tip to attain its blunted profile) and Q defines the contribution to the growth rate made by the spasmodic elements of fast fracture (i.e. elements of cracking, additional to the minimum geometrical amount required each cycle, due to structural inhomogemitics not permitting the crack tip to blunt until it has passed beyond them). It is not improbable, that Q might be related to the fast-fracture characteristics of the material as a whole. Thus summarising, if $Q = 0$, the growth rate is independent of mean stress and the growth-rate characteristics of an alloy will be similar to those of the base metal. If Q has a positive value, the growth-rate characteristics vary with mean stress and the rate of growth of a crack in an alloy will be faster than one of the same length (for a given loading cycle) in the base metal. Q may also vary with crack length, i.e. for some materials its value may not be significant until the crack attains a certain length.

If a macrocrack of a given length does not grow under a given cyclic stress, it could be argued that the changing crack-tip profile during one stress cycle is insufficient to cause cracking at the tip to extend at least one atomic spacing. This could arise either if the changing crack-tip profile was accommodated wholly by cyclic deformation or if the atomic bonds which do break on loading rebond completely on unloading. As mentioned previously it is known that keeping the atmosphere away from the crack tip can certainly result in an increase in the stress required to cause a crack of a given length to grow.

5. DISCUSSION

As a surface microcrack is developing along a crystallographic plane into the body of the material due to cyclic shear stresses, it will, at the same time, be opening and closing due to cyclic normal loads. Until this opening and closing is sufficient to cause cracking at the tip to extend at least one lattice spacing, the crack can only continue along the crystallographic plane by the same mechanism that led to its nucleation. When it does reach this required depth, it will align itself so that the opening is a maximum for a given crack depth (i.e. it will turn normal to the maximum cyclic tensile stress). It can now be considered a macrocrack whose growth characteristics are defined by the experimentally determined parameters A and C. If a cyclic stress sufficient to initiate cyclic slip is insufficient to cause the resulting surface microcrack to penetrate to this changeover depth, it will remain dormant (both Wadsworth(43) and Hempel(4) have found that microcracks can be present in slip bands at stresses equal to and less than the plain fatigue limit). The plain fatigue limit of these specimens will therefore be the cyclic stress necessary to cause a surface microcrack to develop to the changeover length, rather than the cyclic stress necessary to cause surface slip. For a given σ_a, equal to or greater than the plain fatigue limit, an estimate of this changeover crack depth (at zero mean load) can be obtained by substituting σ_a in the $\sigma_a^3 l = C$ relationship, as this will give the maximum length of macrocrack which is just not able to grow under the cyclic stress σ_a. Values of the changeover depth for various materials subjected to stresses corresponding to their plain fatigue strengths (at 50×10^6 cycles) are given in Table 2; obviously the higher σ_a is above the plain fatigue limit, the smaller will be the changeover depth.

As the development of a surface microcrack (by cyclic shear stresses) is slow compared to the rate of growth of a macrocrack after the changeover depth, these values illustrate why surface markings are more easily observed on copper, aluminium and nickel than on the other materials listed. By similar reasoning, these values can also be considered as the maximum depth of a surface fissure, defect or scratch which can be tolerated without causing a decrease in plain fatigue strength.

It is interesting to note that C has the same value for all the ferritic steels tested, irrespective of their different chemical compositions, static strength properties and plain fatigue limits. This suggests that the scatter in results associated with the conventional S/N diagram for a batch of nominally similar steel specimens is related essentially to the number of stress cycles to form and develop surface microcracks to the changeover depth. Although the plain fatigue limit of an alloy steel is higher than that of mild steel (because of the greater difficulty in causing surface slip) there is a limit to which the fatigue limit of a steel can be increased by merely increasing its hardness or tensile strength. This limit will be reached when it is easier for a crack to initiate at and grow from an inherent defect or inclusion than for cyclic slip to occur in a surface grain. It is well known that the fatigue strength of very high tensile steels can only be increased by improved manufacturing techniques, such as vacuum melting, which result in a reduction of inclusion size. If the inclusions or flaws are not too numerous and are remote from the surface, then cracks initiated by such flaws will require a higher stress to grow than that given by simply substituting an approximate value for l in the $\sigma_a^3 l = C$ relationship. This is due, firstly, to the fact that an internal crack requires a higher stress to cause it to open the same amount as an edge crack of the appropriate length (e.g. the stress required to open an internal disc-shaped crack of radius l, lying normal to the loading direction is about 1.7 times that required to open a through edge-crack of length l the same amount) and, secondly, the fact that the atmosphere will be excluded from the tip of an internal crack and hence some value of C greater than the in-air value may be appropriate. However, if flake-like flaws having sharp root radii are numerous and distributed randomly, then it is highly probable that one flaw will meet the free surface and be normal to the loading direction. In this case, the use of the in-air value of C and the appropriate flaw depth will give the minimum value of σ_a needed to cause it to grow. Flaws in the form of graphite flakes are so distributed in ordinary grey cast iron. Assuming their lengths to be in the range 0.01 to 0.04 inch, then substitution in $\sigma_a^3 l = 5.5$ gives σ_a in the region of ±8 to ±5 tons/in^2, which correspond to the fatigue strengths found experimentally. Thus fatigue failure is not necessarily a surface phenomenon; materials which contain inherent flaws will fail by the propagation of cracks from such flaws if their length is such that the required cyclic propagation stress is less than that required to cause surface slip and subsequent cracking.

C is also useful in interpreting notched fatigue data[48]. Unfortunately, for the designer much laboratory notched fatigue data supposedly show that local stress amplitudes, at the root of the notch, greater than the plain fatigue limit of the material can be tolerated without the specimen breaking. Much effort has gone into establishing relationships[49] which supposedly enable designers to predict the nominal stress amplitude that a notched specimen or component can withstand. The fact that notched specimens of some materials seem able to withstand, without breaking, nominal cyclic stresses which would give rise to local stress amplitudes greater than the plain fatigue limit, while similarly notched specimens of other materials will not, has led to the former materials being termed 'notch insensitive' and the latter 'notch sensitive'. It is emphasized that such a classification of materials can be very misleading.

The root radii of machined notches of practical interest are large compared to the radius at the tip of a crack, and it is reasonable to assume that crack initiation in the material at the notch root will be as on the surface of a plain specimen. Due to the presence of a stress gradient at the notch root, the surface layers of material are not subjected to a uniform stress distribution. This and other factors[48], including the effect of work-hardening and the introduction of residual compressive stresses during machining, result in the nominal alternating stress required to initiate a crack at the notch root being somewhat greater than the plain fatigue limit divided by K_t (where K_t is the geometric elastic stress concentration factor) the difference increasing as K_t increases. When a crack is present at the root of a notch, its effective length will, for the usual vee-notch, be increased by the depth of the notch, and whether or not the crack grows (at zero mean load) depends on whether $\sigma_a^3 l_d$ is greater or less than C, where l_d is the notch depth plus crack length. If a notch is sufficiently sharp to initiate a crack at its root at a nominal alternating stress $\pm\sigma_n$, where $\sigma_n^3 l_d < C$, then specimens unbroken after testing should contain small cracks. As mentioned previously the existence of non-propagating

cracks at the roots of certain sharply notched specimens is well known. For notches in which the length of crack at the notch root is small compared to the notch depth, l_d can be taken as the notch depth, d. For a given specimen, non-propagating cracks will exist if

$$K_t > \frac{\text{plain fatigue limit}}{(C/d)^{\frac{1}{3}}}.$$

If K_t is such that non-propagating cracks exist, then the conventional notched-fatigue limit $\pm\sigma_n$ of specimens of a given notch depth is given by

$$\sigma_n = (C/d)^{\frac{1}{3}}.$$

Consider now batches of sharply vee-notched specimens of different materials but all having a notch depth of 0.1-inch, then

$$K_f = \frac{\text{plain fatigue limit}}{(10C)^{\frac{1}{3}}}.$$

The values of K_f shown in Table 4 are the maximum that can be attained (at zero mean load) for the notch depth considered, irrespective of notch root radius. They indicate clearly why ignorance of the presence of non-propagating cracks has led to some materials (e.g. the high-strength steels) being termed notch sensitive, whereas others (e.g. copper, mild steel) are commonly referred to as notch insensitive. It is thus apparent that the procedure of carrying out notched-fatigue tests on batches of specimens containing notches of fixed depth but of different root radii, comparing the K_f values obtained with the corresponding K_t values and concluding that the material is either notch sensitive or insensitive can be very misleading. For example, Mann[50] described tests on copper specimens containing vee-notches 0.075-inch deep and of varying K_t values between 1.8 and 10. He found, irrespective of the K_t value, that K_f did not exceed 1.5 and therefore concluded that the material was notch insensitive. It is obvious that, had deeper notches of the same K_t values been tested, corresponding higher values of K_f would have been obtained.

Many fatigue failures in service are a result of cracks forming in areas where fretting has occurred. When two pieces of material, clamped together under a certain contact pressure transmit cyclic tangential stresses, the oxide layer on contacting asperities is destroyed and they weld together. Although the continual welding and breaking of contacting asperities produces debris which causes abrasive wear, fatigue cracks are found to form at the edges of the locally welded areas. Because surface cracks can, under severe fretting conditions, form at nominal cyclic stresses insufficient to cause complete failure the fretting fatigue limit is governed by the cyclic stress necessary to propagate these cracks. For example, it is found that the fretting-fatigue limits of various ferritic steels under the most damaging fretting conditions are about the same, i.e. increasing the tensile strength of the steel does not result in a higher fretting-fatigue limit than that obtained with mild steel. This would, of course, be expected from the fact that C has about the same value for most ferritic steels.

A considerable improvement in the fatigue strength of a material can be obtained by such processes as shot-peening or surface rolling, which induce residual compressive stresses in the surface layers of the material. Such residual stresses would not be expected to have a marked effect on the cyclic stress necessary to initiate surface microcracks (although this would be increased if the process hardened the surface) but they may prevent them opening and growing as macrocracks. Provided the residual compressive-stress layer is deep enough to prevent surface microcracks penetrating it, the fatigue strength will be increased until the applied cyclic stress is either sufficient to cause the cracks to open and grow as macrocracks, or to cause the cracks to penetrate as microcracks (under the action of resolved cyclic shear stresses), through the affected surface layer.

Discussions of these and other fatigue phenomena have been given in previous papers (51,52). Although the macrocrack growth mechanism proposed in these papers has now been revised, the original arguments are not invalidated, as they do not depend on the detailed mechanisms of surface microcrack initiation and macrocrack growth, but only on the fact that the former is governed by cyclic shear stresses and the latter by the amount the crack opens and closes during each stress cycle.

To conclude, it has been shown that for a complete understanding of all fatigue phenomena, both the plain fatigue properties and the fatigue-crack growth characteristic of materials must be known; the parameters discussed in the present paper provide a simple means of describing the latter behaviour.

ACKNOWLEDGMENTS

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TABLE 1

VALUES OF CRACK GROWTH RATE PARAMETER A FOR DIFFERENT MATERIALS

A defines the initial rate of growth of a crack from a slit in a sheet specimen subjected to the loading cycle $T \pm \sigma_a$ where $T > \sigma_a$, i.e. $dl/dN = A\sigma_a^3 l$, where N is in millions of cycles

σ_a is the semi-range of alternating stress in tons/in^2

l is the half-crack length in inches

A is a material constant which may vary with tensile mean stress.

Material	Tensile strength (tons/in^2)	Tensile mean stress (tons/in^2)	A
Mild steel	21	2-13	0.09
Low alloy steel	54	5	0.10
Cold-rolled mild steel	45	5-7 15-25	0.12 0.22
18/8 austenitic steel	43	5-9 15	0.06 0.11
Copper (either annealed or cold rolled)	20 max.	2-12	0.37
Brass	21	4-8	0.35
Aluminium	7	2-3 5-7	1.1 2.0
Aluminium alloy HS 30WP (1% Mg, 1% Si, 0.7% Mn)	20	3 5 8 12	3.2 3.6 4.4 5.0
5% Mg-aluminium alloy	20	3 8 11-12	2.8 5.0 8.0
4½% Cu-aluminium alloy BS L71	31	2 3-4 15 23	1.3 2.8 12 18
4½% Cu-aluminium alloy Clad. BS L73	31	3-4 15-17	3.0 12
5½% Zn-aluminium alloy Clad. DTD 687A	35	3 4-5 14-16 14-15 (transverse specimens)	14 18 27 65
Commercially pure titanium	36	3½-22	1.2
5% Al-titanium alloy	54	8-28	0.8
15% Mo-titanium alloy	75	8-22 27.5	1.3 2.2

TABLE 2

VALUES OF C AND l_c FOR DIFFERENT MATERIALS

C is the limiting value of the crack growth parameter $\sigma_a^3 l$ where $\pm\sigma_a$ is the nominal gross area stress (zero mean load) applied to a plate specimen containing an edge through crack of length l; if $\sigma_a^3 l > C$ a crack will grow; if $\sigma_a^3 l < C$ a crack will remain dormant. It has the units of $\sigma_a^3 l$ where σ_a is in tons/in^2 and l is in inches. l_c is the change over depth between a micro and macrocrack for σ_a equal to the plain fatigue strength (50×10^6 cycles), i.e. $l_c = C/(\text{plain fatigue strength})^3$.

Material	Tensile strength (tons/in^2)	Plain fatigue strength (50×10^6 cycles) (tons/in^2)	C	l_c
Inconel	42½	±14½	8.0	0.002 6
Nickel	29½	±9	7.5	0.010
18/8 Austenitic steel	44½	±23	5.8	0.000 48
Low-alloy steel	54	±30	5.5	0.000 20
Mild steel	28	±13	5.5	0.002 5
Ni-Cr alloy steel	60	±32	5.5	0.000 17
Monel	34	±15¾	3.8	0.000 97
Phosphor bronze	21	±8½	1.7	0.002 8
Brass	21½	±6¾	1.0	0.003 2
Copper	14½	±4	0.6	0.009 4
4½% Cu-aluminium alloy	29	±9	0.2	0.000 27
Aluminium	5	±1¾	0.04	0.006 5

TABLE 3

COMPARISON OF A AND $20/E^2$

Material	E (tons/in^2)	A	$20/E^2$
Mild steel	1.34×10^4	0.09×10^{-6}	0.11×10^{-6}
Copper	7.51×10^3	0.37×10^{-6}	0.35×10^{-6}
Aluminium	4.46×10^3	1.1×10^{-6}	1.0×10^{-6}

TABLE 4

CALCULATED MAXIMUM VALUES OF K_f FOR SHARP VEE NOTCHES, 0.1 INCH DEEP

Material	Plain fatigue strength (50×10^6 cycles) (tons/in^2)	C	Calculated σ_n (tons/in^2)	Calculated K_f
Inconel	±14½	8.0	±4.3	3.4
Nickel	±9	7.5	±4.21	2.1
18/8 Austenitic steel	±23	5.8	±3.87	5.9
Low alloy steel	±30	5.5	±3.8	7.9
Mild steel	±13	5.5	±3.8	3.4
Ni-Cr alloy steel	±32	5.5	±3.8	8.4
Monel	±15¾	3.8	±3.36	4.7
Phosphor bronze	±8½	1.7	±2.57	3.3
Brass	±6¾	1.0	±2.16	3.1
Copper	±4	0.6	±1.82	2.2
4½% Cu-aluminium alloy	±9	0.2	±1.26	7.1
Aluminium	±1¾	0.04	±0.74	2.4

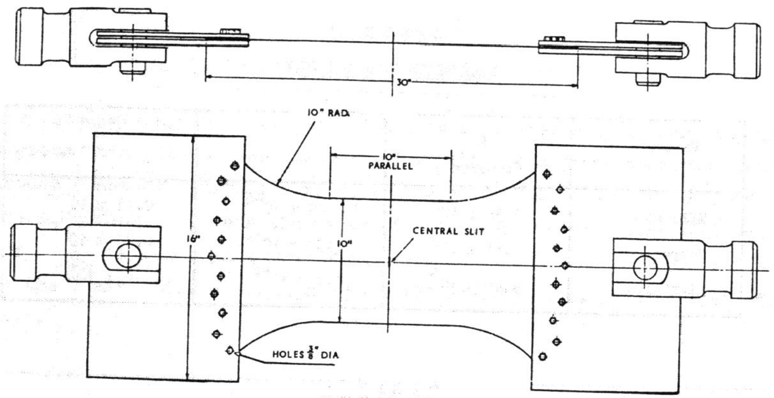
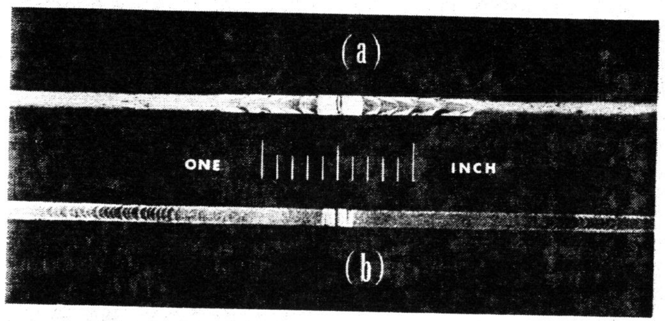


FIG I TEST SPECIMEN AND METHOD OF CLAMPING IN 60 TON SCHENCK FATIGUE MACHINE

(a) $\frac{1}{2}$ inch FROM CENTRE LINE OF SHEET, x 940 (b) $1\frac{1}{2}$ inches FROM CENTRE LINE OF SHEET, x 940

FIG 2 STRIATIONS ON FRACTURE FACE OF COMERCIALLY PURE TITANIUM SHEET SPECIMEN TESTED AT 7 ± 5 tons/in^2



(b) $14\frac{1}{2} \pm 1\frac{1}{2}$ tons/in^2 (a) $2 \pm \frac{1}{2}$ tons/in^2

FIG 3 FRACTURE FACES OF $5\frac{1}{2}$ % Zn ALUMINIUM ALLOY SHEET SPECIMEN

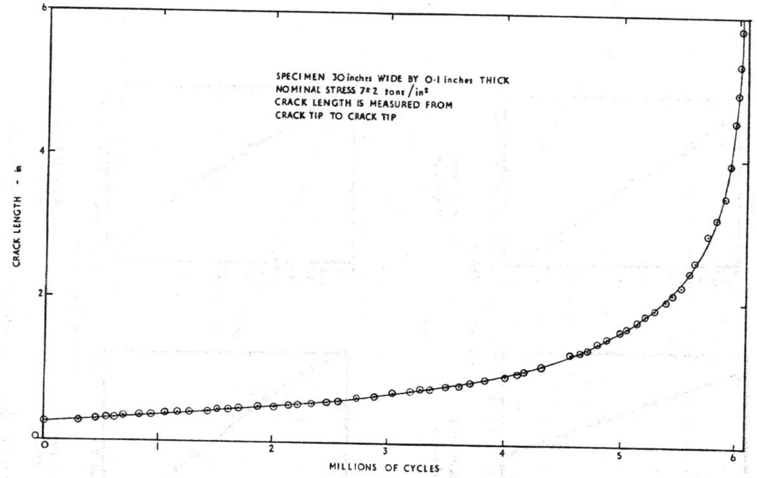


FIG 4 GROWTH CURVE FOR A CENTRAL CRACK GROWING IN A MILD STEEL SPECIMEN

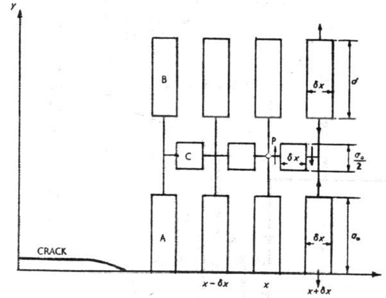


FIG 5 HEAD'S[30] MODEL OF MATERIAL AHEAD OF CRACK TIP

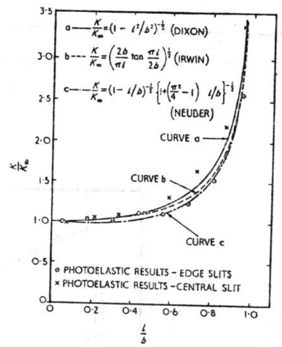


FIG 6 EFFECT OF CRACK-LENGTH / SHEET-WIDTH RATIO ON STRESS DISTRIBUTION AROUND TIP OF SLIT

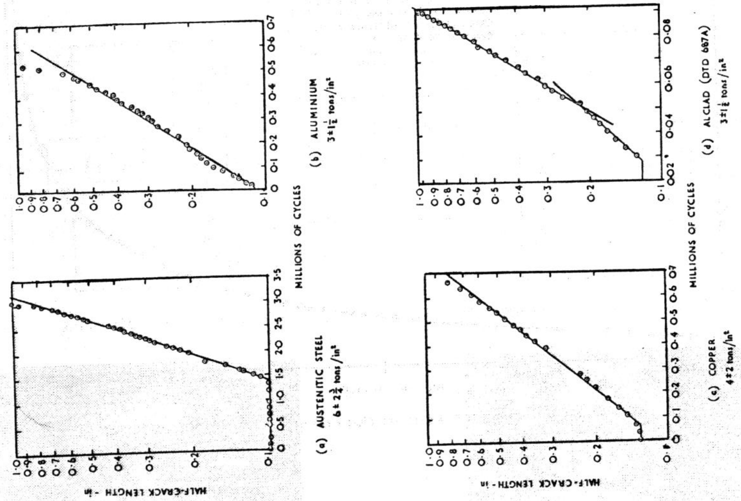


FIG 8 LOG HALF-CRACK LENGTH / NUMBER OF CYCLES

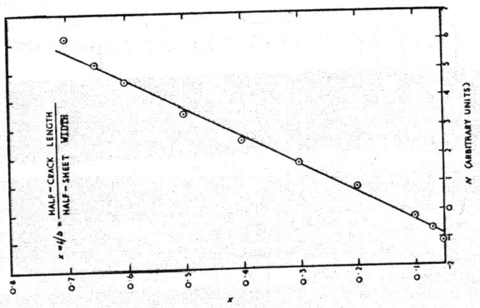


FIG 7 PREDICTED x/N CURVE BASED ON $dl/dN = A\sigma^3 l$ CRACK GROWTH TEST WITH STRESS ON NET AREA REMAINING CONSTANT

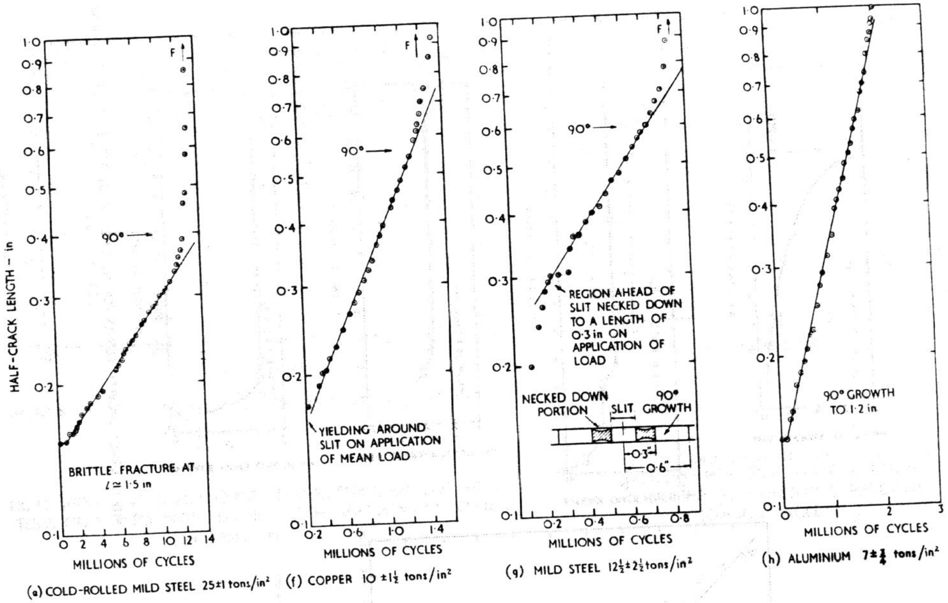


FIG 8 LOG HALF-CRACK LENGTH / NUMBER OF CYCLES

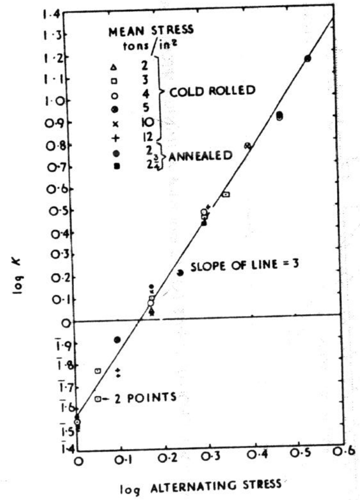


FIG 9 LOG K / LOG ALTERNATING STRESS FOR COPPER

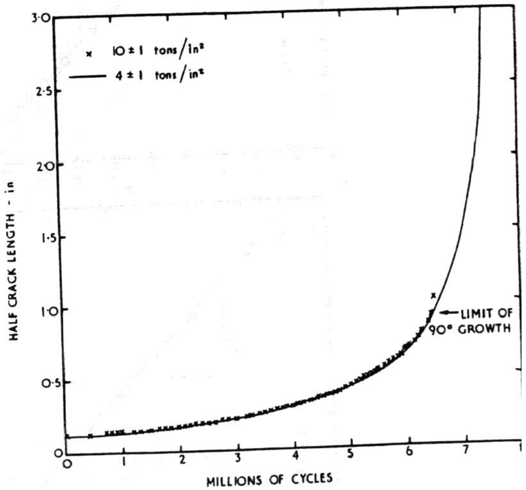


FIG 10 CRACK GROWTH CURVES FOR COPPER

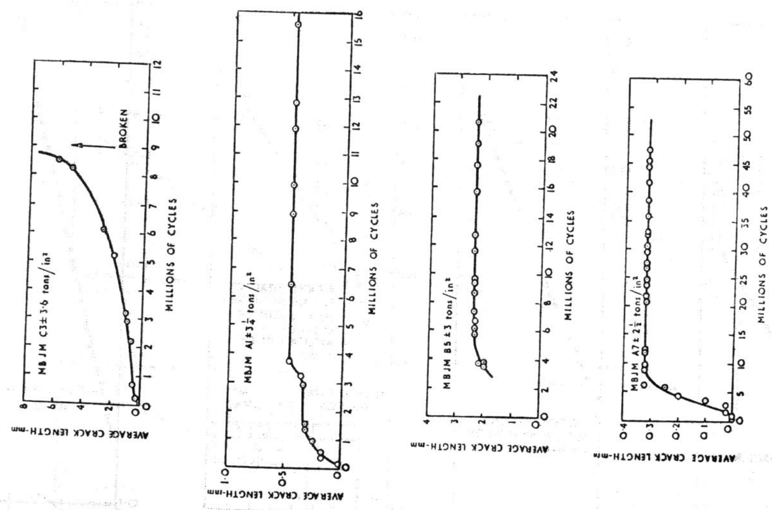


FIG 12 CRACK GROWTH CURVES FOR NOTCHED MILD STEEL PLATE, 0·010 inches ROOT RADIUS

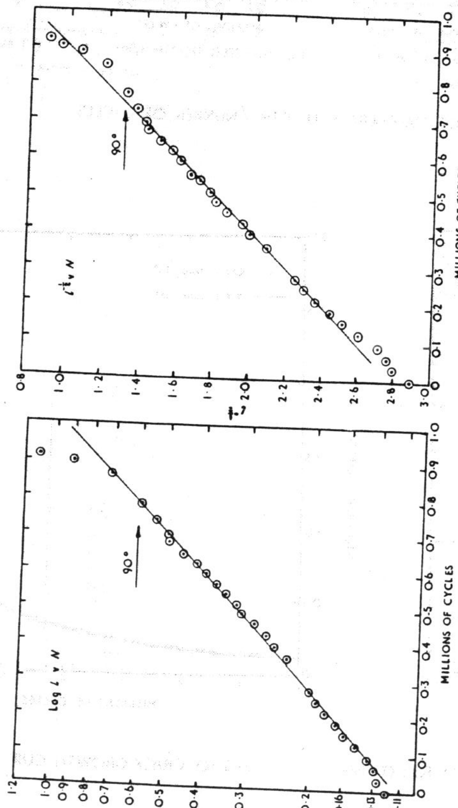
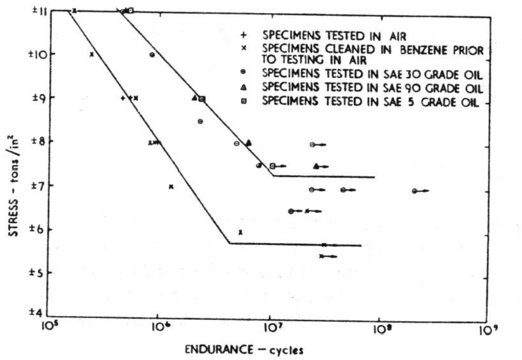


FIG 11 CRACK GROWTH CURVES FOR 10 inch WIDE ALUMINIUM SPECIMEN AT 3 ± 1¼ tons/in²



SPECIMENS 0·5 in o.d., VEE-NOTCH 0·05 in DEEP, ROOT RADIUS AS SHARP AS POSSIBLE.

FIG 13 STRESS/ENDURANCE CURVES FOR ROTATING BENDING FATIGUE TESTS ON SHARPLY NOTCHED MILD STEEL SPECIMENS COMPLETELY IMMERSED IN OIL

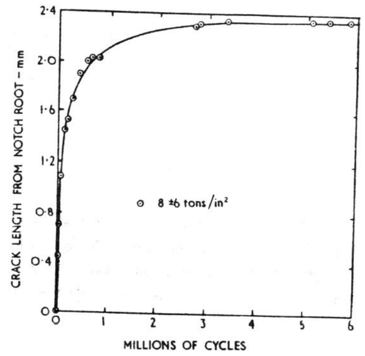
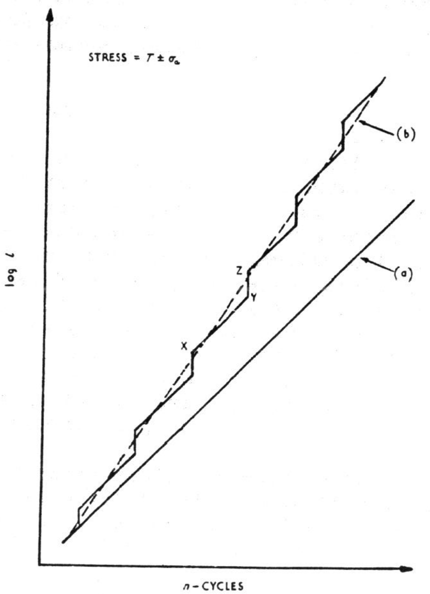


FIG 14 CRACK GROWTH CURVE FOR EDGE-NOTCHED MILD STEEL PLATE SPECIMEN SUBJECTED TO A WHOLLY COMPRESSIVE LOADING CYCLE



(a) GROWTH CURVE FOR ALUMINIUM

(b) MEAN GROWTH CURVE FOR ALUMINIUM ALLOY COMPOSED OF PORTIONS OF TRUE FATIGUE CRACK GROWTH (X Y) PARALLEL TO (a) AND INSTANTANEOUS ELEMENTS (Y Z) OF FAST FRACTURE

FIG 15 DIAGRAMMATIC GROWTH CURVES FOR ALUMINIUM AND AN ALUMINIUM ALLOY