

DI-12 Crack Propagation of Notched Specimen with
Residual Compressive Stress

Takeshi KUNIO*, Hiroshi NAKAMURA**
and Masao SHIMIZU***

Abstract

The remarkable improvement of fatigue strength of low carbon steel notched specimen induction hardened can be considered mainly due to the existence of residual compressive stress in the skin of it.

To clarify the above qualitatively as well as quantitatively, the characteristic of macro crack growth was investigated which is closely related to the residual stress. Consequently, it was found that the compressive residual stress acts effectively to prevent the crack from propagating. Also the new concept concerning consecutive crack development was proposed, introducing "the crack tip stress". Thus the theoretical crack depth can be predicted which agrees well with experimental data.

§1. Introduction

It is well known that the fatigue limit of the notched specimen having the residual compressive stress on the surface shows remarkably higher value than that of the specimen without it. And the fact can be recognized from the experiments, in this case, that there exists a considerable difference between the crack initiation stress σ_{w1} and the crack propagation stress σ_{w2} of such a specimen. As a numerical example, σ_{w1} and σ_{w2} of the induction hardened notched specimen of low carbon steel come up to 16 kg/mm² and 52 kg/mm² for the rotary bending fatigue test, though their values in the normalized state are 8kg/mm² and 12kg/mm² respectively before induction hardening (1). Such an increase of the fatigue limit may be regarded as the result of the remarkable uprise of the crack propagation stress, which is due to effect of residual compressive stress produced by induction hardening process on the surface of the notched specimen from the results of following experimental study (2, 3): 1) the crack propagation stress of 0.37% carbon steel notched specimen induction hardened reaches about 56 kg/mm², while its crack initiation stress remains 18 kg/mm², and 2) the difference of their stress values for a hollow cylinder

*, ***) Department of Mechanical Engineering, Keio University, Tokyo
**) Railway Technical Research Institute, JNR, Tokyo
Numbers in Parentheses refer to the References at the end of the paper.

relieved from the compressive stress is not so much as $\bar{\sigma}_{w1} = 17 \text{ kg/mm}^2$ and $\bar{\sigma}_{w2} = 18 \text{ kg/mm}^2$. Similar results have been obtained through the fatigue tests concerning 0.17% carbon steel specimen (4).

The reason why the notched specimen induction hardened has such a very wide non-propagation stress range seems due to the fact that the residual compressive stress at the crack tip or notch root prevents effectively the crack from opening, thus propagating. As a matter of fact, it is observed that the greater part of the fatigue life of such a specimen is occupied by the propagation period of the crack (1, 2).

Generally, the residual stresses produced by ordinary methods such as shot peening and cold rolling, are relaxed during load repeated in fatigue test. However, the residual stress by induction hardening usually would not be released so easily under repeated stress.

As known in the above-stated descriptions, behavior of the fatigue crack seems to be closely related to the residual compressive stress.

In this paper, it will be described in what manner the crack develops at the notch root of the specimen with residual compressive stress and what interpretation can be given for the situation of the crack propagation.

In addition, it will be shown that the curves obtained theoretically for the growth of the crack agrees well with those given experimentally. And finally, it will be concluded that the interpretation proposed here is appropriate qualitatively as well as quantitatively for extremely high fatigue strength of such an induction hardened notched specimen.

§2. Experimental Procedure

Every specimen tested of the two kinds of materials viz. 0.16% and 0.17% carbon steels was annealed at 850°C for 2 hours. The specimens which were machined into the shape as shown in Fig.1, were induction hardened from quenching temperature as high as 1100°C. As a well-known fact, the residual compressive stress remains on the outer surface of the specimen by means of induction hardening treatment. The chemical compositions of the materials of two groups employed in this test are tabulated in Table 1. The fatigue testing machine for rotary pure bending whose capacity is 10 kg-m was used.

The crack depth λ was measured by following manner: first, the cracked specimen was heated for 30 minutes in an electric furnace whose temperature was kept about at 400°C high, so as to make the cracked part colored, and then the specimen was re-set to the machine until they broke down completely. Thus, the crack depth can be measured from the colored figure by means of projecting apparatus which has 50 times in magnitude. At that time, the mean depth at 8 positions on the circumference of circular section was employed as the data used.

The crack propagation stress $\bar{\sigma}_{w2}$ was decided by finding out the maximum nominal stress level at which the specimen can endure the infinite life containing crack at the notch root, while the crack initiation stress $\bar{\sigma}_{w1}$ was settled by extrapolating the curve of the non-propagating crack length λ_{np} versus the applied nominal stress $\bar{\sigma}_0$ to the line on the λ_{np} is equal to zero.

§3. Results and Discussion

3.1 The remarkable uprise of the crack propagation stress $\bar{\sigma}_{w2}$ due to the effect of residual compressive stress and the characteristics of the crack growth.

The observations as for rising of the crack propagation stress $\bar{\sigma}_{w2}$ due to the residual compressive stress and the characteristics of the crack growth concerning the 0.16% carbon steel specimen will be summarized briefly as follows: Fig.2 shows the experimental results that the crack propagation stress $\bar{\sigma}_{w2}$ of the notched specimen rises up remarkably from 12 kg/mm² to 52 kg/mm² by treatment of induction hardening. Since the crack initiation stresses $\bar{\sigma}_{w1}$ before and after induction hardening are 8 kg/mm² and 16 kg/mm² respectively, it is understood obviously that there produces a considerable difference between $\bar{\sigma}_{w1}$ and $\bar{\sigma}_{w2}$, in other words non-propagating stress range is extended largely by this treatment. Fig.3 shows the state of the crack growth of such specimens for various stress amplitudes. It should be noted from the figure that the crack appears in very early stage of the fatigue process at every stress amplitude and almost all parts of the fatigue life of the specimen is actually occupied by that of the growing process of this macro crack. For example, at higher stress level of 70 kg/mm², the crack was recognized after 150 cycles of load.

However, the final fracture at this stress level occurred at about 3×10^4 cycles. Also the noteworthy point seen here is that the rate of crack growth decreases rapidly as the crack grows even if the stress amplitude is beyond the fatigue limit. In Fig.4 the similar crack growth curves (5) concerning the annealed specimens (0.17%C) whose fatigue limit $\bar{\sigma}_{w1}$ and $\bar{\sigma}_{w2}$ are 7 kg/mm² and 12 kg/mm² respectively, are presented. Comparing these results with the previous ones, it is seen that the crack growth characteristics of the notched specimen with large compressive residual stress differ very much from those of the specimen without the residual stress.

3.2 Estimated stress at the crack tip by which the experimental results may be interpreted.

The crack growth characteristics of induction hardened notched specimens of low carbon steel, as stated in the previous section, can not be satisfactorily explained from the reduction of effective cross sectional area, stress concentration at the crack tip and so on.

Assuming that there is a sufficient residual compressive stress at the notch root and the tip of the crack, the stress at the tip of the crack due to external force may be evaluated by the following form:

$$\sigma' = \sigma_0 \cdot \delta \dots\dots\dots(1)$$

where σ_0 : nominal stress at the notch root before the crack initiates,
 δ : ratio of the calculated stress at the crack tip to the nominal stress σ_0 .

The above relation may be sufficiently precise, provided the two following assumptions are allowed: that is, first the facial pressure is maintained at the crack surface of the tension side even under applied force, second the crack

appears on the surface where there is no shear stress, in other words it grows at the notch root in the plane making right angle to the axis. In connection with the first assumption, the following fact has been known through the experiments (6): that is, after making the specimen of 50 mm diameter having circumferential crack of 1.7 mm depth due to rotary bending fatigue, the bending strain across the crack was measured under the statical bending. Fig.5 shows the results of such experiments. As seen in the figure, the longitudinal strains have the same values at tension and compression sides for the induction hardened specimen. For the non-hardened one, however, the remarkable difference is seen at the beginning stage of loading. This implies that the crack of the specimen having compressive residual stress would not open as long as applied load is below a certain value. Therefore, it can be supposed that no large stress concentration or no strain concentration occurs at the crack tip, and also that the crack surface sustains the part of load equivalent to the reduced facial pressure.

As for the second assumption, the extending direction of the crack can be regarded as the right angle to the axis within a scale considered here, and in fact this has been often observed by ordinary microscope. One of the authors has derived the expression (1) in somewhat different way and discussed successfully the problem concerning the non-propagating crack (2, 7). Now, if the σ' calculated above be the stress at the crack tip, the decrease of crack propagation rate with the growth of the crack may be explained by saying that the stress σ' at the crack tip diminishes rapidly as the crack advances. Fig.6 can be obtained by finding the rate of the crack propagation $d\lambda/dN$ and the stress σ' at the crack tip for various stress levels σ_0 from λ - N curves in Fig.3, and $d\lambda/dN$ may be considered to depend only upon the stress σ' . However, it will be shown from later discussion that $d\lambda/dN$ depends more essentially upon other variables than the stress σ' .

The value of σ' where $d\lambda/dN$ comes down to zero corresponds to the stress for non-propagation of crack and it is approximately 50 kg/mm², which is nearly equal to the plain fatigue limit σ_{wo} of the present material. This fact should be noted.

Fig.7 shows the relationship between σ' and N obtained from Fig.3. In Fig.7, the S - N curve for the specimen (stress concentration factor $\alpha = 1.26$) of similar material having the same hardness numbers is also drawn. It is seen clearly that σ' at the crack tip for various stress levels is nearly equal to the plain fatigue limit of the material for $N = 10^7$, where the crack propagation has already halted. Further, it can be observed that σ' - N and S - N curves coincide with each other for sufficiently many numbers of stress cycles applied ($N > 10^4$).

This shows the following interesting fact: Fig.8(b) shows schematically the S - N curve of the present material which shows that the fatigue fracture occurs after N_1 cycles with stress amplitude of σ'_1 .

Also Fig.8(a) gives the distribution of actual stress σ' in the vicinity of the notch root in which the ordinate of a curve of the stress σ' versus λ is drawn with the same scale as that of curve in Fig.8(b). And also this curve corresponds to a particular case of the applied nominal stress σ_0 by which the actual stress σ'_1 produced at the distance of λ_1 from notch root. From previous discussion, this stress distribution remains unchanged after the

crack occurs at the notch root. Judging from comparison of both Fig.8(a) and 8(b), after N_1 cycles with the nominal stress σ_0 applied, the fatigue fracture must be present at a point $\lambda_2 (< \lambda_1)$ because $\sigma'_2 > \sigma'_1$. In other words, the crack develops to the crack depth λ_1 . On the contrary, since $\sigma'_3 < \sigma'_1$ for $\lambda_3 > \lambda_1$, fatigue fracture does not occur, hence no crack appears in the range of λ_3 . Consequently, there may exist a law of crack propagation, that is, after repeating with the nominal stress of σ_0 in numbers of N_1 cycles, the crack propagates until σ' equals the stress S at N_1 on the S - N curve for the material concerned. Also the more essential relationship between the calculated stress σ' and the rate of crack propagation $d\lambda/dN$ can be derived based on the above law of crack propagation. Now, consider a crack depth of λ has developed at the notch root after n cycles of nominal stress σ_0 applied, and put the increment of the crack depth by $d\lambda$ after additional cycles dn of the same stress level σ_0 . Then, from the law of crack propagation the following relationship holds as known from Fig.9:

$$\frac{d\sigma'}{d\lambda} d\lambda = \frac{dF(n)}{dn} dn$$

$$\frac{d\lambda}{dn} = \frac{dF(n)/dn}{d\sigma'/d\lambda}$$

Using the relation (1) $\sigma' = \sigma_0 \cdot \delta(\lambda)$, we have

$$\frac{d\lambda}{dn} = \frac{dF(n)/dn}{\sigma_0 \cdot d\delta(\lambda)/d\lambda} \dots\dots\dots (2)$$

The numerator $dF(n)/dn$ in the right side of the above is the function of σ' only, and denominator $d\delta(\lambda)/d\lambda$ represents the stress gradient for unit nominal stress in the vicinity of the crack tip. Therefore the previous description that $d\lambda/dN$ might be decided by only σ' , independent to σ_0 is not always exact one essentially, but it seems approximately or apparently reasonable, because of the constancy of $\sigma_0 \cdot d\delta(\lambda)/d\lambda$ from the consideration as follows: the crack depths λ for a σ' will have different values according to nominal stresses applied. However, for specimens having the same dimension of notch, λ for a σ' increases as σ_0 becomes larger and vice versa. On the other hand, it is obvious that the gradient $d\delta(\lambda)/d\lambda$ will decrease as λ increases. So the products of σ_0 and $d\delta(\lambda)/d\lambda$ seem to be approximately constant for various values of σ_0 and the previous description can be concluded.

3.3 Prediction of crack growth curve and its experimental verification.

From previous discussions, if the S - N curve and stress distribution at the notch are known, one can predict the crack behavior of specimen having sufficiently large residual compressive stress at notch root which does not vary so much with the repeated stress. However, at this time the following two points should be mentioned: (1) the S - N curve for final fracture of present induction hardened material is employed as the fracture condition at the notch root, that is the condition of crack propagation, and (2) the maximum stress only at notch root is taken into consideration with respect to the fracture, but the steep stress gradient in vicinity of it is not. As the condition for the fracture at the notch root, the S - N relation for the crack initiation should be taken up more essentially rather than that for final fracture, since

the repeated cycles N of the ordinary $S-N$ curve is total number of cycles of crack initiation and propagation process. In connection to this, after how many cycles of repeated load the crack actually initiates was studied concerning the present induction hardened specimen without a notch groove.

As a result, it was found that the crack initiation could not be recognized within 70% of final life on applying repeated stresses of 120% and 140% of fatigue limit. Therefore the $S-N$ curve for the final fracture was used as the fracture condition at the notch root.

Next, as for the fracture condition at a point having steep stress gradient such as at notch root and crack tip, it should be usually decided under considering stress and or strain state in vicinity of that point(8). However, the reasons for taking only the maximum stress into consideration are as follows: first, the experimental fact has been known that crack initiation stress σ_{w1} of low carbon steel notched specimen induction hardened is close to the stress value, divided the plain fatigue limit σ_{w0} by stress concentration factor α of the notch form. This means that the fracture at the stress concentrated point occurs dependently only upon the maximum stress value there, in other words, that fatigue fracture occurs when the stress at the point attains to the critical value, or the fatigue limit in present case. Second, even if the fracture at the stress concentrated point depends upon the stress or strain state in the vicinity of it, such a range related to the crack propagation remains uncertain.

From the reasons mentioned, the discussions were preceded, paying attention upon only maximum stress at the point concerned. Finally, by use of the law of crack propagation stated previously, predictions of crack growth curves for induction hardened 0.17% carbon steel notched specimens with various stress concentration factors were made and compared with experimental data. The results are shown in Fig.10, 11 and 12. These figures show the predicted (dotted) curves and the experimental data for factors $\alpha = 3.95, 3.07$ and 2.20 respectively. It will be seen there that the predicted and the experimental results coincide well with each other.

§4. Conclusion

The authors performed various experiments for the investigation on characteristics of crack propagation of specimen treated by induction hardening and discussed about the obtained results. It was shown that the experimental results can be clearly explained quantitatively by using the proposed concept of the stress σ' at the crack tip, and also that the prediction of the crack growth curves is possible under several assumptions, which agrees well with the experimental ones.

Consequently, it is concluded that the proposed concept here might give possible interpretation for the uprise of fatigue limit of notched specimen with the surface residual compressive stress.

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Table 1 Chemical Compositions of the Materials (%)

| Material | C | Si | Mn | Pb | S |
|--------------|------|------|------|-------|-------|
| 0.16%C Steel | 0.16 | 0.18 | 0.45 | 0.014 | 0.031 |
| 0.17%C Steel | 0.17 | 0.20 | 0.56 | 0.016 | 0.021 |

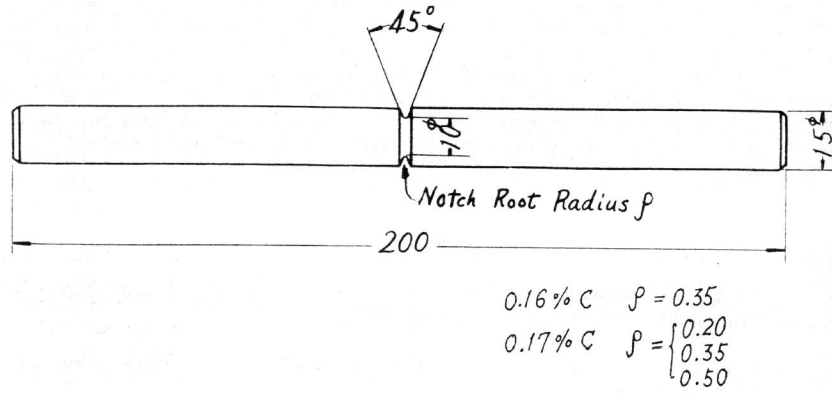


Fig. 1 Profile of specimen (in mm)

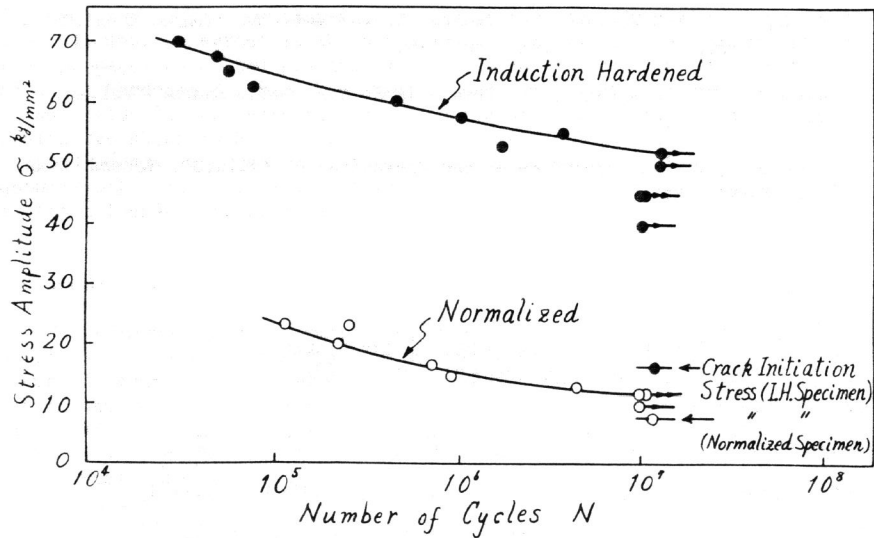


Fig. 2 An example of remarkable uprise of fatigue limit of notched specimen due to induction hardening (0.16% carbon steel, $\alpha = 3.07$)

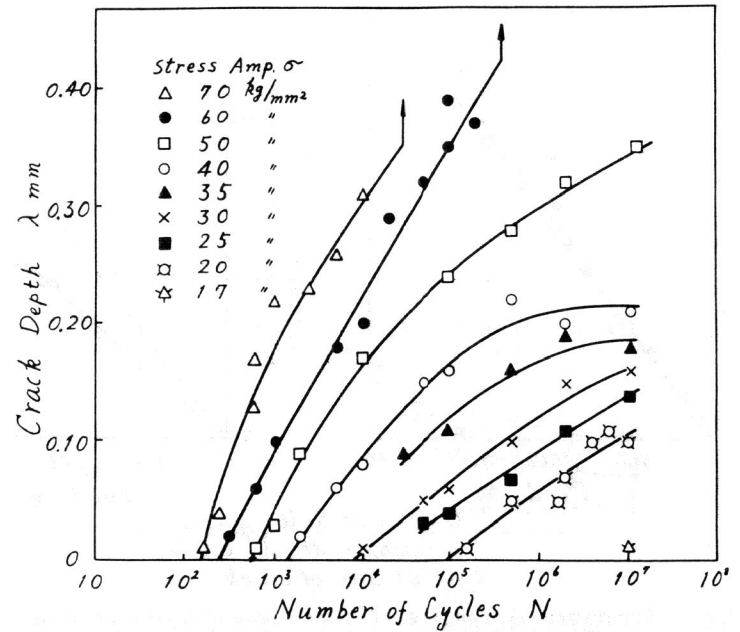


Fig. 3 Crack growth curves of the induction hardened notched specimens (0.16% carbon steel, $\alpha = 3.07$)

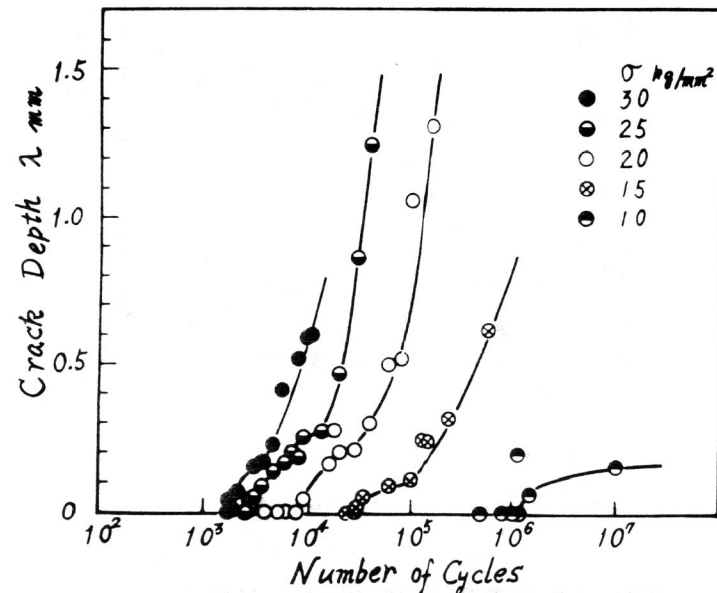


Fig. 4 Crack growth curves of the annealed notched specimens (0.17% carbon steel, $\alpha = 3.07$)

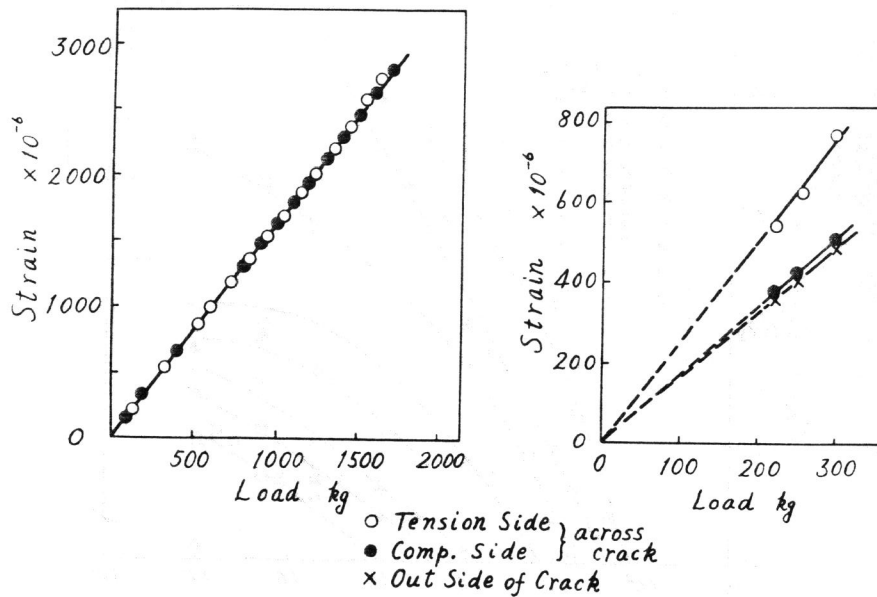


Fig.5 Experimental results of strain measurement showing the effect of the residual compressive stress.

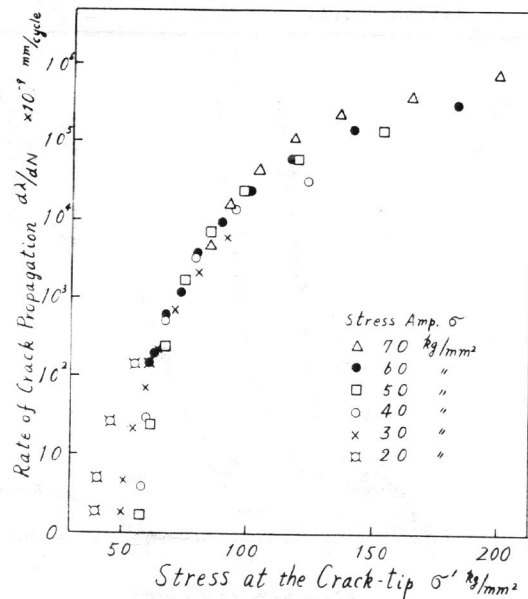


Fig.6 Experimental data showing that $d\lambda/dN$ is approximately decided by only σ' .

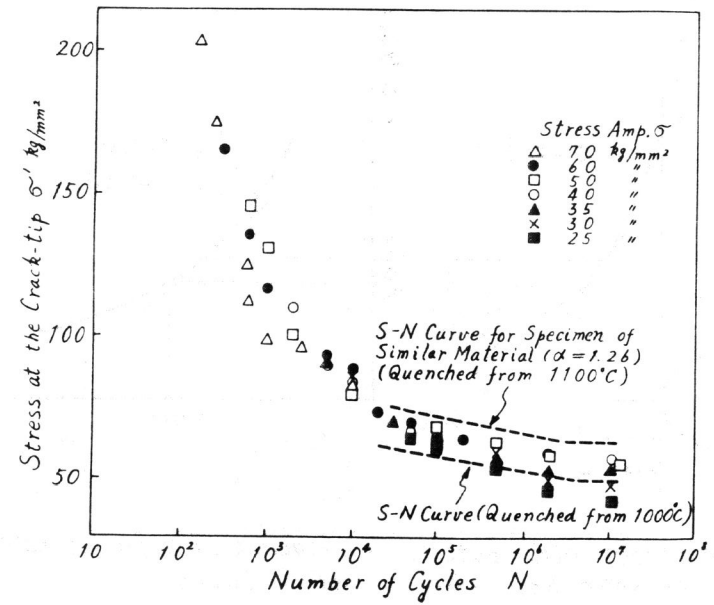


Fig.7 Experimental data showing the relation between $\sigma'-N$ and $S-N$.

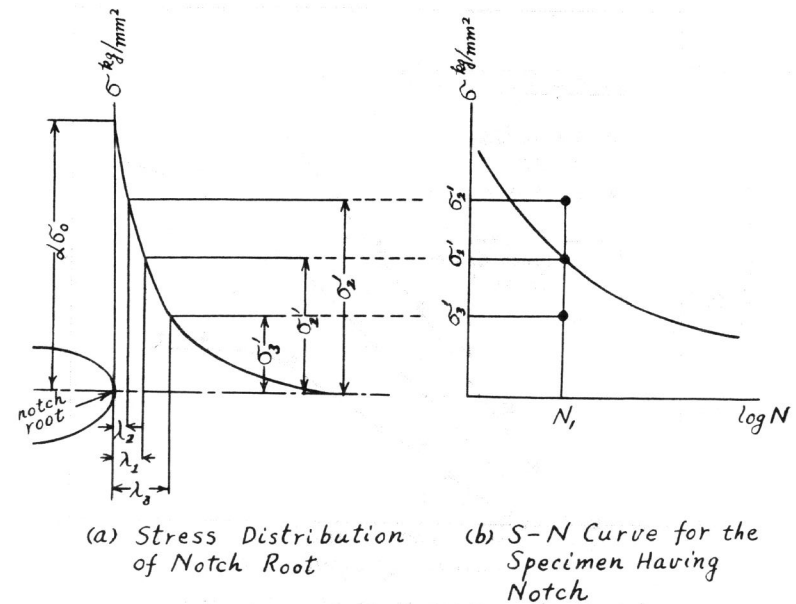
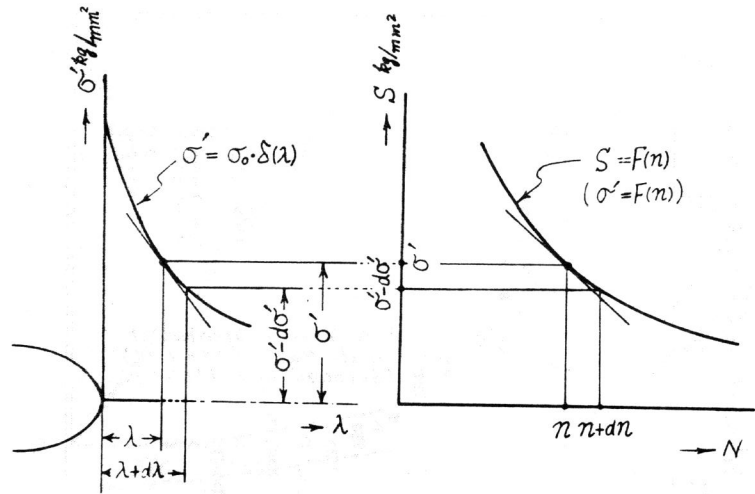


Fig.8 Sketch showing explanation of the proposed low of consecutive crack development (A)



(a) Stress Distribution of Notch Root (b) Fracture Curve for the Material (S-N Curve)

Fig.9 Sketch showing explanation of the proposed law of consecutive crack development (B)

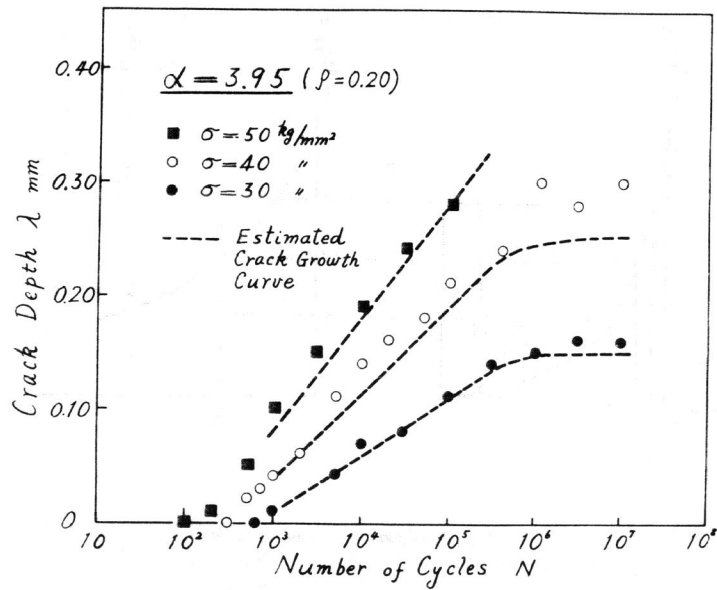


Fig.10 Comparison of predicted growth curves with experimental data ($\alpha = 3.95$)

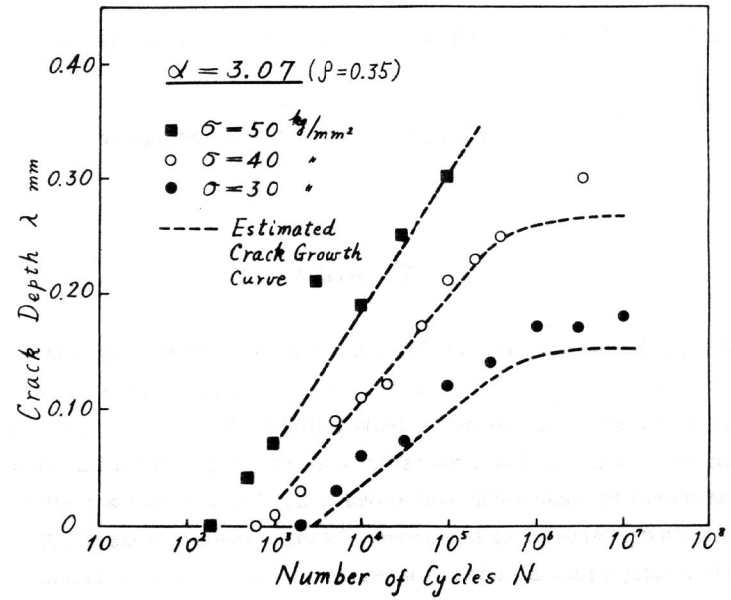


Fig.11 Comparison of predicted growth curves with experimental data ($\alpha = 3.07$)

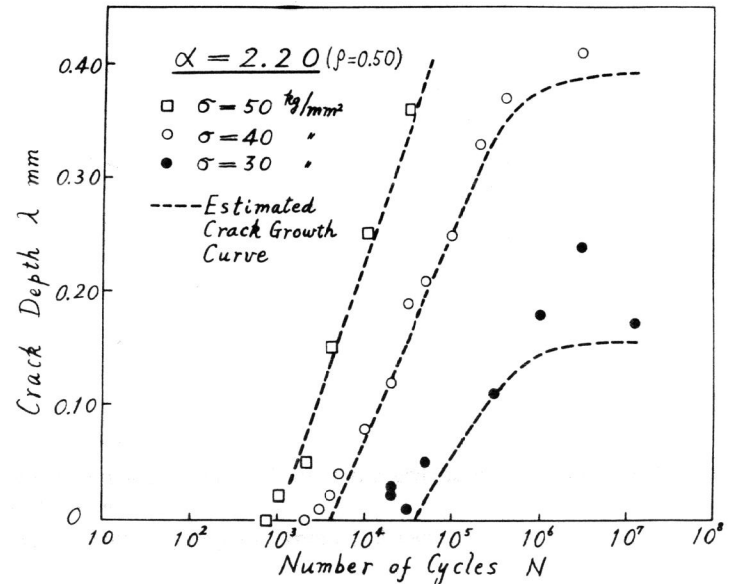


Fig.12 Comparison of predicted growth curves with experimental data ($\alpha = 2.20$)