STRUCTURE AND THEORETICAL

STRENGTH OF GLASS

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A new method is proposed for estimating the value and temperature dependence of the theoretical strength of glasses by taking into account the effect of the microheterogeneous structure on the strength. The method is based on the analysis of the curve of thermal linear expansion and the temperature dependence of sound velocity. A curve has been calculated for the temperature dependence of the strength of alkali-silicate sheet glasses. The theoretical strength of this glass at absolute zero is equal to 800 kg/mm², and at 300°K to 650 kg/mm². From the comparison of the calculated and experimental results it follows that the temperature dependences of both theoretical and real strengths are in agreement.

In a work [1] an equation has been obtained establishing a relationship between the theoretical strength and elastic modulus

$$\sigma_m = \varkappa E \tag{1}$$

where E is Young's modulus, $\varkappa$ is a certain coefficient

$$\varkappa = \frac{1}{n+1}\left(\frac{m+1}{n+1}\right)^{\frac{m+1}{n-m}} \tag{2}$$

m and n are power parameters of the binomial formula for the potential of interaction between two atoms.

$$U = -\frac{A}{\zeta^m} + \frac{B}{\zeta^n} \tag{3}$$

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The practical value of this equation may be realized only in the case where an accurate method will be found for determining experimentally the constituent E, m and n parameters for every specific material.

It is known that the experimental determination of Young's modulus presents no difficulty, therefore the problem of evaluating the theoretical strength is confined to a reliable determination of the m and n power parameters.

The calculations given below are based on the assumptions that

1. Glasses have microheterogeneous structure, thereby under the structural group is meant a submicroscopic element interacting with those similar to it and forming the continuum of a glass-like substance.

2. The theoretical strength of glasses is dependent on the energy of interaction between structural groups.

3. Bonds between structural groups are predominantly of ionic nature.

In a state when the solid is not stressed the minimum of the potential energy of interaction according to (3) may be written as follows:

$$U_o = - \frac{B}{\tau_o^n}\left(\frac{n}{m}-1\right)$$

Or, which is the same, in the form:

$$U_o = - \frac{\tau_o^3}{mn} \cdot \frac{nB}{\tau_o^{n+3}} (n-m) \tag{4}$$

After expanding the function (3) near the τ_o point into Taylor series to powers $\xi = \tau - \tau_o$ up to a third term and subsequently differentiating the left and right sides of the equation, we shall have:

$$\mathcal{F} = -\frac{dU}{d\xi} = -\left(\frac{d^2 U}{d\tau^2}\right)_{\tau_o} \cdot \xi = -\gamma \cdot \xi$$

or, otherwise,:

$$\mathcal{F} = \gamma \xi = -\tau_o^2 \cdot E \cdot \frac{\xi}{\tau_o} \tag{5}$$

where $\mathcal{F}$ is the quasi-elastic force of the interaction between two atoms, γ is the elastic coefficient, E is Young's modulus, τ_o is the equilibrium distance between atoms.

From further calculations it follows that

$$E = \frac{nB}{\tau_o^{n+3}} (n-m) \tag{6}$$

According to equations (4) and (6) the interaction energy can be written in the form

$$U_o = - \frac{E \cdot \tau_o^3}{mn} \tag{7}$$

By substituting into equation (7) τ_o^3 for its approximate ratio $\tau_o^3 \simeq \frac{M}{\rho N_A}$ and multiplying the leftside and rightside parts by Avogadro number N_A we shall obtain:

$$U_o = -\frac{M}{mn} \cdot \frac{E}{\rho}$$

where ρ is the density, M is a grammolecule of the substance.

Considering that $\frac{E}{\rho} = c^2$ is a square of the sound velocity alongside of the thin rod, we may write

$$U_o = - \frac{M \cdot c^2}{mn} \tag{8}$$

The second equivalent expression for this quantity can be obtained from equation (7), using Frenkel expression [2] for the coefficient of thermal linear expansion

$$\alpha = \frac{gK}{\tau_o \gamma^2} \tag{9}$$

where $g = -\frac{1}{2}\left(\frac{d^3 U}{d\tau^3}\right)_{\tau_o}$, K is Boltzmann constant. From (9), we shall obtain

$$\tau_o^3 = \frac{K}{2} \cdot \frac{m+n+3}{\alpha \cdot E} \tag{10}$$

Putting this expression in (7) permits the second form of expressing the potential energy to obtain

$$U_o = - \frac{R}{2\alpha} \frac{m+n+3}{mn} \tag{11}$$

Examining the process of plastic deformation in crystals on a model of two interacting chains of identical atoms, Frenkel and Kontorova [3] have obtained a correlation associating the ultimate energy of the shear deformation in the chain with a mass of interacting atoms and the velocity of sound alongside the chain. This expression written for one pair of atoms has the form:

$$w \simeq \frac{2}{\pi^2} m_o c^2 \tag{12}$$

where w is the ultimate shear energy per one pair of atoms (one kinetic unit), m_o is the atomic weight (of kinetic unit), c is the velocity of

sound.

By recalculating per a grammolecule of kinetic units, equation (12) will be

$$W_o \simeq \frac{2}{\pi^2} M c^2 \qquad (13)$$

Formulae (8), (11), (13) may be considered as three approximate expressions determining the three sought quantities: M, m, n.

Assuming that the strength of glass depends on the energy of the bonds between structural groups, we shall take it into account in the obtained equation substituting the "0" subscript for the "g" subscript.

The following expressions will then be obtained:

1. $$U_g = -\frac{Mc^2}{mn}$$

2. $$U_g = -\frac{R}{2\alpha}\frac{m+n+3}{mn} \qquad (14)$$

3. $$W_g = \frac{2}{\pi^2} M c^2$$

The general solution of equation (14) enables us to write down

$$\left. \begin{array}{l} m \cdot n = 4.94 \dfrac{W_g}{U_g} \\[2mm] m + n = 4.99\alpha \cdot W_g - 3 \end{array} \right\} \qquad (15)$$

where α is the coefficient of thermal linear expansion above glass transition.

The test checking of equations (15) indicates that with a sufficiently high accuracy it can be assumed that $m=const=1$, and the n parameter can be considered as a function of temperature.

Then for $\mathcal{æ}$ in equation (1) we shall have:

$$\mathcal{æ}(T) = \frac{1}{n(T)+1}\left(\frac{2}{n(T)+1}\right)^{\frac{2}{n(T)-1}} \qquad (16)$$

where

$$n(T) = 4.94 \cdot \frac{W_g(T)}{U_g(T)} \qquad (17)$$

The plotting of the $U_g(T)$ and $W_g(T)$ functions entering into equation (17) is made according to the method discussed in a work [4].

According to (16) and (17), equation (1) can be written in such a generalized form:

$$\sigma_m(T) = \mathcal{æ}(T) \cdot E(T) \qquad (18)$$

Thus, on the basis of the experimental results expressing the temperature functions $\alpha(T)$, $C(T)$ and $E(T)$, it becomes possible to calculate the theoretical strength of glass within the total temperature interval (from absolute zero to a point close to the softening temperature). This calculation involves short-duration testing when the non-thermal mechanism of fracture plays a principal part.

As an example for illustrating the possibilities offered by the proposed calculating procedure, let us take the results of the analysis of the acoustic-dilatometric measurements on an alkali-silicate sheet glass.

A dilatometric curve for this glass is presented in Figure 1. Symbols T_1, T_g, T_2, and T_f designate the lower temperature boundary of the glass transition range, the glass transition temperature, the upper temperature boundary of the glass transition range and the temperature of softening, respectively.

Figure 2 shows the temperature dependence of the group interaction energy $U_g(T)$ (Curve 1), the temperature dependence of the shear deformation energy $W_g(T)$ (Curve 2), the sloped line 3 indicating to the temperature dependence of the activation energy $U(T)$ according to Bartenev and Lukjanov [5]. The difference between the ordinates on Curves 3 and 1 is determined by the energy contribution from the structural stresses initiated at the boundaries of interacting groups, which at a temperature below the A point become frozen. Their energy contribution W at these temperatures (down to absolute zero temperature) is assumed to be constant.

In Figure 3, Curve 1 shows a temperature dependence of the theoretical strength of the sheet glass, plotted according to the calculated data. It indicates that the theoretical strength has a maximum at the temperature of absolute zero. With raising the temperature, its value decreases passing near temperature T_1 through a small maximum.

Curve 2 is the result of a graphical averaging of experimental values of the real strength for similar glasses, which were obtained by Vonnegut [6], Jones and Turner [7]. The latest results for glass rods are characterized by higher strengths than those by Vonnegut for sheet glasses. The data obtained by Jones and Turner are multiplied by a factor of 0.28 to determine the unique temperature dependence of strength.

The comparison of Curve 1 and 2 shows that the temperature dependences of theoretical and real strengths are in agreement.

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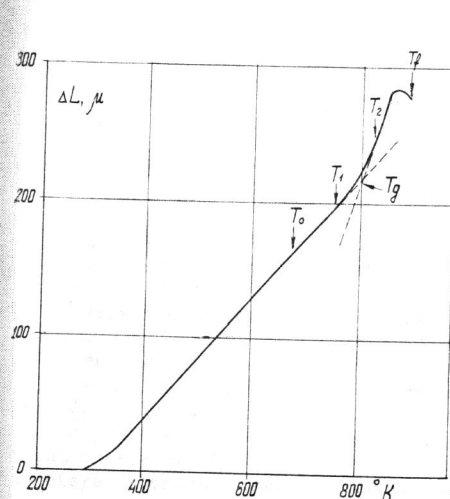7. G. Jones, W. Turner, J. Soc. Glass. Techn. 26, 35, 1942.

Figure 1.
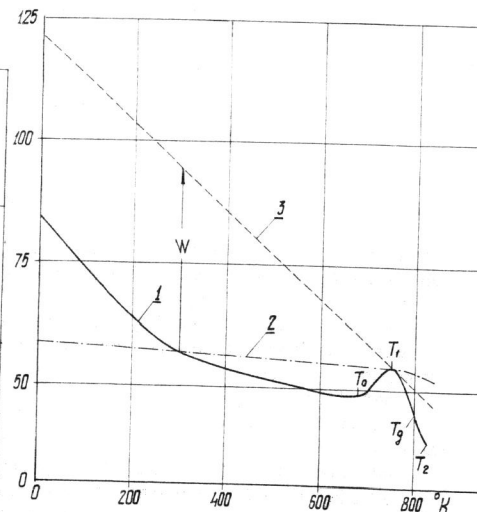Curve of thermal linear expansion of sheet glass.

Figure 2.
Dependence of energies: 1 - $U_g(T)$; 2 - $W_g(T)$ and 3 - $U(T)$ on absolute temperature.
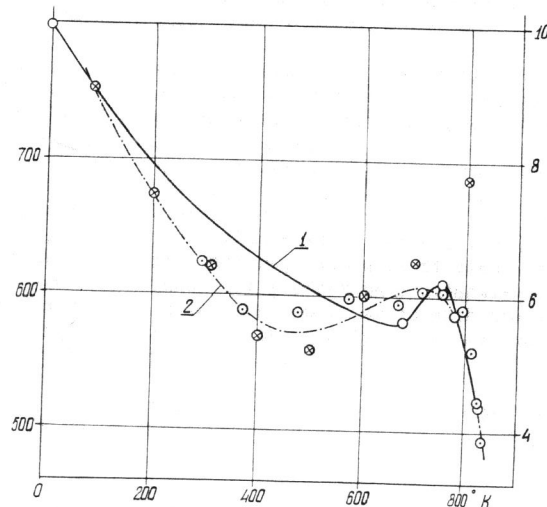
Figure 3. Type of temperature dependences: 1 - theoretical $\sigma_m(T)$ and 2 - real strength of glass.