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GROWTH OF FRACTURE IN METAL UNDER RANDOM CYCLIC LOADING

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ABSTRACT

Too few investigators have attempted by experimentation to show a correlation in the fatigue characteristics of metals under pure random loading with those under sinusoidal constant-amplitude loading. The majority of investigations that have been made should be classified as quasi-random load tests. Although realistic random load patterns may have been initially adopted for test purposes, they were either programmed into a frequency spectrum of discrete load steps or else possessed periodic load patterns derived from small random samples. The application of the cyclic loads described in this paper was continuously random. Also differing from past investigations, this program designed a series of discrete load tests from the measured response of a structure under random loading. And, in addition, fatigue crack-growth rates were determined for each of the above test load types. This was accomplished on simple notched test panels and is believed to be the first time crack-growth characteristics under random cyclic straining have been presented.

By means of an ordinary analysis, it was demonstrated that the growth of fracture due to random load straining could be predicted from the results of discrete load experiments.

Suggestions for further research are given. A similar program is recommended for tests of a real structure possessing more than one fracture mode and many critical locations in which cracking can occur. For the fatigue design evaluation of a complex structure, it has not yet been established whether a series of discrete load tests, as described in this program, can be used successfully. The characteristics of importance are crack-growth rates, fatigue life, and the identification of the critical structural element for a specific random load spectrum.

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INTRODUCTION

It is well known that the phenomenon of progressive fracturing from fatigue can adversely affect the life and strength of flight vehicle systems. Therefore, knowledge concerning the behavior of materials susceptible to this phenomenon is important, if efficient structures are to be designed.

It has been the usual practice to define the fatigue characteristics of metals, or of structural components, by subjecting test specimens of the structural elements to repeated or alternating loads. By this procedure, design data are obtained to evaluate the effect of these alternating loads on structures. However, such fatigue programs have not always been wholly satisfactory for obtaining estimates of probable structure life or for defining rates of non-linear accumulative damage to structures. One of the principal reasons for this is that load cycles in fatigue testing usually are applied regularly, whereas in practice the loads on structures fluctuate indiscriminately.

In the design phase of flight vehicles, current and future, it is necessary first to define the response of structures to random alternation of stress. These stress alternations arise from a disordered distribution of loads which are realistically encountered within vibration, acoustic, and turbulent-atmosphere environments.

An analytic method has been developed (References 1 and 2) for calculating the approach of failure of a structure subjected to these complex environments. In theory, the method predicts damage imposed under random loads from the observed damage measured under discrete amplitude loading. Although the method tentatively has been accepted, the experimental data existing are insufficient to demonstrate that the calculated behavior under random cyclical loading agrees with the observed phenomenon. The only data that can be found in the literature are those of Head and Hooke (Reference 1). Although there was reasonably good agreement between theory and their experimental results, a need for additional laboratory investigations in this field still exists.

Notably, this lack of sufficient proof exists for such characteristic material properties as fatigue life, fatigue-crack progression, fracture strength, and fracture mode.

The objectives of the analytic and experimental program discussed in this paper are to provide some additional proof and uncover new knowledge in these areas.

EXPERIMENTAL PROCEDURE

The test panels in this investigation were 0.091-in. gauge x 6 in.-wide x 8-in. long 2024-T3 (Cu - Mg - Al, F_{TU} = 63,000) bare aluminum alloy plates. The panels were flexed in bending from tension to compression about a zero mean stress. Fatigue cracking originated and propagated from a small starter notch centrally located in the width direction of a panel. Periodically, during the course of fatigue testing, the growth of the cracks and the corresponding number of stress cycles were measured and recorded. This procedure was followed for both the random-load and the discrete- or constant-amplitude-load fatigue-cracking tests.

RANDOM LOAD CRACK-GROWTH TESTS

The random load tests were conducted on an MB Electrodynamic Shaker, Model C-100. The test panels were mounted to the head of the shaker and vibrated along the vertical axis of excitation. The noise bandwidth for all random tests was from 20 to 2000 cps. To establish the resonant frequency of the beam specimen, a 0.1 G peak input sinusoidal frequency sweep was conducted from 20 to 1000 cps at a sweep rate of 1.0 octave per minute. A fundamental frequency of 47.3 cps was determined for the beam specimens during this test. It was necessary to define the fundamental frequency of the panels in order to evaluate the number of stress cycles as a function of test time on the vibrator.

A random noise signal was input to the shaker-driver amplifier. The energy input (acceleration spectral density) to the test beam was uniform at all frequencies from 20 to 2000 cps. However, since the beam specimen is a lightly damped single-degree-of-freedom system it responds primarily at its resonant frequency and ignores all others. The situation in this investigation was one of random loading with no periodicity but at a discrete frequency. Figure 1 shows a view of the beam specimen mounted to the shaker table. Figure 2 is a view looking down on the test panel. An accelerometer attached to the end of the beam was monitored throughout the duration of the test run and was maintained at a constant level of rms G's. This was controlled by an additional accelerometer attached to the shaker head which recorded an input random vibration of 0.045 G^2/cps acceleration density. A response level of 21 G_{rms} was selected for these tests. After calculating the rms displacement at the end of the excited beam, it was then a simple task to calculate the rms stress level. The rms stress level at the critical section of the panel, by the use of simple beam theory, was calculated to be 9510 psi (6.7 Kg/mm^2). At the present time, for vibration fatigue testing, it is believed that the level of loading expressed in terms of rms values is the most convenient factor in defining the content of a random load response.

CONSTANT-AMPLITUDE CRACK-GROWTH TESTS

The constant-amplitude load tests were conducted on a 15,000-lb. capacity Krouse fatigue-testing machine. The frequency of flexural reversals was 20 cps. Nine stress ranges were selected for this series of tests. Two panels were tested at each of nine discrete values of the rms stress level previously established for the random load testing. The maximum alternating stresses for the constant-amplitude test loading series ranged from levels of 1.4 to 3.0 times the rms stress level used in the random load tests.

Figure 3 is a trace from an oscillograph tape of the recorded constant-amplitude and random-loading patterns.

SUMMARY OF RESULTS

In both types of tests, the exact number of stress cycles to which the panel specimens were subjected at any given length of fatigue crack during its progression was known. In the constant-amplitude load tests, the number of cycles was recorded with a counter. In the random load tests, the excited panel specimen resonated at its fundamental frequency. Therefore, the number of cycles accumulated was arithmetically equal to the product of its natural frequency times the minutes and seconds on the vibrator.

A representative sample was taken from the tape recording of the random load test to define the distribution of loads. Of the peak-to-peak loads that were measured, there were extremely few loads having the same value. In every case a load wave cycle was followed by a load wave of a greater or lesser level. Dividing the counts into small class intervals, it was established that the load frequency content possessed a Gaussian distribution. This was anticipated since the relatively small test specimens were not expected to have an effect on the randomly excited armature of the large shaker. If a distribution of random measurements is normal (Gaussian), it is well known that their proportion can be calculated from the Rayleigh probability distribution function. Figure 4 shows the agreement between the frequency of the measured loads with the predicted number of loads calculated from the Rayleigh equation. The Rayleigh function has been used in the analytical phase of this investigation to define the proportion of peak loads with respect to the rms load level.

The results of the fatigue crack-growth measurements under constant-amplitude stressing are shown in Figure 5. Each curve represents the average of two tested panels. The scatter in the data does not exceed 10%. This variability may appear abnormally low, but it should be remembered that the large scatter in fatigue test results is associated only with the crack nucleation period in unnotched specimens. In these investigations the visible crack growth in the panels, at the severe starter notch, comprised over 95% of the total test time.

Once the fracture progression characteristics have been established under discrete loading, it is a simple task to predict these same characteristics for random loading. This is accomplished with the aid of the following equation:

$$\frac{1}{n_r} = \int_0^\infty \frac{P_{(x)} d_x}{n_S} \sum_{\ell = o}^{\ell = i} \frac{x_i e^{-\frac{x^2}{2}} \Delta x}{n_{S(x_i)}} = \frac{1}{n_{r(x_i)}}$$

where

n_r = number of random load cycles to propagate a crack from ℓ_o to ℓ_i

$P_{(x)} = xe^{-\frac{x^2}{2}}$ (the Rayleigh probability density function)

x = ratio of peak load to rms load

Δx = weighting factor dividing the distribution into class intervals

n_S = number of cycles under discrete loading to propagate a crack from ℓ_o to ℓ_i.

Successive calculations are made with the use of the above equation for the time or cycles required to grow a crack under fatigue loading, increasing in length from its initial starter size to longer lengths in increments of 0.1 in.

The proportion of the number of cycles of discrete loading (within a class interval) that occur within the random process can be assumed from the Rayleigh equation. The proportion of the damage imposed by each of these discrete steps is then linearly accumulated, even though the rate of damage accumulation as witnessed by the flaw growth curves is not. Table 1 presents a sample calculation for the predicted number of cycles of random load needed to propagate a crack a given length from the discrete load test results.

The predicted growth rate of progressive fracture under random cyclic loading was calculated for small and increasing increments of length. The result is shown in the graph of Figure 6. The experimentally determined crack growth in the randomly flexured test panels is also shown on Figure 6. There appears to be a good agreement between the predicted and measured results.

DISCUSSION

In the pre-notched test panels of this investigation, and on a macroscopic scale, there was only one critical location for fracture to occur. The location was common to all levels of cyclic stress that produced cracking. The fracture origin also was identical for the two types of cyclic test loading; e.g., constant-amplitude or random stressing. This situation is not necessarily true in the testing of large complex structures. On a finer microscopic scale both Williams (Reference 3) and Frost (Reference 4) have indicated the existence of a discontinuity in the fatigue behavior of even the simplest of test specimens. They suggested and then demonstrated the overlap of two types of fracture mechanisms that can occur during the fatigue process. The two types of fracture, although occurring at the same location in the specimen, appeared to be dependent upon the level of stress.

Slipband cracking occurred at the lower stresses and cracking at sub-grain boundaries occurred at the higher stresses above the discontinuity. These important observations are analogous to the multimode fatigue fractures that often are witnessed in the testing of complex structures. Figure 7 graphically depicts differences which have been observed in the testing of single elements as compared to complex structures (References 5, 6). One of the reasons for this apparent anomaly is the nonlinear distribution of straining within the individual elements of a composite structure as it is subjected to gradually increasing loads. It is also unlikely that the individual S/N curves for any two elements of a built-up structure could be alike. The coupling of these conditions results in fatigue fractures occurring at different locations within the overall structure, each time the cyclic stress level or stress range is altered by an appreciable amount.

The above discussion has been included for many reasons. It is to caution the unwary that accelerated fatigue testing, in the high stress region only, can often lead to an invalid identification of the critical member within a structure. The preceding discussion is also a natural introduction into the subject on the equivalency of constant-amplitude sinusoidal testing for random load testing. Now there are two views on this matter: one view maintains that an equivalency exists; the other maintains that it does not. From the limited experimental results of this investigation, it is believed that good engineering structure, designed to operate in a random loading environment, can be safely evaluated by means of a series of so-called equivalent sinusoidal loading tests.

A single test-stress range is not recommended as an equivalent to random testing. In some cases, it may be considered sufficient for first approximation answers. Even in these cases, however, an approximate single-test stress range cannot be accurately predicted in advance for real structure.

CONCLUSIONS

1. The close agreement between predicted and experimental results indicates that a series of less costly discrete tests can be used to predict behavior of structures subjected to random loads.

2. Although there appears to be no single stress-producing damage at rate equal to that measured under random loading (Figures 5 and 6), it is believed that the accuracy of the predicted behavior is adequate for use in design.

3. Additional experimentation is required. Complex structure possessing many modes of fracture should be investigated in future programs. It should be determined if fatigue life (crack propagation characteristics, as well as the critical fractured member in a built-up structure and under natural random loading) can be predicted from a series of discrete load tests.

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Table 1
SAMPLE CALCULATION FOR NUMBER OF RANDOM LOAD CYCLES TO GROW CRACK FROM 0.5 TO 0.8 INCHES

$x = \frac{\text{peak stress}}{\text{rms stress}}$	$xe^{-\frac{x^2}{2}} = P_{(x)}$	$xe^{-\frac{x^2}{2}}\Delta x$	σ, psi	$n_S \times 10^{-6}$ at ℓ = 0.5 in.	$n_S \times 10^{-6}$ at ℓ = 0.8 in.	Δn_S from ℓ = 0.5 to 0.8 in.	damage (ℓ = 0.8 - 0.5) $= \frac{xe^{-\frac{x^2}{2}}\Delta x}{\Delta n_S}$
(1)	(2)	(3)=(2) x 0.2	(4)	(5)	(6)	(7)=(6)-(5)	(8)= (3)/(7)
0.8	0.581	0.1162	7610	∞	∞	∞	0
1.0	0.606	0.1212	9510	∞	∞	∞	0
1.2	0.584	0.1168	11410	∞	∞	∞	0
1.4	0.525	0.1050	13310	0.540	0.900	0.360	0.291
1.6	0.445	0.0890	15210	0.160	0.272	0.112	0.795
1.8	0.356	0.0712	17110	0.118	0.205	0.087	0.820
2.0	0.271	0.0542	19020	0.097	0.160	0.063	0.860
2.2	0.196	0.0392	20920	0.081	0.124	0.043	0.910
2.4	0.135	0.0270	22820	0.062	0.092	0.030	0.900
2.6	0.089	0.0178	24720	0.044	0.069	0.025	0.710
2.8	0.054	0.0108	26620	0.024	0.046	0.022	0.490
3.0	0.033	0.0066	28525	0.015	0.030	0.015	0.430
							$\Sigma = 6.206 \times 10^{-6}$

Predicted Δn_r from ℓ = 0.5 to 0.8 in. $= \frac{1}{6.206 \times 10^{-6}} = 160{,}000$ cycles.

Measured Δn_r from ℓ = 0.5 to 0.8 in. = 140,000 to 154,000 cycles.

FIG. 1 BEAM PANEL MOUNTED ON ELECTRODYNAMIC SHAKER

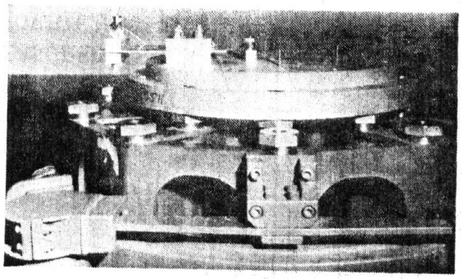


FIG. 2 TEST PANEL

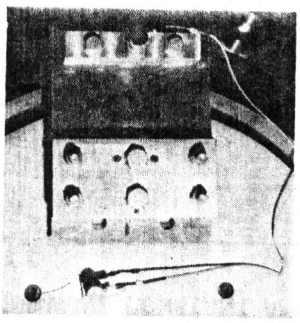
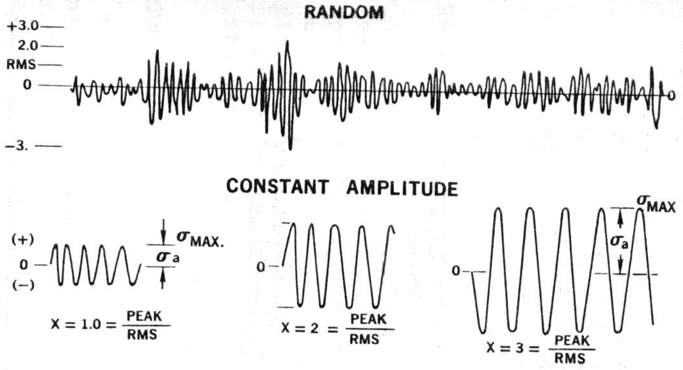


FIG. 3 TEST PANEL LOADING



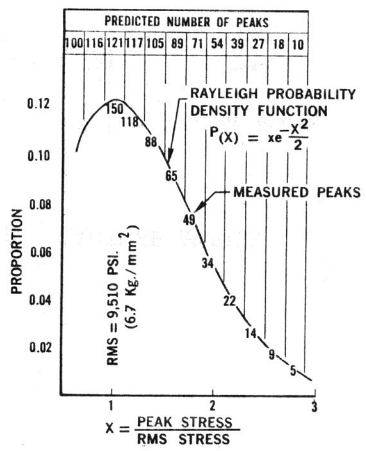
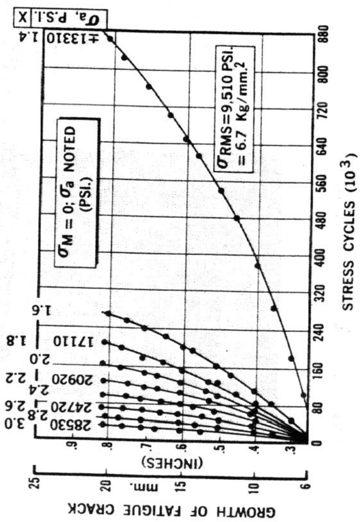


FIG. 4 MEASURED STRESS RESPONSE TO RANDOM VIBRATION

FIG. 5 SINE-WAVE CONSTANT AMPLITUDE TESTS



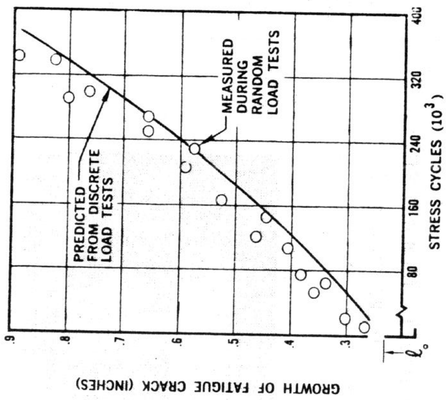


FIG. 6 COMPARISON OF PREDICTED AND MEASURED CRACK GROWTH UNDER RANDOM VIBRATION

FIG. 7 FRACTURE MODES