BII-1 Micro and Macro Crack Formation

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ABSTRACT

The formation of cleavage microcracks with a length of the order of one grain diameter is considered to be the initial step in fracture. It is assumed that the stress concentration required for cleavage is supplied by thick slip or twin bands, and the critical width of these yield bands is calculated. For example, in iron with a grain radius of 10^{-2}cm, the critical slip band width is 2×10^{-5}cm, and this value is compatible with observations in the vicinity of microcracks. The second stage of crack formation involves the semi-continuous propagation of microcracks to form unstable macroscopic cracks. We postulate that plane strain fractures occur under conditions where thick slip bands are formed in the yielded region in front of an advancing crack. Work is required to extend the initial microcracks, and this incremental work is used to calculate the crack extension force, G_c, which is required in linear fracture mechanics. In the case of iron, the microcrack extension force, γ, is calculated to be 5×10^3dynes/cm, and the minimum value of the crack extension force under plane strain, G_{Ic}(min), is calculated to be 2.5×10^6dynes/cm. This approach emphasizes the three conditions required for fracture: 1) a combination of stress and yield band width sufficient to cause local cleavage; 2) sufficient mechanical energy in the system to propagate the crack; 3) the development of a critical value of the initiation stress in order to continue crack extension.

These concepts may be used to estimate the plane strain transition and the nominal stress for fracture in plates. The nominal plate fracture stress, σ_n, is estimated from an elastic-plastic stress analysis to be $\sigma_c/4$, where σ_c is the value of the yield stress at the tensile transition temperature. The tensile transition is chosen as the point at which the yield and fracture stress are about equal, and the plate transition temperature corresponds to the temperature, T_c, at which the yield stress has a value of $\sigma_c/4$.

We also estimate that crack arrest in steel plates corresponds to an energy absorption, G_c(arrest) = 2.5×10^4 t/d, where G_c (arrest) is the crack extension force at the transition between plane strain and plane stress (dynes/cm), t is the plate thickness (cm), and d is the grain radius (cm). Reasonably good correlations for our calculated values of G_{Ic} and G_c(arrest) are obtained with the available data. We also use tensile transition data to estimate a plate transition temperature and a critical tensile stress for crack propagation. These are combined with a suggested minimum value of the crack extension force, G_c(arrest), to provide the basis for a fracture-safe design criterion.

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1. Introduction

The fracture process in metals has been considered in detail on two rather different dimensional levels. The theoretical strength of a crystal is estimated on an atomic basis with the assumptions that the preceding deformation is elastic and that a force equivalent to the surface tension is all that is necessary to extend an atomically sharp crack[1]. It is recognized, however, that plastic flow precedes fracture, and the mechanism of plastic deformation is considered in terms of elementary steps of the order of 10^{-8}cm. A sequence of these elementary dislocation displacements has been used to account for the formation of cleavage microcracks with the dimension of about a grain diameter, i.e. about 10^{-3}cm[2,3]. Cleavage microcracks of this size have been observed frequently, but many of these cracks go no farther[4,5]. Tensile specimens with as many as two percent cracked grains have been observed to remain intact at the yield stress[5], and it is apparent that the formation of a microcrack does not immediately produce failure.

On the other hand, failures can occur under conditions where the structural member does not exhibit general yielding, and the plastic deformation is confined to a narrow region in the vicinity of a notch or a propagating crack. Failures of this type are considered to occur under plane strain conditions and are frequently treated in terms of linear fracture mechanics on a macroscopic continuum basis[6,7,8,9]. These macroscopic concepts envision a plastic zone which must remain smaller than the plate thickness, in order to maintain plane strain conditions. The macroscopic crack is extended by the cracking of this plastic zone and the maintenance of this restricted plastic zone ahead of the propagating crack. The plastic zone is, in effect, a suppressed extension of the actual crack and contains the very sharp crack defect required for the fracture of the material.

There is an evident discontinuity in these approaches to fracture. The microscopic viewpoint does not indicate how failure can result from a microcrack. On the other hand, the macroscopic

theory does not show how the plastic zone in front of a large crack is converted into a thin cleavage crack. Furthermore, there is little guidance on when plane strain fracture, rather than general yielding should be expected. The initiating notch, or the travelling crack, cannot part the material in front of it without some intermediate steps which probably involve the formation of microcracks and the extension of these small cracks to form an unstable defect.

In this paper we attempt to make the interconnection between the microscopic and the macroscopic viewpoints. We consider how a microcrack is formed and how it can continue to propagate to form an unstable defect of macroscopic size. The optimum set of conditions for the crack extension corresponds to plane strain fracture, and we thus attempt to define the crystallographic requirements for this mode of fracture. We have found it helpful to reconsider the process of microcrack formation. The dislocation approaches must be modified for extension into the macroscopic region, and we have returned to an older hypothesis that a coarse slip band or a mechanical twin provides the mechanism by which the elastic stress field is concentrated into a tension stress large enough to cause local cleavage[10]. We use dislocation methods to calculate this stress concentration and indicate the critical band width required to produce a microcrack. The extension force for a microcrack in iron is estimated to be of the order of 5×10^3dynes/cm. This development is presented in the next section.

In the third section we consider the microscopic sequence which can produce an unstable macroscopic crack. The minimum crack extension force corresponds to the formation of a crack with the smallest possible plastic zone in front of it, and we postulate that this also corresponds to a mode of deformation wherein coarse slip bands or twins are formed in the plastic zone. We estimate this macroscopic crack extension force to be of the order of 2.5×10^6dynes/cm for iron, and associate this with the minimum values of the parameter, G_{Ic}, used in linear fracture mechanics.

It is interesting to note that these approaches reach back to the Griffith formula for a completely brittle isotropic material. The modifications arise because metals undergo plastic flow which is discontinuous on a microscopic scale. Furthermore, we consider that all fracture in metals is crystallographic and occurs by cleavage on well-defined lattice planes. The descriptions brittle and ductile refer only to the amount of plastic flow which has preceded the cleavage. If the plastic flow occurs by the formation of a few coarse slip or twin bands, local cleavage occurs at the yield stress, and individual grains part after the formation of a few slip or twin bands along the fracture path. We consider such a fracture brittle. If the flow occurs by the formation of many narrow slip bands, the stress must be raised locally to the ultimate stress, or time must be allowed for sufficient thickening of the slip bands by creep. This results in much more flow than in the previous case. The cleavage path is now much more tortuous and the crack extension force is much greater, and we label such a fracture ductile. However, the same cleavage process is involved, and we postulate that shearing or tearing fractures still occur by cleavage on a fine scale. The distinction is further confused in some materials which exhibit identical slip and cleavage planes. This feature is observed in several non-ferrous materials(11), and perhaps in martensitic steels(12), but we consider the mechanism of fracture to be the same.

2. Microcrack Formation

Let us consider a polycrystalline material and a particular grain which is subjected to a shear stress sufficient to cause yielding. The yielding occurs by slip or twin formation; in bcc metals and in some fcc and hcp alloys the slip is discontinuous. The discontinuity occurs because the glide or twin shear does not occur on uniformly spaced planes within a given grain but occurs in packets of planes. The deformation within such a packet is of the order of 10^{-2}, whereas the surrounding material exhibits micro-

strains of the order of 10^{-4}. Twin formation is favored over slip in iron at lower temperatures or at higher strain rates, but the relative strain conditions are quite comparable. The widths of the slip and twin bands increase as the temperature is lowered, probably because of the higher yield stresses at low temperatures. The width of a slip or twin band will also increase during creep, and some relaxation of band width has been observed on the removal of the stress. It is also evident that the width of the largest slip band or twin is probably related to the grain size, but this geometric relationship has not been determined.

Many low carbon ferritic materials exhibit the characteristic form of the tensile properties as a function of temperature shown in Figure 1(4). We focus our attention on the two low temperature regions. In the lowest temperature region, the yield and fracture strengths are about equivalent. However, the discontinuous yielding at these temperatures occurs primarily by twinning, and fracture occurs by cleavage along (100) planes with little ductility. We label this as the brittle-twin temperature region. At somewhat higher temperatures, but still within the brittle cleavage range, yielding is initiated primarily by slip band formation, although some twins are also formed as yielding proceeds. We label this the brittle-slip region. As indicated in Figure 1, cleavage microcracks are observed in these brittle regions. The number of unsuccessful microcracks increases as the yield strength increases in the brittle region, and the apparent maximum is observed only because fracture occurs before the discontinuous yield strain is completed. At the lowest test temperature, it appears that the first microcrack may propagate to failure.

We now assume that a slip band or mechanical twin, we shall call these yield bands, can provide sufficient stress concentration to raise the stress locally to the theoretical fracture stress. Thus, as indicated schematically in Figure 2, we apply a tensile stress, σ_y, sufficient to initiate yielding in a polycrystalline material with an average grain diameter, 2d; the maximum shear stress will be of the order, $\tau_y = \sigma_y/2$. We consider a grain with a favorably oriented slip system in which a slip band of width, p, forms. For convenience, we assume that slip is along (110) planes and that the maximum shear stress is at 45 degrees to the tensile axis. We consider that the elastic stress is unloaded locally by the formation of the slip band. If the slip band cannot propagate into the next grain at the same stress, a shear stress concentration occurs at the boundary; a tensile stress concentration will also be produced, and a cleavage crack normal to the tensile axis results if the stress is large enough. Thus, the elastic stress is relieved by local shear, and this is relieved by a microcrack if the shear is stopped. Some of the mechanisms for stopping the shear at the boundary are: 1) an unfavorable orientation in the neighboring grains; 2) the presence of carbides or other hard particles, and 3) the presence of other phases. The slip band need not be blocked completely to produce this stress concentration, and any hindrance to free slip or twinning will produce a tensile stress concentration.

In Figure 2 we show schematically a yield band of critical width, p_c, which cannot propagate into grain B. The shear displacement within the band (1234) is converted into a crack with a normal displacement, u, and a shear displacement, v, which protrudes into grain C. A narrower slip band is formed at point 6 to relieve the shear displacement. Thus, points 5 and 6, as well as 4 and 7, which were coincident before shear, are now separated. A microcrack with this approximate configuration is shown in Figure 3. This microcrack was formed within a Lüders band in a mild steel (0.22 C, 0.36 Mn) at -196°C(5). The microcracks were always observed well within the Lüders band and were not observed at the interface with the unyielded region. A similar situation occurs when a massive twin is stopped by another twin or by a grain boundary. Microcracks of this type are shown in Figure 4 which were observed in single crystals of iron tested at -196°C(13). It thus appears that microcracks are formed to relieve the tensile displacements which can occur when a massive slip or twin band is blocked.

It is evident that the stress concentration factor resulting from a yield band is proportional to its thickness, for the greater the thickness, the greater the shear associated with the band and the greater the tensile displacement, u, at the barrier. We estimate the stress concentration factor, q, by analogy with dislocation theory. The passage of one dislocation results in a unit displacement, b; the stress concentration for a number of dislocations pushing against a barrier is given by the number of dislocations. The equivalent picture here is a stack of planes being sheared away from a boundary. We regard this packet of sheared planes as a macrodislocation, and the stress concentration factor, q, becomes the number of planes in the slip band. The width of the slip band becomes p = qb, where b is now the spacing between slip planes. We do not insist that a slip band consist of q planes, each with a displacement, b; we assume only that the total displacement of the slip band is given by the quantity, qb. We follow the analogy with dislocation theory further by assuming that the shear stress at the head of a slip band, τ_q, is given by,

$$\tau_q = q(\tau_y - \tau_i) \qquad (1)$$

where τ_i is the stress required to drive a slipping plane against the resistance of lattice friction, dispersed impurity atoms, precipitate zones, particles and lattice defects. This friction stress resists the initial shear motion of the lattice planes and is thus not a part of the stress concentration at the end of the slip band.

We define a slip band of critical thickness, p_c, where

$$p_c = q_c b \qquad (2)$$

and q_c corresponds to the critical stress concentration which is large enough to raise the tension stress at the head of the band to the theoretical stress. If we assume that $\tau = \sigma/2$ along the shear plane and neglect the hydrostatic component of the tension force, we may write

$$\sigma_f = q_c(\sigma_y - \sigma_i) = \left(\frac{2E\gamma_o}{b} \right)^{1/2} \tag{3}$$

where E is Young's modulus and γ_o is the true surface energy. We have used the Orowan estimate of the theoretical strength, σ_f, but the exact value of the lattice strength is not critical to the argument. We can neglect the hydrostatic component for materials of low yield strength, since this term is small in comparison with the shear term. However, this term should not be neglected where large compression stresses are involved or in the case of high strength steels where the shear and the hydrostatic terms may be of comparable size. Equation (3) should then be modified in these cases to include a term of the order of $\sigma_y/3$ in addition to the shear concentration term. The absolute value of the term $(\sigma_y - \sigma_i)$ should be used, since the direction of the slip is of no consequence.

Neglecting the hydrostatic term, equation (3) becomes

$$q_c = \frac{1}{(\sigma_y - \sigma_i)} \left(\frac{2E\gamma_o}{b} \right)^{1/2} \tag{4}$$

The frictional term σ_i may be evaluated experimentally in a number of ways. Some authors have assumed that σ_i corresponds to the stress at which the first plastic strain is observed, and both σ_y and σ_i can thus be determined in a single tension test. The value of the stress at which a permanent set of 10^{-6} is observed is indicated as σ_i in Figure 1. However, in many bcc materials it has been shown that

$$\sigma_y = \sigma_i + k_y d^{-1/2} \tag{5}$$

where 2d is the grain diameter, and k_y is the grain size factor. The appropriate values of k_y for slip and twinning must be used, and a typical plot of equation (5) is shown in Figure 5[14]. It should be noted that k_y, and thus $(\sigma_y - \sigma_i)$ is independent of temperature in either the slip or the twinning region. The entire temperature dependence of the yield stress appears to reside in the frictional term; this is also evident in

Figure 1, where σ_y and σ_i vary in the same way with temperature. Using equations (4) and (5),

$$q_c = \frac{d^{1/2}}{k_y} \left(\frac{2E\gamma_o}{b} \right)^{1/2} \tag{6}$$

and

$$p_c = \frac{bd^{1/2}}{k_y} \left(\frac{2E\gamma_o}{b} \right)^{1/2} \tag{6a}$$

Equations (6) provide an estimate of the critical stress concentration, and the critical band width required to start a cleavage crack. The value of q_c appears to be almost invariant with temperature, even though the yield stress may vary greatly with temperature. However, the experimental values of the slip band width, p, and the corresponding stress concentration, q, are treated here as independent variables, although they may be dependent on temperature and applied stress. At room temperature it appears that the initial slip bands are too narrow at the yield stress to produce microcracks (Figure 1), and the stress must be raised to the ultimate before local stress concentration becomes large enough. The slip band may thicken as the stress is raised, but the major effects probably come from the increase in stress and the devious path for the final fracture because of the multiple slip at large deformation. However, the critical combination of band width and stress occurs within the slip-cleavage and twin-cleavage regions at lower temperatures, and we shall use these data to estimate the value of p_c.

Values of k_y have been measured for a number of irons and steels[14,15], and within the temperature range wherein yielding is initiated by slip a value of $k_y = 5 \times 10^7$ dynes/cm$^{3/2}$ is very close to most determinations*. Using values of $E = 2 \times 10^{12}$ dynes/cm^2, $b = 2.5 \times 10^{-8}$ cm, $\gamma_o = 10^3$ ergs/cm^2, the critical band width may be calculated for any grain radius d, and these values are listed in Table 1. For example, $d = 10^{-2}$ cm (about ASTM 3), $q_c = 0.8 \times 10^3$ and $p_c = 2 \times 10^{-5}$ cm. It should be emphasized that p_c is the smallest slip band width which can produce a cleavage microcrack. On the

*Care must be exercised in taking values of k_y from the literature. Some authors define d as the grain diameter, and others use 2d as the grain diameter. We have chosen the latter convention.

other hand, if we extend this calculation to a martensitic steel with a grain size of 10^{-4}cm, then $q_c = 80$ and $p_c = 2 \times 10^{-6}$cm. Thus, a very narrow slip band will produce microcracks in martensitic steels, but the corresponding yield stress will be quite high because of the small grain size.

Several observations of slip band widths, p, in the vicinity of microcracks are listed in Table 2 and compared with the corresponding calculated values of p_c. Unfortunately, only a few values of slip band widths are available, and these have been taken from a number of optical and electron micrographs. The metallographic observations overestimate the band width, because the angle of observation is seldom normal to the slip band and because of shadowing effects. The data on polycrystalline iron are taken from the work of McMahon[16], who observed that cleavage microcracks were associated with cracked carbides at the grain boundaries. These boundary carbides cracked during local yielding, however, and the cracks may be regarded as being a consequence of the blockage of slip and twin bands by the carbides. The slip band widths in the two steels were estimated from optical photomicrographs in the vicinity of microcracks and the listed widths are undoubtedly overestimates. Although the observed values of slip band width, p, are larger than the critical values, p_c, by factors of two to ten, they lie within a reasonable range of the calculated critical widths.

Observations of k_y in the twin-cleavage region[14] indicate that k_y for twinning is of the order $k_y = 20 \times 10^7 \text{dynes/cm}^{-3/2}$. Values of p_c(twin) are thus about 1/4 the value of p_c(slip), but otherwise the picture is similar. Figure 1 indicates that σ_y(twin) is almost independent of temperature, whereas σ_y(slip) is strongly temperature dependent. A slip band is thus more effective than a twin band in promoting cleavage, as shown in Table 1, but the competition between slip and twinning is determined by other factors.

It would appear from Figure 5 that twin formation is favored in iron at very large grain sizes at low test temperatures. Tension tests on single crystals bear this out and cleavage in single crystals appears to be initiated by twin formation. Recent observations have shown that cleavage in single crystals occurs when a massive twin intersects another thick twin or a parasite grain boundary[13]. Cleavage by slip intersections was not observed in pure iron, although it has been observed in iron-silicon crystals[17]. Values of observed twin width, p, are listed in Table 2 for single crystals which fractured by cleavage below the transition temperature. We have used the value of k_y(twin) obtained from measurements on polycrystals and have assumed that the thickness of the crystal corresponds to the grain size. These approximations give reasonable values for the quantities $(\sigma_y - \sigma_i)$. The observed values, p, are larger than the calculated values, p_c, by a factor of 10. However, it should be noted that cleavage occurs almost immediately on the blockage of a thick twin. Furthermore, the resultant cleavage in a single crystal is not a microcrack, but a macroscopic fracture of the entire specimen.

A. Microcrack Extension Force

It is useful to consider the energy required for the formation of a microcrack. We may estimate this by considering a macrodislocation with q_c dislocations which is unloaded into a microcrack. The energy balance becomes,

$$q_c(\tau_y - \tau_i)b = \gamma \quad (7)$$

where γ is the energy of formation per unit area of crack, or the microcrack extension force. The work term, or microcrack extension force, γ, now includes the plastic flow required to produce the stress concentration and is much larger than the true surface energy, γ_o. Using equation (4), this becomes

$$\gamma = \left(\frac{E\gamma_o b}{2}\right)^{1/2} \quad (8)$$

Using the values, $E = 2 \times 10^{12}$ dynes/cm^2, $b = 2.5 \times 10^{-8}$cm, $\gamma_o = 10^3$ergs/cm^2 for iron, the microcrack extension force becomes, $\gamma = 5000$ ergs/cm^2.

The crack formation energy may be estimated in another way. Since local yielding must occur in the grain undergoing cleavage, we may estimate the work term by calculating the energy required to yield one grain, recognizing that only the non-frictional portion of the stress is transferred into the crack formation. For a grain size of 10^{-3}cm, $(\tau_y-\tau_i)$ is of the order of 6.9×10^8dynes/cm^2 (10^4psi). The work required to yield a region one grain deep is, $W = (\tau_y-\tau_i)\varepsilon_y t$, where ε_y is the local yield strain and t is the thickness of the cold worked region. If we assume $\varepsilon_y = 10^{-2}$ and $t = 10^{-3}$cm, $\gamma = 7000$ ergs/cm^2, which is close to the previous estimate.

We thus conclude that the initiation of cleavage by the formation of a microcrack requires about 5000 ergs/cm^2 under the most favorable conditions. This estimate is close to those obtained by other investigators using somewhat different mechanisms of crack formation. It is lower than some estimates, because we have assumed that only the non-frictional deformation is effective in crack formation. The picture used here also differs in requiring a slip or twin band of some minimum thickness, p_c, at a given slipping stress, $(\tau_y-\tau_i)$, to create the proper combination of events for crack formation. The energy criterion is thus a necessary but not sufficient condition for microcrack formation.

There are a number of observations in other systems which lend some support to the assumption that microcracks are initiated by yield bands. Stubbington[18] has shown by transmission electron microscopy that persistent slip bands, up to 500A, form during reversed glide in an aged Al -7.5 Zn -2.5 Mg alloy, and that fatigue microcracks are associated with these thick slip bands. Price and Kelly[11] have shown that coarse slip bands are formed in single crystals of the aged alloys, Al -3.7 Cu, Al -20 Ag, and Al -13 Zn by shear on a (111) plane in a [110] direction, followed by crack propagation. The coarse slip bands first appeared at a constant resolved shear stress; these were followed by cracks at the foot of the slip bands when the shear stress was increased. The cleavage occurred on the slip planes. Price and Kelly observed that the step heights of the individual slip lines (which correspond to the bands discussed here) varied between 500 and 15,000A, and their electron micrographs indicate that the widths of the slip bands are of the same order. Since the step height is an indication of the local shear displacement, we consider this as some corroboration for the assumption that the stress concentration is proportional to the width of the band. We may estimate the value of p_c for these aged aluminum alloys from equation (4). If we assume that the difference in yield stress as the temperature is lowered arises mainly from the friction term, τ_i, the quantity, $(\tau_y-\tau_i)$ is about 2 Kg/mm^2 (2×10^8dynes/cm^2) in these aged alloys. Taking $E = 7 \times 10^{11}$dynes/cm^2, $\gamma_o = 10^3$ergs/cm^2 and $b = 3 \times 10^{-8}$cm, we calculate $p_c = 7500$A, which is within the range observed by Price and Kelly.

Rather similar observations have been made by Argon and Orowan[19] on crack nucleation in MgO single crystals. They observed cracks resulting from the blockage of mutually perpendicular slip bands. The blockage occurs because the slip systems are quite restricted and it is difficult for one slip band to penetrate another. The resultant geometrical incompatibilities result in stress concentrations which are relieved by the formation of microcracks. The overall picture appears to be the same as that described in the metallic crystals.

3. Formation of Macroscopic Cracks

The formation of a microcrack with a length of one grain diameter is not a sufficient condition for the failure of the specimen. It is necessary that a microcrack continue to propagate through surrounding grains until the growing crack either parts the specimen or meets other expanding cracks which have started from other sources. Let us first consider an unnotched tensile bar tested in either the brittle-slip or the brittle-twin region indicated in Figure 1. The entire gage section is brought to the yield stress, and we have postulated that yield bands of more than critical width are produced under these conditions. We assume that microcracks form in every favorably oriented grain, i.e. in grains with a slip or twin band oriented in the maximum shear direction and with a cleavage plane normal to the tensile axis. This is the crack initiation step, and we have calculated the microcrack extension force, γ, in the previous section. We now investigate the requirements for extending this crack into the surrounding grains with less favorable orientations, and assume that the extending crack becomes unstable when it meets other similar cracks. The case of a plate with a crack starting at a notch is quite similar. A region at the root of the notch reaches the yield stress, and we postulate that microcracks will form in this yielded region if the yield bands are of more than critical width. The crack will travel to the notch from the nearest microcracks, the yield zone will move forward, new microcracks will form and the crack will be extended in a somewhat discontinuous fashion as the expanding microcracks travel back toward the main crack. The microscopic mechanism is the same in both cases and involves the joining of expanding microcracks. The macroscopic behavior depends on the size of the yield zone.

We now consider whether the macroscopic crack will propagate under plane strain or plane stress conditions. Under plane strain conditions, the strain in a plane normal to the plane of the crack is negligible and the specimen does not exhibit necking or large overall deformation at failure. These failures are usually described as brittle even though there is local yielding in the vicinity of the crack. In plane stress failures, the stress in a plane normal to the fracture plane is negligible, and considerable overall deformation is observed prior to fracture. Plane strain

requires that the yielded zone in front of the crack remain smaller than the plate thickness. We assume that the yielded zone remains small when the widths of the yield bands are greater than the critical value and microcracks form. Thus, all of the favorably oriented grains within the yield zone cleave, and we require that these microcracks expand at about the same stress. The smallest yield zone is thus one grain, and this occurs when the yield band in every grain is wide enough to cause cleavage.

Plane stress fractures occur when the initial yield bands are narrower than p_c. It is then necessary to raise the stress, to allow creep, or to introduce many stress cycles in order to thicken the bands sufficiently to produce fracture. This requires additional deformation energy and allows the yielding to spread over a larger volume in the specimen. Final fracture in a notched specimen will still occur in the notch region because of the stress concentration, but the yielded region can be quite large. In our view, therefore, the propagation of a cleavage microcrack for a distance equivalent to the average distance between microcracks is required to produce a sustained travelling crack.

The minimum crack extension force, or crack energy, is required under plane strain conditions where the initial yield bands are thicker than the critical value, p_c, and the resulting yielded region is small. The maximum crack energy is required when the slip or twin bands are very narrow and the entire plate must be deformed to a strain approximately equivalent to the ultimate strain before fracture can propagate. We shall estimate these minimum and maximum energies in this section and attempt to relate these calculations to the crack extension force, G_c, introduced in the treatment of macroscopic fracture by the method of linear fracture mechanics.

A. Linear Fracture Mechanics

Let us consider an edge crack of length c, or a similar included crack of length 2c, in a much larger plate. The stress concentration in front of such a defect has been worked out by both elastic and plastic methods[20,21]; the method of linear fracture mechanics considers the influence of a plastic zone at the head of a crack on the force required to propagate the fracture. The normal tensile stress close to the crack tip is written as

$$\sigma = K(2\pi r)^{-1/2} \tag{9}$$

where r is the distance from the tip of the crack on the crack plane, and K is a stress intensity factor. The factor K is a function of the geometry and of the plastic behavior of the material, and it is determined experimentally by methods which have been developed by Irwin and coworkers for a variety of specimen shapes[6,7,8,22]. The measured values of the stress intensity factor, K_c, are considered to be characteristic of the material, and the minimum value corresponding to crack propagation under plane strain conditions, K_{Ic}, is assumed to be a material constant independent of the thickness. The stress intensity factor is relative to the crack extension force, and

$$K_c^2 = EG_c \text{ (plane stress)}$$

$$K_c^2 = EG_c/(1-\nu^2) \text{ (plane strain)} \tag{10}$$

where E is Young's modulus, ν is Poisson's ratio and G_c is the crack extension force. The quantity G_c plays the same role as the surface energy γ_o in the Griffith equation or the microcrack extension force, γ, used in equation (7) in the previous section. The fracture strength, σ_f, becomes

$$\sigma_f = \left(\frac{2EG_c}{c}\right)^{1/2} \tag{11}$$

where c is now the size of the critical flaw which will permit continuous crack growth.

The radius of the plastic zone at the tip of the crack has been calculated by a number of procedures for the condition that the normal stress is small relative to the yield stress, σ_y. The calculation of McClintock and Hult[20] for plane stress conditions gives this plastic zone radius, r_y, as

$$r_y = EG_c/(2\pi\sigma_y^2) \tag{12}$$

and the other calculations give substantially the same results. Another parameter which has been calculated by the method of linear fracture mechanics is the crack opening extension, δ. This has been estimated by Wells[9] as

$$\delta = \frac{4G_c}{\pi\sigma_y} \tag{13}$$

where δ is evaluated at the value of σ where the crack extends. This provides a possible experimental method for the evaluation of the stress intensity factor, which may now be written as

$$K_c^2 = \frac{\pi}{4}\,\sigma_y E\delta \tag{14}$$

This method has been tested by Wells who found rather good correlations in thick plates between values of K measured from crack opening displacements and values obtained by the usual techniques.

A particular situation of interest occurs at the transition between plane strain (brittle) and plane stress (ductile) behavior. The transition is usually obtained at a transition temperature which depends on the test conditions as well as on the material. The transition is assumed to occur when the plastic zone radius, r_y, approaches the plate thickness, t, and it is evident that both the crack extension force and the crack opening displacement depend on the specimen thickness.

The method of linear fracture mechanics has been described in a series of ASTM papers[22] and determinations of K_c and G_c have been made for various materials. The values of G_c, or K_c for plane strain fracture conditions are of particular interest since, at a given plate thickness, the higher the value of G_c, the more difficult it becomes to achieve brittle fracture. Plane strain fractures have been induced in high strength materials by introducing a notch, extending the notch by forming a fatigue crack, and then testing the specimen in tension as a function of temperature. As the temperature is lowered, a transition to plane strain is observed. Plane strain conditions are more difficult to achieve in mild steels because of the greater ductility. Some investigators have used thick plate tests(9) and double tension tests(23,24). A recent study has used nitrided notch bend and tension specimens(25), and these values of the minimum value of the crack extension force, G_{Ic}, and the transition temperature T_c, agree quite well with values obtained by the other methods. A few representative values of G_{Ic} and T_c are listed in Table 3 for several mild steels. The values of G_{Ic} appear to fall in the range 3-15 x 10^6dynes/cm, in marked contrast to the value $\gamma = 5 \times 10^3$dynes/cm calculated for the microcrack.

B. Microcrack Propagation

Let us consider the case where a local yielded zone of radius r_y has been formed at the tip of a crack or a sharp notch and the widths of the yield bands within this zone exceed the critical value in every suitably oriented grain in the region. Every favorably oriented grain in the yield zone cleaves with a microcrack extension force γ. We now define the probability, w, of finding a favorably oriented grain. This can be estimated experimentally from the measurements of microcrack frequency in a number of irons and steels which have been homogenized to remove preferred orientation. It is evident that a non-random grain orientation could lead to the formation of relatively large cracks in a string of similarly oriented grains, but we shall confine ourselves to the case of random grain orientation. Hahn et al(4) and McMahon(16) have shown that microcracks appear on yielding, and that the number of microcracks increases as the yield strain increases. Figure 6 shows the frequency of microcracks in iron(16). The number of microcracks increases with increasing elongation at temperatures below -140°C; above this temperature the number of microcracks decreases because of the decrease in yield stress. Since we are concerned with the number of microcracks produced on the formation of the initial yield bands, we consider only the number formed at the onset of yielding and estimate a value of $w = 10^{-4}$. These observations were made on one plane, the surface, and since we are concerned with the number of microcracks in the

plane of the extending crack, it appears that the quantity (1/w) is a good estimate of the number of grains which must be traversed in front of the crack tip before a new microcrack is reached.

If we now consider the microcrack opening for each initial microcrack, it is apparent from the geometry shown in Figure 2 that the crack opening, u, is approximately equal to the horizontal component of the shear displacement, v. We may estimate the total shear displacement as $(q_c b)$ and the displacement components thus become,

$$u = v = p_c/\sqrt{2} \tag{15}$$

It is unlikely that the neighboring grains are suitably oriented for cleavage, and we may estimate the linear distance to the next microcracked grain as about $w^{-1/2}$ grains. However, the next grain, C, in Figure 2 will be required to accommodate the shear and tensile displacements, and this will result in a yield band. The accommodation yield band will be smaller than the initial yield band, p_c, and we estimate that its width will be of the order of v; i.e. $p = p_c/\sqrt{2}$. Other accommodation bands will also be of the same width.

In order to propagate the microcrack from grain A to grain C, it will be necessary to increase the width of the slip band in grain C by a factor of $1/\sqrt{2}$. This can be done by continuing the yield strain and thus supplying an additional crack extension energy. We define the incremental microcrack extension force, $\Delta\gamma$, as

$$\Delta\gamma = \left(1 - \frac{p}{p_c}\right)\gamma = 0.4\gamma \tag{15a}$$

In the case of iron, this additional strain energy is of the order of 2000 ergs/cm^2 for each grain which must be cracked in this progressive fashion. We now assume that the growing crack becomes unstable when it has propagated halfway to the next microcrack in the yield zone. For a circular crack, the critical radius r_c thus becomes

$$r_c = \frac{d}{2} w^{-1/2} \tag{16}$$

The number of grains with subcritical yield bands within this critical radius, n, becomes

$$n = 1/4w \tag{16a}$$

If we neglect the energy required to crack the first grain, the minimum crack extension force G_{Ic} becomes the energy required to widen the slip bands in $1/(4w)$ grains. Since 0.4γ is required to widen the slip bands in each grain, the energy balance for crack extension becomes,

$$2G_{Ic} = 0.1\left(\frac{\gamma}{w}\right) \tag{17}$$

Introducing the expression for γ (equation 8),

$$G_{Ic}(\text{min}) = \frac{0.05}{w}\left(\frac{E\gamma_o b}{2}\right)^{1/2} \tag{18}$$

Taking the same values used previously for iron and using $w = 10^{-4}$, $G_{Ic}(\text{min}) = 2.5 \times 10^6$ dynes/cm.

The quantity G_{Ic} calculated in equation (18) represents the minimum work required to extend a crack under the most favorable plane strain conditions. The plastic flow is confined to one grain diameter in front of the crack. Observations of plates with brittle cracks have shown that the yielded region is indeed confined to a few grain diameters in the vicinity of the fracture. We now define a transition at the point where the yielded zone approaches the thickness of the plate. In the treatment above, the yielded zone was only required to spread a distance, r_c. If we now define the value of the crack extension force $G_c(\text{arrest})$ at which the yield zone spreads to the plate thickness, then

$$G_c(\text{arrest}) = \frac{G_{Ic}(\text{min})\,t}{2r_c} = \frac{G_{Ic}(\text{min})\,tw^{1/2}}{d} \tag{19}$$

Thus, for a 2cm plate with $d = 10^{-3}$cm, $G_c(\text{arrest}) = 20G_{Ic}(\text{min})$. We consider the quantity $G_{Ic}(\text{min})$ as the smallest feasible plane stress value of the crack extension force, and we assume that $G_c(\text{arrest})$ thus corresponds to the value for crack arrest. The transition between $G_c(\text{arrest})$ and $G_{Ic}(\text{min})$ will not necessarily be sharp, and the experimental values of G_{Ic} will depend on the experimental definition of the transition. We suggest that the transition will usually be chosen under conditions where severe plastic flow will occur in a region at least one grain deep on each side of the fracture, i.e. for a depth of three grains, including the cracked grain, corresponding to an experimental value, $G_{Ic}(\text{exp}) = 3G_{Ic}(\text{min})$.

Several experimental determinations of the crack extension force are listed in Table 3. The values of G_{Ic} obtained by means

of the nitrided notch test appear to approach our calculated value quite closely, and this is probably due to the close approximation to plane strain fracture conditions. The experimental values of G_{Ic} determined from thick plate and double tension tests are usually closer to our estimate of $G_{Ic}(\text{exp})$, and in the case of mild steels these values are in the neighborhood of 10^7dynes/cm. The value of $G_c(\text{arrest})$ has been determined by Wells[9] in a 7.5 cm (3 in.) plate of a mild steel at a level of about 9×10^9dynes/cm. For a plate of this thickness, and $d = 10^{-3}$cm, we estimate $G_c(\text{arrest}) = 1.9 \times 10^8$ dynes/cm. Although this is lower than the measured value, it approaches the right order of magnitude, and it should be recognized that the crack extension force under plane stress conditions can rise much above our calculated value of $G_c(\text{arrest})$ at the transition.

The crack extension, δ, corresponding to the various values of G_{Ic} may be estimated in the following way. In accordance with Figure 2, the microcrack extension is given by u. The opening for an unstable macrocrack involves the opening of $1/w$ grains, and the corresponding crack opening is given by

$$\delta_c = u/w = 0.7\,p_c/w \tag{20}$$

Attempts to measure δ have been made by Wells, and he lists values of the order of 2×10^{-2}cm at the transition. From Table 2, at $d = 10^{-3}$cm, $p_c = 0.64 \times 10^{-5}$. Taking $w = 10^{-4}$, $\delta_c = 4.6 \times 10^{-2}$cm, and the calculated value is reasonably close to the measured crack openings.

C. The Transition Temperature

The temperature at which the transition from plane stress to plane strain occurs is difficult to estimate a priori in a given type of specimen. The effective slip stress, $(\tau_y - \tau_i)$ is independent of temperature and changes only when the deformation mode changes from discontinuous slip to twinning. Thus, only the width of the yield band will determine whether cleavage occurs and we cannot yet write the explicit conditions for the thickness of the band. However, we see in Figure 1 that the 10^{-6} yield stress is approximately equivalent to the values of σ_i obtained by extrapolating the yield stress to $d^{-1/2} = 0$. It is evident that σ_i varies with temperature in about the same way as the yield stress and the entire temperature dependence is thus associated with the initiation of flow.

We now suggest that thin slip bands form when the yield stress, σ_y, is small, and that thick slip bands are formed only when σ_y reaches a critical value, σ_c. This critical value may be estimated in the tension test at the temperature in the brittle slip region where the yield approaches the fracture stress. We postulate that the slip bands which form at this stress are thick enough to produce immediate fracture. In Figure 7, for example, this corresponds to a value of σ_c = 55,000 psi at about -90°C. The stress concentration factor for a crack which is about to become unstable and propagate through a plate may be estimated from a calculation of Hahn et al[21] in the following way. At the point of instability the crack has a critical radius, r_c, and the yield zone in front of the crack is confined to one grain diameter, 2d. The relative size of the crack to the yield zone, $r_c/2d = 1/4w^{1/2}$, and this has a value of 25 if we assume that $w = 10^{-4}$. We would like to find the stress concentration at the elastic-plastic interface, i.e. at $x = 1 + (2d/r_c)$ = 1.04 in terms of the crack length. This corresponds to a stress concentration of $\alpha \approx 4$, using the crack model of Hahn et al[21] and is not much different for a more recent model[26]. Thus, a value of the critical initiation stress in a tension test, σ_c = 55,000 psi becomes a nominal fracture stress, σ_n = 14,000 psi for a plate with a critical crack. From Figure 7, we see that σ_y has this value at a temperature of about 30°C. If we assume that this stress concentration corresponds to the maximum constraint in a thick plate, the plate transition temperature becomes T_c = 30°C, at a nominal fracture stress of 14,000 psi.

It is evident from Figure 7 that a nominal stress of 14,000 psi with a stress concentration factor of four will raise the stress at the notch above the yield at the transition temperature. It is thus not sufficient to produce yielding at the notch, but it is also necessary that the yield stress be increased sufficiently that σ_y reach a critical value. If the plastic constraint is greater than four, it is evident that the plate transition temperature will be higher and the critical nominal stress lower than our estimated

values. We thus use the measured values of the smooth bar tensile test data, which are sensitive to metallurgical variables and prior strain history to predict the behavior of a thick plate.

Table 4 lists values of the critical yield, σ_c, and tensile transition temperature selected from tensile data, along with the corresponding values of the plate transition temperature and the critical nominal fracture stress, σ_n. Although a direct comparison with the steels in Table 3 cannot be made, the transition temperatures are reasonably close, considering the nature of the assumptions. A comparison with the Charpy V-notch 15 ft-lb transition indicates a fortuitous agreement in view of the extrapolations required in the values for σ_y near room temperature.

4. Fracture-Safe Design Criterion

This paper has attempted to bridge the gap between microcracks of one grain diameter and plane strain brittle cracks in thick plates. One key assumption is that microcracking occurs only when the slip or twin bands are thick enough to raise the local tension stress to the theoretical fracture value. It is also obvious that slip bands or twins of this width are favored in systems which undergo discontinuous yielding, and that many fcc metals will not meet this condition except at the ultimate stress. Another key assumption is that the formation of a critical yield band requires a yield stress, σ_c, above a minimum value, and we have selected this minimum value from tensile measurements.

With these assumptions we have calculated a crack extension force for microcracks of $\gamma = 5 \times 10^3$ dynes/cm, and a minimum crack extension force for macrocracks of $G_{Ic}(\text{min}) = 2.5 \times 10^6$ dynes/cm for iron. We have then shown how a tensile transition in iron at -90°C and σ_y = 55,000 psi results in a plate transition temperature of 30°C at a nominal fracture stress of 14,000 psi.

These assumptions require refinement and modification, and additional attempts should be made to bridge the gap between microscopic and macroscopic behavior. However, we may use these concepts to consider materials and fracture criteria for design purposes. Combining our equations to express the condition for crack arrest, we obtain

$$G_c(\text{arrest}) = \frac{0.05}{w^{1/2}}\left(\frac{E\gamma_o b}{2}\right)^{1/2}\frac{t}{d} \quad (21)$$

$$= 2.5 \times 10^4 \ t/d \ (\text{for iron})$$

This provides an estimate of the crack arrest value for a given grain size, 2d, and plate thickness, t. If tensile measurements of σ_y as a function of temperature are available, we may estimate the critical conditions for brittle fracture in a plate from the relationship, $\sigma_n = \sigma_c/4$, and pick the corresponding critical temperature from the tensile curve. A fracture-safe design criterion might thus be summarized as follows:

1. The plates should be thick enough and the grain size small enough to develop a specified minimum crack arrest force. A value of G_c(arrest) = 20 x 10^6dynes/cm would probably be suitable for the structural steels considered here.
2. The nominal fracture stress is obtained from tensile data as a function of temperature. The critical stress, σ_c, is the value of σ_y for which the yield stress approaches the fracture stress. The maximum normal stress for the plate is then, $\sigma_n = \sigma_c/4$.
3. The plate transition temperature, T_c, is then $T(\sigma_c/4)$, i.e. the temperature at which σ_y has a value, $\sigma_c/4$.

The metallurgical and the design characteristics of the steel are thus described by three parameters, G_c(arrest), σ_n and T_c.

Let us use the data in Table 4 and apply these criteria to a 20 mm plate. For the steel containing 0.16 C, 1.30 Mn, and for a grain size of ASTM 7, G_c(arrest) = 26 x 10^6dynes/cm, T_c = -20°C, and σ_n = 37,000 psi. On the other hand, for the steel containing 0.22 C, 0.36 Mn, and for a grain size of ASTM 3.5, G_c(arrest) = 7.5 x 10^6dynes/cm, T_c = 100°C, and σ_n = 20,000 psi. The first steel is obviously superior for a fracture design.

These criteria should only be considered a first approximation and it is expected that the underlying assumptions will be refined with additional experience.

Acknowledgements

The author would like to acknowledge the support of the Ship Structure Committee and the assistance of the Ship Hull Research Committee of the National Academy of Sciences. This work has been drawn from research on the metallurgical aspects of brittle failure which has been sponsored by this group in the Department of Metallurgy at MIT over a period of years, and the author is grateful to his colleagues and associates for their contributions, and for many stimulating discussions of the fracture problem.

The author would also like to acknowledge the assistance of the Structural Mechanics Branch of the Office of Naval Research in making possible the presentation of this paper at the current International Conference on Fracture. The opportunity for interaction with the other research workers in this field is of great value in stimulating new avenues of approach to a difficult problem.

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Table 1. Critical slip and twin band size in iron

grain radius d(cm)	slip q_c	slip $p_c(10^{-4}$cm)	twin q_c	twin $p_c(10^{-4}$cm)
10^{-1}	2600	0.64	650	0.16
10^{-2}	800	0.20	200	0.05
10^{-3}	260	0.064	65	0.016
10^{-4}	80	0.020	20	0.0050
10^{-5}	26	0.0064	6.5	0.0016
10^{-6}	8	0.0020	2	0.0005

$$q_c = \frac{d^{1/2}}{k_y}\left(\frac{2E\gamma_o}{b}\right)^{1/2}$$

$$p_c = q_c b \;=\; 2 \times 10^{-4} d^{1/2} \text{ (slip)}$$

$$= 0.5 \times 10^{-4} d^{1/2} \text{ (twin)}$$

Table 2. Stress concentrations at slip and twin bands

material	grain diameter 2d(cm)	q_c	p_c $(10^{-4}$cm)	p (observed) $(10^{-4}$cm)
a. microcracks originating at twin bands				
α-iron single crystal (0.002 C)	0.1	440	0.11	1.0
α-iron polycrystal (0.035 C)	0.03	240	0.06	0.5
b. microcracks originating at slip bands				
α-iron polycrystal (0.035 C)	0.03	980	0.25	0.5
steel (.22 C, 0.36 Mn)	0.012 0.004	620 360	0.15 0.09	0.5 0.3
steel (.16 C, 1.3 Mn)	0.014 0.0034	670 330	0.17 0.08	0.5 0.3

Table 3. Crack extension force, G_{Ic}, in mild steels

	steel	experimental method	G_{Ic} (exp) (10^6dynes/cm)	transition temp (°C)
1.	0.19 C, 1.12 Mn, 0.23 Si	nitrided notch	5.6	-50
2.	0.24 C, 1.33 Mn, 0.27 Si 0.40 Mo	"	5.8	-60
3.	0.21 C, 1.15 Mn, 0.51 Si 0.4 Cr, 0.2 Ni, 0.01 Al 0.08 Ti (ASTM 9)	"	6.9	-35
4.	0.26 C, 0.66 Mn, 0.41 Si	"	3.5	-60
5.	.23 C, 0.61 Mn	double tension	2.0	-
		calculated values		
		G_{Ic} (min)	2.5	
		G_{Ic} (exp)	7.5	
		G_c (arrest), 2 cm plate	50	

Table 4. Critical stresses and transition temperatures

steel	grain size		tensile		plate		V-15
	ASTM	2d (10^{-3}cm)	T_c (°C)	σ_c (10^3psi)	T_c (°C)	σ_n (10^3psi)	transition (°C)
0.16 C, 1.30 Mn	3.1	13.9	-180	90	20	22	18
	7.2	3.4	-210	140	-20	37	-22
0.22 C, 0.36 Mn	3.5	11.9	-160	80	100	20	72
	6.6	4.1	-180	95	60	24	52
.039 C	-	40.9	-90	55	30	14	-
	-	11.3	-150	85	-10	22	-

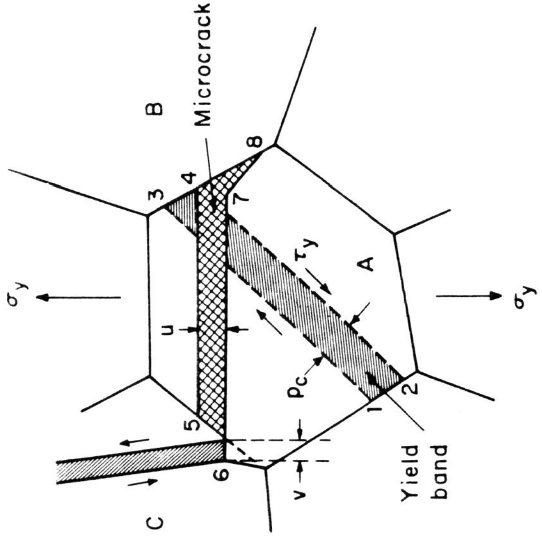

Fig. 2 Relief of slip or twin band displacement by microcrack

Figure 3. Microcrack in ferrite tested in tension at -140°C **170X**

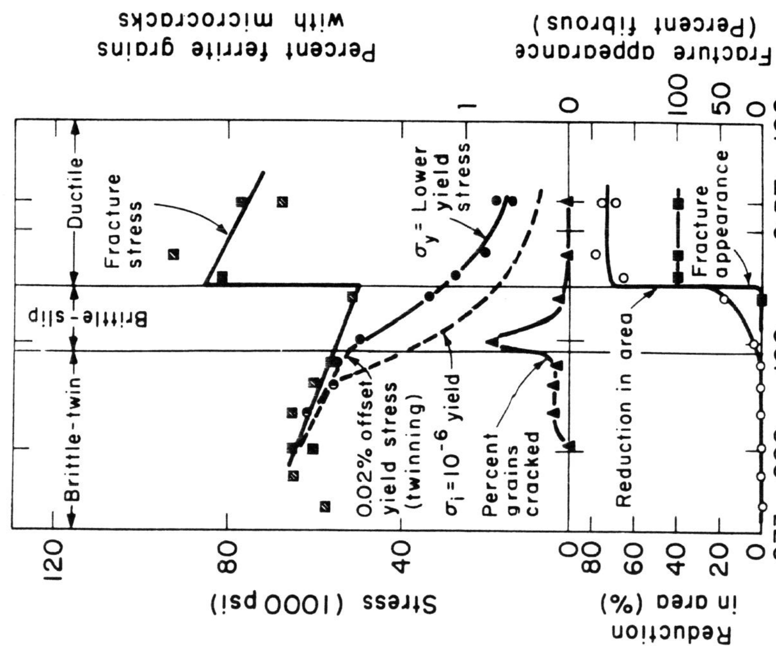

Fig. 1 Tensile properties of coarse-grained ferrite, 0.039 pct C, d=0.041 cm. (Hahn et al.)

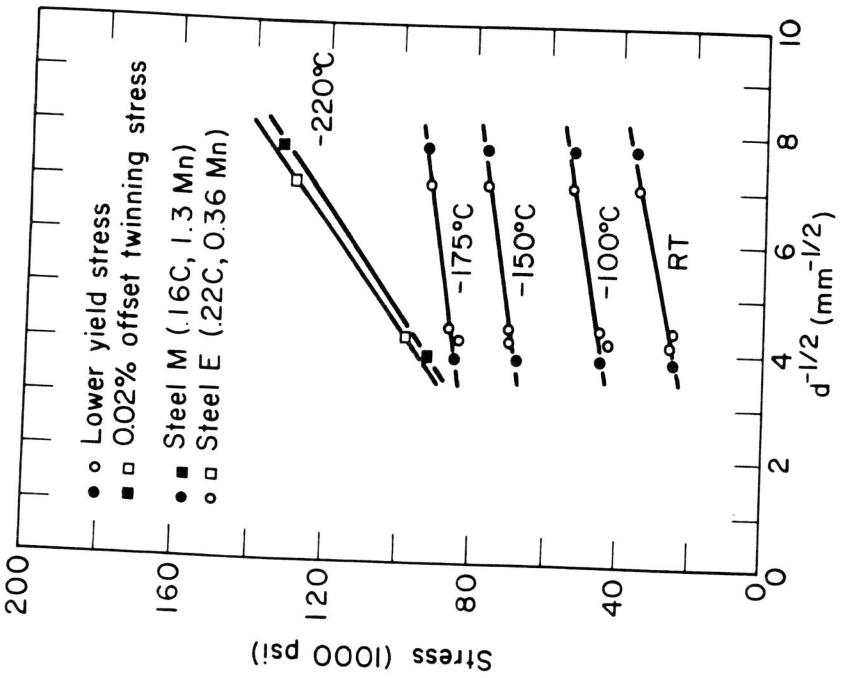

Fig. 5 Grain size dependence of lower yield and twinning stress (Hahn et al).

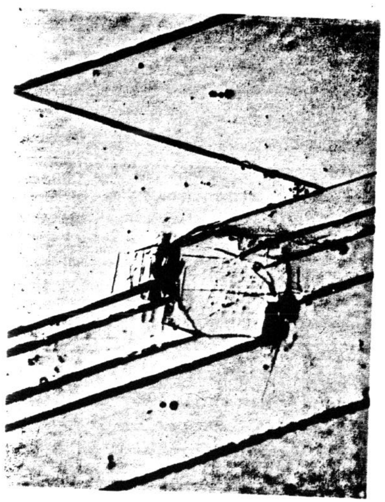

Figure 4. Microcracks formed in a single crystal of iron at -196°C by blockage of large twins by a parasite grain boundary. 170X

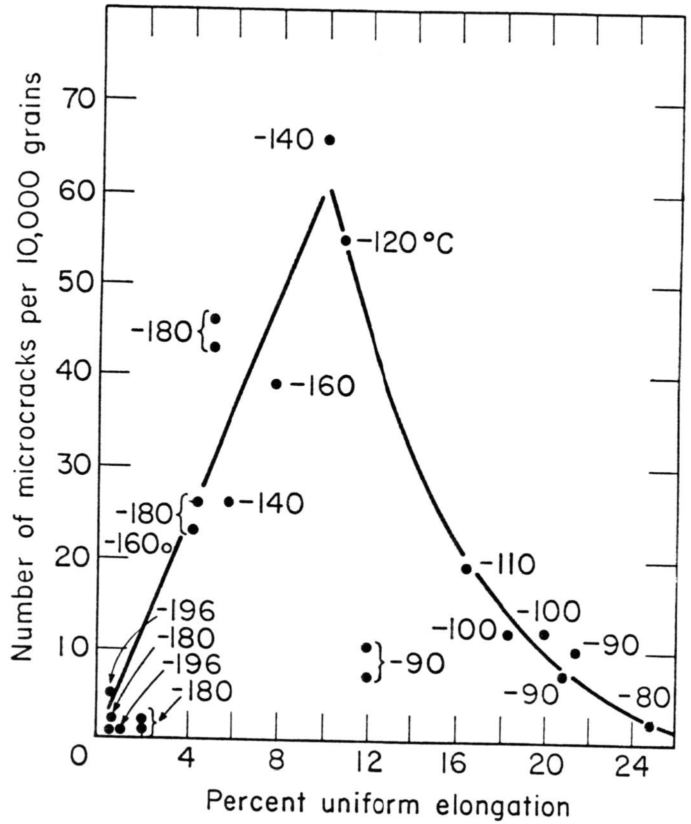

Fig. 6 Number of microcracks as a function of uniform strain in ferrite. Numbers next to points refer to test temperatures. (McMahon)

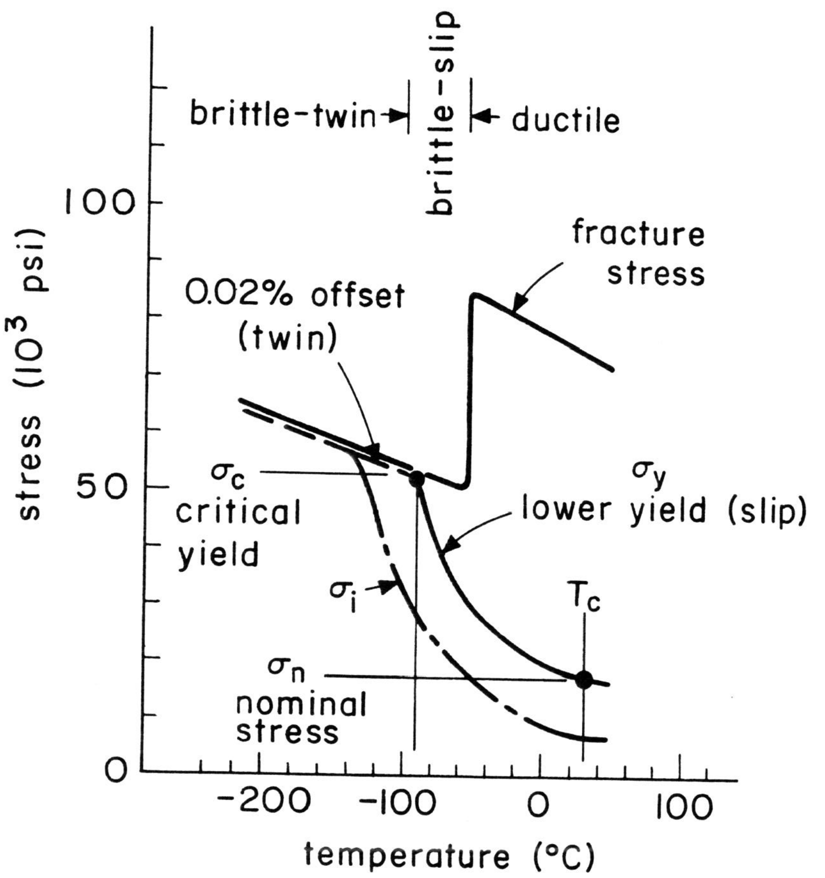

Figure 7. Plate transition temperature,
$\sigma_n = \sigma_c / \alpha$ with $\alpha = 4$
0.039 C ferrite, d = 0.041 cm