A-12

YIELDING INITIATION PROCESS AND ITS EFFECT ON FRACTURE OF CRACKED PLATE AND FATIGUE DAMAGE NUCLEATION

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ABSTRACT

The fracture stresses of cracked plates and the flow stresses of polycrystalline aggregates are related to the stresses and strains at the ends of plano-discontinuities, i.e. cracks and dislocation arrays. Cracks and linear dislocation arrays are equivalent to each other. The stresses and strains at the end of a crack or a dislocation array are characterized by a $r^{-1/2}$ singularity, r being the distance from the end. Based on the characteristic singularity, a unified concept of fracture stress and flow stress can be formulated with the stress and strain environment at the end of the discontinuity as mechanical criterion.

The stresses and strains near the spearhead of a dislocation array in a surface grain is higher than that in an interior grain, provided that the grain sizes are the same. Therefore, yielding is normally initiated in the surface layer of a polycrystalline aggregate. Furthermore, the flow stresses in the surface layer is lower than that in the interior. The effects of yielding initiation in the surface layer and the inhomogeneity in flow stress on the fracture stress of cracked plate and fatigue damage nucleation, are discussed.

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I Introduction

The fracture stress of a cracked plate is related to the stresses and strains at a crack tip, and the flow stress of a polycrystalline aggregate is related to the stresses and strains at the spearhead of a dislocation array. The cracks in a continuum medium are equivalent to linear dislocation array. Therefore a unified concept can be established for fracture and flow stresses. Starr (1) used a shear crack to simulate a slip band. Eshelby, Frank, and Nabarro (2) compared the stresses adjacent to the spearhead of a dislocation array with that at a crack tip. Bibly, Cottrell and Swinden (3) discussed the relationship between dislocation arrays and cracks. The stresses and strains at the end of a plano-discontinuity, i.e. crack or dislocation array, have the singularity of $r^{-1/2}$, r being the distance from crack tip, and the stresses and strains are characterized by a coefficient often called stress intensity factor, (4, 5). The characteristic singularities give rise to Griffith's fracture stress (6, 7, 8) and Petch's flow stress of polycrystalline aggregate (9).

For a polycrystalline aggregate, the stress intensity factor of a dislocation array in a surface grain is higher than that in an interior grain. Therefore, yielding will initiate in the surface layer forming slip bands and propagate into the interior as the applied stress is increased. Consequently, on the average, the flow stress in the surface layer of a polycrystal is lower than that in the interior. The yielding initiation processes affect the stress profile at the crack front and thereby affect the fracture stress of a cracked plate.

Fatigue damage has been observed to nucleate in the surface layer, such as observed by Forsyth (10) and Wood (11). The fatigue damage nucleation is associated with the plastic deformation in the surface layer of a polycrystalline aggregate. The mechanical environment for fatigue damage nucleation can be characterized by the relative displacement on a slip-plane or by the stresses and strains at the end of a dislocation array.

II Dislocation Arrays and Cracks

Eshelby, Frank and Nabarro (12) studied the equilibrium positions of a set of like discrete dislocations in a common slip-plane under the influence of applied stress. Eshelby (13) has indicated that discrete dislocations can be approximated by a continuous distribution of elementary infinitesimal dislocations with the same total Burgers vector. This approximation, valid if the discrete dislocations are closely spaced, was used by Leibfried (14), Head and Louat (2) to calculate the continuous dislocation distributions of linear arrays. A dislocation is in static equilibrium if the force on the dislocation is zero. Therefore the condition for equilibrium dislocation distribution in an array is zero force on every dislocation in the slip-plane. With Eshelby's approximation of elementary infinitesimal dislocations, the condition of zero force on every dislocation is equivalent to one of zero stress at every point in a slip-

plane. The later condition is the boundary condition of a crack in a continuum. Consequently, an equilibrium linear dislocation array corresponds to a crack in a continuous medium.

Following Chou and Louat's notation (15), a dislocation function, f(x), of a linear array is in equilibrium under the applied stress $\sigma(x_0)$, if the equation

$$A \int_D \frac{f(x)}{x - x_0} = \sigma(x_0) \qquad (1)$$

holds for all x_0 in the region D, with the possible exception at the ends. A is a material constant, A = Gb/2πk, where G is shear modulus; b, the Burgers vector; k = (1-ν), for edge dislocation and unity for screw dislocation; and ν, Poisson's ratio. The three elementary types of dislocation arrays are of freely-climbing and freely-gliding edge dislocations and of screw dislocations. If the dislocations are parallel to Z axis and in X-Z plane, the corresponding stress components, i.e. σ(x) for these three types of dislocation arrays, are respectively σ_{yy}, σ_{xy}, σ_{yz}. The conditions of zero force on every dislocation of the above three elementary types of arrays are equivalent to $\sigma_{yy} = 0$, $\sigma_{xy} = 0$, and $\sigma_{yz} = 0$ in the corresponding slip-planes. These are the same boundary conditions for tensile cracks and lateral and longitudinal shear cracks.

Dislocation arrays has been compared with cracks by Starr (1), Eshelby et. al. (12), Head and Louat (2), Bilby et. al. (3), and many others. The dislocation distribution function at the spearhead of an array and the stresses at the tip of a crack are generally unbounded and are related to each other. The relationship between a freely-gliding edge dislocation array and a lateral shear crack was discussed by Liu (16). In this section, the relationship will be extended to all three types of arrays and cracks.

The stresses and strains of cracked plate have been investigated by Inglis (17), Starr (1), Sneddon (18), Williams (4), Irwin (5), Sih et. al. (19). Since the stresses at a crack tip are unbounded, they are of special interest with respect to fracture and yielding intitiation. The stresses adjacent to a crack tip are approximately given by

$$\sigma_{xx} = \frac{K_1}{\sqrt{2r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] - \frac{K_2}{\sqrt{2r}} \sin\frac{\theta}{2}\left[2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right] \qquad (2)$$

$$\sigma_{yy} = \frac{K_1}{\sqrt{2r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] + \frac{K_2}{\sqrt{2r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \tag{2}$$

$$\sigma_{xy} = \frac{K_1}{\sqrt{2r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{K_2}{\sqrt{2r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_{xz} = \frac{K_3}{\sqrt{2r}}\left[\sin\frac{\theta}{2}\cos\theta - \cos\frac{\theta}{2}\sin\theta\right]$$

$$\sigma_{yz} = \frac{K_3}{\sqrt{2r}}\left[\sin\frac{\theta}{2}\sin\theta + \cos\frac{\theta}{2}\cos\theta\right]$$

where K_1, K_2 and K_3 are stress intensity factors respectively for a tensile crack, a lateral shear crack and a longitudinal shear crack. The co-ordinates X, Y, θ and r are shown in Fig. 1. The stress intensity factors prescribe the stresses and strains as well as the displacement fields adjacent to crack tips. Eshelby (13), has indicated that the continuous dislocation distribution functions are related to the Burgers vectors and the derivatives of the displacements of the crack surface with respect to x.

$$f_{1,2,3} = -\frac{2}{b}\,\frac{\partial u_{2,1,3}(x,0^+)}{\partial x} \tag{3a}$$

The dislocation distribution functions near spearheads are given by

$$f_{1,2,3} = \frac{1}{\pi A r^{1/2}} K_{1,2,3} \tag{3b}$$

where f_1, f_2 and f_3 are dislocation distribution functions near the spear-heads of arrays of freely-climbing and freely-gliding edge dislocations and screw dislocations; and u_1, u_2 and u_3 are displacements along the crack surfaces in the x, y, and z directions. Equations (2) and (3) relate the dislocation distribution functions adjacent to a spearhead to the stresses adjacent to a crack tip. The stresses at the array spearhead or the crack tip are unbounded; therefore, the stress intensity factors are important to both fracture and yielding of crystalline solids.

Equations (1) and (3) indicate that the stress intensity factors are the same for all three types of cracks if the corresponding dislocation arrays are governed by Equation (1). This has been shown to be true for isolated cases such as an infinite row of equally spaced collinear cracks under uniformly applied stresses, Paris and Sih (20), and for concentrated forces on the crack surfaces; Sih et. al. (19) and Sih (21).

The force, on the leading dislocation of a single-ended array, F_s, under a uniform applied stress σ has been calculated by Cottrell (22) and Eshelby et. al. (12).

$$F_s = n\sigma b = \frac{\pi k \sigma^2 L}{G} \tag{4}$$

where n is the total number of dislocations in the array, and L is the length of the pile up. Equation (4) is applicable only to the single-ended array under a uniform applied stress. In the general case, the force on the leading dislocation can be calculated from Equation (4) and the consideration of the stress field in the vicinity of a spearhead.

Equation (2) gives the intensity as well as the distribution of stress adjacent to a spearhead. The intensity of the stress fields are completely specified by the stress intensity factors and the distribution of stresses is independent of applied stresses and geometric configuration. The accuracy of Equation (2) improves as r approaches zero. Within a very small region near the spearhead, the stresses can be considered as given by Equation (2). If the region is considered as separated from the rest of the solid, the force on the leading dislocation is determined by the stresses on the boundary of the region, which are specified by stress intensity factors. Therefore, for a given value of K, the force on the leading dislocation is the same. Consequently Equations (3) and (4) and the stress intensity factor for a single-ended array lead to

$$F = \frac{\pi k K^2}{2G} \qquad (5)$$

where F is the force on the leading dislocation of an array. It is interesting to note that this force is the same as elastic energy reduction rate of a cracked plate. This force causes the leading dislocation to penetrate barriers.

Equations (2), (3) and (5) indicate that the stresses and the dislocation distribution functions adjacent to a spearhead as well as the force on the leading dislocation are prescribed by stress intensity factors. The stress intensity factors depend not only on the applied stress but also on the geometric configuration. In a polycrystal the lengths of the active slip bands in the interior grains and the surface grains can be considered as the same. The two dimensional calculations, (23, 24, 25, 20) indicate that

$$K_{1(s)} = 1.58\, K_{1(i)}$$
$$K_{3(s)} = 1.41\, K_{3(i)} \qquad (6)$$

where K_1 and K_3 are stress intensity factors for tensile and longitudinal shear modes respectively, and the subscripts "s" and "i" in the parentheses refer to surface and interior grains. The stress intensity factor for the lateral shear surface crack is not available. Therefore, similar comparison cannot be made for lateral shear mode. Since the stress intensity factor for the surface grain is higher than that for the interior grain, it is reasonable to expect that macro-yielding is initiated on the surface and it propagates into the interior of a polycrystal as applied stress is increased. This phenomenon has important effects on fracture criterion and fatigue crack initiation. These effects will be discussed in detail in subsequent sections. However, a unified concept of yielding and fracture initiations will be discussed first in the next section.

III Fracture Stress of Cracked Plate and Flow Stress of Polycrystal

Zener (26) has proposed that slip-band propagation is caused by stress concentration at the spearhead. Petch (9) has used the force on the leading dislocation of an array as the criterion for slip band propagation. The force on the leading dislocations must overcome the strength of dislocation barriers. Armstrong et. al. (27) hypothesized that when the stress field at the end of a slip band activates a new dislocation source in the neighboring grain, yielding propagates through grain boundary. All of these theories predict that the yield strengths or flow stresses of polycrystals are proportional to $L^{-1/2}$, L being grain size.

Griffith (6) formulated a fracture criterion for brittle materials based on energy considerations. A crack will propagate when the elastic strain energy released is larger than the surface energy. Subsequently, Orowon (7) and Irwin (8) extended Griffith's criterion to ductile materials. The surface energy was neglected, and the plastic energy absorption rate was assumed constant. Later Williams (4) and Irwin (5) recognized that the stress field near a crack tip is characterized by a $r^{-1/2}$ singularity, and the stress distribution is independent of loading and geometric configuration of the specimen. Therefore, the elastic stress "environment" can be used as a fracture criterion. Neuber (28) hypothesized a maximum fracture stress for notched specimens. Weiss (20) extended the theory to fracture of ductile materials. All of these theories predict that fracture stress of a cracked member is proportional to $a^{-1/2}$, a being the half crack length.

When a theory on fracture or yielding is proposed and an experiment is carried out, one seeks a mechanical parameter or parameters controlling the fracture or yielding processes. The classic case of yield criterion is a typical example. Shear stress or a combination of shear stresses is the controlling mechanical parameter for macroscopic yielding initiation. The reason for the success of yield criterion is that plastic deformation is the result of dislocation gliding, and the gliding forces on dislocations are related to shear stresses. A given level of shear stresses provides a stress "environment" which exerts, on the average, a corresponding force on dislocations. For the same shear stress environment, the forces on dislocations, on the average, are the same even though the exact values of the forces are unknown. Therefore, if plastic deformation is caused by dislocation gliding and if dislocations are moved by the forces exerted on them, yield criterion should consist of shear stress or a combination of shear stresses.

Following the similar reasoning discussed above, a unified concept can be formulated for fractures of cracked plates and the grain size effect on yield strengths of polycrystals. Equation (2) indicates that for a given value of stress intensity factor, the elastic stress environment at a spearhead or a crack tip, i.e. the intensity and the distribution of stresses is completely determined. If a polycrystalline aggregate or a cracked plate remains linearly elastic prior to yielding and fracture, the yield strengths of polycrystals and fracture stresses of cracked plates can be established without further complication. In both cases, the stress intensity factor is a constant, because for a given value of stress intensity factor, the stresses and strains, whatever they may be, at a spearhead or a crack tip must be the same.

As has been noted, stress intensity factors not only control the stress environment at a crack tip, but also control the displacement and strain fields there. Irwin (5) has shown that the strain energy release rate, G, is related to K. Williams (30) has shown that the radius, R, of the deformed crack tip is related to K. Wells (31) has shown that the relative displacement δ, of the crack surfaces (i.e. crack opening displacement in the case of tensile mode) is also related to K. The sizes of the regions of high stresses, r_σ, and of high strains, r_γ, are also related to K. r_σ and r_Y prescribe the regions of high stresses and strains that exceed certain levels. Furthermore, both the force, F, on the leading dislocations, and the dislocation distribution functions near a spearhead, f, are related to K. In other words K characterizes the stress and strain environment as well as all the quantities, G, R, δ, r_σ, r_Y, F, and f. Or if one likes, any one of these quantities characterizes all the others. Therefore, any one of these parameters can be used for fracture criterion of cracked plates and yielding criterion of polycrystals. They will lead to the same expressions for fracture stress and flow stress if the elastic solutions are used. Since K characterizes so many physical quantities, and its characteristics are derived from the properties of a crack tip or a spearhead, it would be more appropriate to generalize the name to singularity coefficient of plano-discontinuity.

It has been recognized that multiple slip does take place at the spearhead of a dislocation array, and that plastic deformation does take place at a crack tip. Both multiple slip and plastic deformation relax the stresses in a region near the spearhead or the crack tip. Under such conditions it is surprising that linear elastic solutions are still applicable to yielding as well as to fracture initiations. Liu (32) has shown that if the size of the region of plastic deformation is small, the stress intensity factor still characterizes the stress and strain environments at a crack tip. The same conclusion is applicable to the effect of multiple slip.

It has been noted that within a small region near a crack tip, the elastic stresses are given by Equation (2). Let this region be prescribed by r'. If the size of plastic zone, r_p, is very small in comparison with r', the stresses within r' are hardly affected by the plastic deformation within r_p, except in the immediate vicinity of r_p. Therefore, the general stress environment within r' is still characterized by K, even though plastic deformation has taken place. Furthermore, if $r_p \ll r'$, the size of plastic zone is proportional to K^2. This conclusion is substantiated by the calculations by Hult and McClintock (33), Dugdale (34), and Bilby, Cottrell and Swinden (3) for special cases. Since r_p is directly related to K, it can also be used to characterize the stress and strain environments at a crack tip.

At the same values of stress intensity factors, or the same size of r_p, the stress and strain environments at crack tips are the same, even though their exact values are unknown. Therefore, if one specimen fails under a condition of stresses and strains at its crack tip, so will the others. This is similar to the situation where shear stress or stresses are used as yield criterion, because the shear stresses provide a stress environment for yielding initiation, even though the exact values of the forces on dislocations are unknown.

If the stress intensity factor or any of the related parameters is used as fracture criterion, the fracture stress of a cracked wide plate is given by

$$\sigma_F = \frac{K_c}{\sqrt{a}} \tag{7}$$

where K_c is an empirical constant. The above conclusions are valid if the size of the plastic zone is small, e.g. if the applied stress is low in comparison with the yield strength of the material.

The above concept of stress and strain environment as a mechanical criterion was developed for fracture. The same concept and conclusions can be derived for the flow stresses of polycrystals. If a critical stress and strain environment is hypothesized for yield penetration through grain boundaries and an internal stress τ_i is assumed on slip-planes, the critical shear stress for yield penetration is given by

$$\tau_Y = \tau_i + K_Y \left(\frac{L}{2}\right)^{\frac{1}{2}} \tag{8}$$

Since the orientations of grains are random, the tensile flow stress is given by

$$\sigma_Y = m\left[\tau_i + K_Y \left(\frac{L}{2}\right)^{-\frac{1}{2}}\right] \tag{9}$$

where m is Taylor's orientation factor (35) and L is the grain diameter. Equation (9) is identical to the expression derived by Armstrong et. al. (27).

An expression similar to Equation (8) can be derived for the fracture strengths, σ_f, of polycrystalline aggregates. The expression for fracture stress is identical to that given by Petch (9).

$$\sigma_f = \sigma_i + 2K_f \left(\frac{L}{2}\right)^{-\frac{1}{2}} \tag{10}$$

Therefore, equations of the type depicted in (7), (9) and (10) can be derived from the elastic solutions with the unified concept of stress and strain environments for fracture and yielding. A detailed physical model is not necessary.

IV Yielding Processes and Fracturing of Cracked Plates

In the preceding section, the concept of stress and strain environment for fracture was discussed without knowing the details of the stresses and strains within the plastic enclave. Such an approach provides an adequate solution to practical engineering problems. However, it does not provide any insight into the mechanisms of fracture, not even in terms of macroscopic parameters.

In this section the stresses and strains within a plastic zone and their effects on fracture will be discussed. The stresses and strains within a plastic zone are affected by the plane strain to plane stress transition and the yielding initiation and propagation processes. Both affect the stresses in the surface and the interior regions of a specimen and thereby affect the fracture stress of a cracked plate.

For a cracked plate, the following have been observed. Fracture stresses of cracked plates decrease as crack lengths and plate thicknesses increase (8). For a thin plate, the region ahead of a crack tip necks down prior to the onset of fracture, (34, 36). For a thick plate, the interior of the plate fractures first, and the crack front there leads the crack front near the specimen surface (37). This is often called tunnelling or pop-in phenomenon. The fracture surface in the interior of a specimen is a plane normal to the applied tensile stress, and the fracture surface near the specimen surface is a plane of maximum shear stress, which is inclined 45° to the applied stress. The region where the fracture surface is a maximum shear plane is usually called shear lip. If the fracture stress is high in comparison with the yield strength of the material the fracture surface consists of shear surface only and the normal cleavage fracture is absent. It is often observed that the sizes of shear lips are more or less constant for a given material, and it is independent of plate thickness and crack length.

It is difficult to explain all of these observed phenomena in terms of classic continuum mechanics, which assumes homogeneity of a solid. The difficulty can be illustrated with geometrically similar specimens. For such specimens, the classic continuum mechanics depicts that at geometrically similar locations, the stresses and strains are identical, and the elastic and the plastic energies are porportional to their volumes. Therefore, if the effect of surface energy is neglected, the continuum mechanics indicate an identical fracture stress for geometrically similar specimens. Furthermore, if fracture processes are controlled by stresses and strains, the shear lip sizes should be proportional to the sizes of the specimens. When Orowon and Irwin adapted the Griffith fracture criterion to ductile materials, the constant plastic energy absorption rate was a fundamental hypothesis which cannot be substantiated by continuum analysis, if homogeneity of the medium is assumed. Therefore, it becomes a task to explain why the modified Griffith criterion works for brittle fracture of metals.

The crack length dependence of the fracture stress as depicted by the modified Griffith criterion, the thickness dependence of the fracture

stresses as indicated by Irwin's experimental observation (8), as well as the constant shear lip size are associated with yielding initiation and propagation processes of polycrystalline aggregates. The yielding processes are surface phenomena, which cause inhomogenous flow stress in a specimen.

Suzuke (38) showed that surface sources can be activated by lower stresses. Cottrell (39) stated that the dislocation network near the surface is freer and the internal stress there is lower. Therefore, it is very likely that micro-yielding will be initiated in the surface layer. Once the sources are activated, the generated dislocations will pile-up against strong barriers. It has been noted, Equation (6), that the stress intensity factor for screw dislocation arrays in interior grains is 0.71 times that in the surface grains. Therefore, under normal conditions macro-yielding will also be initiated in the surface layer of a polycrystal and propagated into the interior. As the applied stress increases, the favorably oriented dislocation sources will be activated, and the generated dislocations will pile-up against barriers. However, only these dislocation arrays, whose length is close to grain diameter, will cause macro-yielding propagation. For a polycrystalline aggregate of randomly oriented and distributed grains, slip-bands will be initiated at favorable locations. Once a slip band is formed on a slip plane, the stresses adjacent to the active slip plane are relaxed except at the ends of the pile-up. The high stresses at the end of a pile-up cause multiple slip, yielding propagation through barriers, and activation of dislocation sources on neighboring slip planes. Therefore, in a uniform stress field, the plastic deformation is concentrated on regularly spaced slip bands. The flow stress in the surface layer and in the interior are determined by the critical stress intensity factor for yielding propagation. Assuming that the average internal stress in the surface layer as well as in the interior is the same, the average flow stress of the surface layer of a polycrystalline aggregate is therefore 0.71 times that of the interior. The flow stress increases from the surface of a polycrystal to the interior. The depth of the transition region is a function of grain size and strengthening mechanisms and it is independent of the size of the specimen, since it is controlled by a surface phenomenon. In a non-uniform stress field, such as near a crack tip, the spacing between slip bands and the amount of plastic deformation on each slip band are influenced by the stress gradient. Slip bands at a crack tip have been observed by Hahn and Rosenfield, (40).

Hill (41) has shown that for a rigid-plastic material, the tensile stress in the plane strain region of a plastic enclave is 3 σ_Y, where σ_Y is tensile yield stress, whereas the tensile stress in the plane stress region of a plastic enclave is equal to σ_Y. Therefore, the maximum tensile stress in the interior region is three times that in the surface region. Hill's solution assumes homogeneity of the material. If the effect of yield initiation process is superimposed on the difference between plane stress and plane strain cases, the maximum tensile stress in the interior region is 5.2 times that in the surface region.

Plastic deformation causes microcracks to form in the deformed region. The formation of micro-cracks as a result of plastic flow has been discussed by Stroh (42), Hahn (43) and Bullough (44). Fig. 2 shows that micro-crack densities and sizes at a crack tip, increase with applied stress, as indicated by the solid curve. In the diagram, for the sake of simplicity, the micro-crack densities and sizes both in surface and interior regions are assumed to be the same functions of applied stress. The fracture strength of the deformed material as shown by the dashed line decreases as microcrack size and density increase. In the interior region, the micro-cracks are imbedded in a tensile stress field of 3 σ_Y whereas in the surface region, the micro-cracks are imbedded in a tensile stress field of 0.71 σ_Y. Because of the high stresses in the interior of a thick specimen, fracture will be initiated there first, and the fracture surface is normal to the applied stress. In the surface region, the normal stress is not high enough to cause cleavage fracture, and shear deformation is necessary to cause tear. As indicated in the diagram, the fracture stress for the interior, σ_{FI}, is lower than the fracture stress for the surface region, σ_{FS}. In general, the fracture stress of a cracked plate is some combination of σ_{FI} and σ_{FS}, and both σ_{FI} and σ_{FS} are controlled by the capability of the material to sustain plastic deformation, i.e. ductility.

The extent of the shear fracture region depends on the strains and stresses along the crack front and the ductility of the material. The stress transition from plane stress to plane strain depends on the geometric size of the specimen, such as crack length, width, and thickness. The stress transition from surface grains to interior grains is independent of geometric size of the specimens. If the material has a low ductility, so that the cleavage fracture extends to the latter transition region, the shear lip size will be more or less constant. Figure 3a shows the fracture surfaces of two geometrically similar specimens. The specimens are sharply notched. The lengths L of the fractured surfaces are 2.45" and 1.40". The measured sizes of the shear lips at geometrically similar locations are shown in Figure 3b. The sizes are nearly equal.

The shear lip is a region of extensive plastic deformation. The limit on its size restricts the plastic deformation energy. Consequently, the plastic deformation energy absorption rate is less than that predicted by a continuum theory which assumes homogeneity of material properties. In such case, the elastic strain energy release rate will eventually surpass the plastic energy rate, as the applied stress increases. From an energy point of view, this constitutes the condition of the onset of fracture propagation.

If the size of the shear lip is very small, r_p is small and the plastic energy absorption rate could be approximately constant, and thereby the modified Griffith criterion is applicable. If the material is ductile, r_p is large. In such case, the modified Griffith criterion predicts a fracture stress higher then the observed value (32).

In the above discussion, it was assumed that slip is not impeded at the surface. If slip is impeded at the surface either by mechanical means such as cold work, or by chemical means such as oxidation, the normal stress in the surface layers will be raised. Therefore, σ_{FS} will

be reduced. On the other hand, the effect of liquid metal will reduce the fracture strength of the material, as shown by the dotted line in Figure 2, with a possible increase of the normal stress in the surface layer. Consequently, a large reduction in fracture stress is expected.

For a very thin plate, the material near a crack tip necks down. The necking down associated with cracks was first discussed by Dugdale (34) and McClintock (36). The extensively deformed zone becomes an elongated and narrow region, as shown in Figure 4. Necking causes strain concentration in a localized region. The extensive plastic deformation causes crack to propagate. Bilby et. al. (3) hypothesized that the plastic displacement for fracture at a crack tip is constant. Hahn and Rosenfield (40) used plastic strain as a fracture criterion for plane stress fracture. If Hahn and Rosenfield's model is extended further, the thickness effect on fracture stresses of thin plates can be calculated.

Dugdale (34) solved the problem of a crack with extended narrow yield regions. Bilby, Cottrell and Swinden (3) solved the corresponding problem of linear dislocation array. Bilby et. al. calculated the plastic displacement, ϕ at such a crack tip.

$$\phi = \frac{\pi a \sigma_Y}{2G} \left(\frac{\bar{\sigma}}{\sigma_Y}\right)^2 \tag{11}$$

where

σ_Y = yield stress

$\bar{\sigma}$ = applied stress

a = half crack length

If a material does not strain harden, the plastic displacement, ϕ, will be spread out to a region its width is equal to the thickness of the specimen. If a material does strain harden, ϕ will be spread out to a region of αT, where α is a constant and T is the plate thickness. On the average, the strain at the crack tip is $\phi/\alpha T$. Let ε_F be the strain at the crack tip when specimen fractures.

$$\varepsilon_F = \frac{\phi}{\alpha T} \tag{12}$$

Equations (11, 12) lead to

$$\bar{\sigma}^2 a = \frac{2G\sigma_Y \alpha \varepsilon_F T}{\pi} \tag{13}$$

Therefore, the fracture stress of a cracked plate is related to the fracture strain and the plate thickness. Irwin (8) has shown the thickness effect on fracture stresses of thin plates of 7075 - T6 aluminum alloy. Using Irwin's data for the thin plate, and with $G = 3.8 \times 10^6$ psi, $\sigma_Y = 68,000$ psi, the estimated value for $\alpha\varepsilon_F$ is equal to 0.46 in comparision with the fracture strain of the aluminum alloy $\varepsilon_F = 0.38$, (45).

As discussed above, the state of stresses and strain are far more complicated than could be calculated. Yet when r_p is small, the stress intensity factors are capable of characterizing all of these events. The reason is that the stress and strain distribution at a crack tip is independent of the geometry of the specimen and the applied stress; only the intensities of stresses and strains are functions of geometry and applied stress. This has been shown to be true for elastic cases (4, 5), general elasto-plastic cases if r_p is small (32), and elasto-perfect-plastic cases of longitudinal shear cracks even if r_p is large (33, 46).

It has been discussed that plastic deformation and plastic energy are affected by surface effects and the plane stress to plane strain transition. In order to formulate an energy criterion, these effects have to be considered. On the other hand, if a physical parameter can characterize the state of stresses and strains at the crack tip, the parameter can serve as a fracture criterion. The complications of surface effects and the transition of states of stresses might be avoided. The size of a highly strained region, r_Y, seems to be a reasonable physical parameter; and the fixed strain criterion at a given distance away from crack tip as proposed by McClintock and Irwin (47) can be interpreted from this physical point of view.

V Fatigue Damage Nucleation

Thompson, Wadsworth and Louat (48) have observed persistent slip bands in the surface layer of a fatigued specimen. The life of a fatigue specimen can be prolonged by removing the surface layer periodically. It has observed that the resistance of metals to fatigue increases greatly at high vacuum (49). Therefore, the contamination of the slip-plane by oxides is important for fatigue damage nucleation. These investigations indicate that fatigue damage nucleation is associated with the properties of a free surface. Indeed Forsyth has observed extrusion and intrusion in silver chloride. Wood (11) investigated surface as well as sub-surface markings. Hills and valleys have been observed on the surface and pores are often formed on slip bands and at grain boundaries. Cottrell and Hull (50), Mott (51), Thompson (52), and McEvily and Machlin (53) have proposed dislocation mechanisms for extrusion-intrusion processes. On the other hand, Hempel (54) has found cracks are nucleated at inclusions as well as at grain boundaries. Hanstock and Forsyth (55) have found cracking at precipitation zones. Undoubtedly in all of these cases the damage

nuclei are caused by stresses and strains. In this section, the parameters that characterize the mechanical environments which control the fatigue damage nucleation will be discussed.

It has been noted that surface sources can be activated by lower applied stress (36), and the internal stresses in the surface layer are lower than those in the interior (39). Furthermore, the stress intensity factor of the dislocation arrays in the surface layer is higher than that in the interior grain. Consequently it is expected that yielding is generally initiated in the surface layer, and the flow stress in the surface layer is lower. Fatigue damage is nucleated by localized plastic deformation. Therefore, it is reasonable to expect that fatigue cracks are usually nucleated in the surface layer.

The amount of extrusion and intrusion, the rate of contamination of the slip-planes by oxide, as well as the rate at which the pores and fissures are formed on a slip band, are all related to the number of dislocations traveled on the slip planes. The total number of dislocations traveled is related to the relative displacement between the two surfaces of a slip plane. The maximum relative displacement is at the surface, and it is proportional to (τL), where τ is the net applied stress on the slip plane and L is the grain diameter. Therefore, the parameter (τL) should characterize the mechanical environment for extrusion, intrusion, slip plane contamination, and pore formation on slip band.

On the other hand, at the spearhead of a pile-up, the density of dislocations as well as stresses and strains are highest. If fatigue damage nucleation is caused by strong interactions between dislocations, or the interaction between dislocations and barriers, the spearhead of a pile-up is a likely nucleation site. The dislocation densities and stresses and strains at the end of a pile-up are characterized by $\tau\sqrt{L}$. Therefore, the two likely nucleation sites are at the ends of a dislocation array, one on the free surface, and the other at the spearhead of a pile-up. The mechanical environments at these two sites are characterized by (τL), and $\tau\sqrt{L}$ respectively. Comparing the characteristics of these two cases, one may conclude that for large grains, the relative displacement might be the dominant effect on fatigue damage nucleation. On the other hand, for small grains, the dislocation density, stresses and strains might be a dominant effect. Figure 5 shows the regions of extensive pore formation away from the grain boundary and at the grain boundary.

Not all of the fatigue damage nuclei lead to final failure. The ones which cause failure must penetrate the dislocation barriers in the surface layer. In the surface layer, fatigue crack propagates on the maximum shear plane as observed by Forsyth (56). Therefore, it is reasonable to assume that shear stress is still the dominant force for crack propagation in the early stage. The stresses and strains at such a crack tip are characterized by K_2 and K_3. If one assumes that macroscopic yielding is associated with the deformation or penetration of barriers by dislocation arrays throughout the polycrystal, one concludes that fatigue limit is proportional to yield stress. Equation (6) indicates that the proportional constant for K_3 is 0.71

Sinclair, Corten and Dolan have observed that fatigue limits of titanium alloys with various mechanical surface preparations are proportional to the hardnesses of the surface layers. The same was found to be true for several other titanium alloys, one steel and one nickel base alloy, (58, 59, 60, 61). Figure 6 shows the measured fatigue limit as a function of surface hardness. The surface hardness measures the bulk yield stress of the surface layer.

Tabor (62) has shown that hardness is related to the yield stress of a material. For a perfectly plastic material, the hardness is 2.8 σ_Y, σ_Y being the yield stress. If Tabor's relation is used, the data in Figure 6 indicate that fatigue limit is 0.42 times the yield stress of the surface layer, instead of 0.71 times. This discrepancy could be caused by the fact that Equation (6) is based on a two-dimensional calculation, the fact that Tabor's relation was derived for perfectly plastic materials and the effect of strain hardening was not taken into consideration, and finally the fact that residual stresses in the surface layer affect the hardness readings.

V Conclusions

Linear dislocation arrays are equivalent to cracks. The stresses and strains adjacent to the end of a crack tip are related to the dislocation distribution function near the spearhead and the force on the leading dislocation of an array.

The stresses and strains at the ends of a crack or a dislocation array are characterized by a $r^{-1/2}$ singularity, r being the distance from the end. Based on the characteristic singularity, a unified concept of fracture stress of cracked plate and flow stress of polycrystalline aggregate can be formulated with the stress and strain environment at the end of the discontinuity as a mechanical criterion. If the size of plastic zone and the region of multiple slip are small, the criterion lead to Griffith's fracture stress and Petch's flow stress.

The stress intensity factor for screw dislocation arrays in a surface grain is 1.41 times that in the interior grain, if the lengths of the arrays are the same. Therefore, yielding is normally initiated in the surface layer of a polycrystalline aggregate forming slip bands. Furthermore, the flow stress of the surface layer is lower than that in the interior. For a cracked plate, the tensile stresses along the crack front change from 0.71 σ_Y on the surface to 3 σ_Y in the interior, σ_Y being the yield stress of the material. The high tensile stress in the interior causes tunnelling, restricts plastic deformation, and lowers the fracture stress of the plate.

The fracture stress of a cracked plate is related to the ductility of the material, i.e. the capability of the material to sustain plastic deformation. For thin plates, the fracture stress is related to fracture strain of the material and the plate thickness.

Fatigue damage can be nucleated by intrusion and extrusion mechanisms and the rate of damage will be accelerated by oxide contamination of the slip plane at the surface. On the other hand, fatigue damage nuclei have been observed sub-surface at dislocation barriers such as grain boundaries and inclusions. The mechanical environments for physical processes that lead to these two types of damages are characterized by τL and $\tau\sqrt{L}$ respectively.

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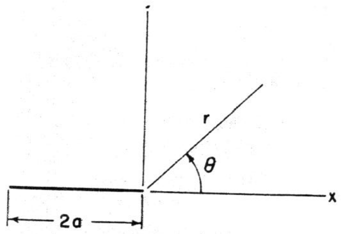

Figure 1 Coordinates at a Crack Tip

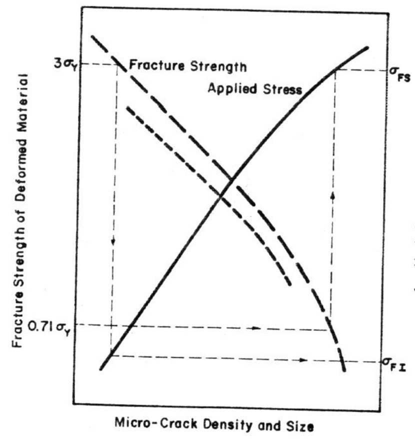

Figure 2 Schematic Diagram Showing Fracture Stress as Influenced by Microcrack Density and Spacing.

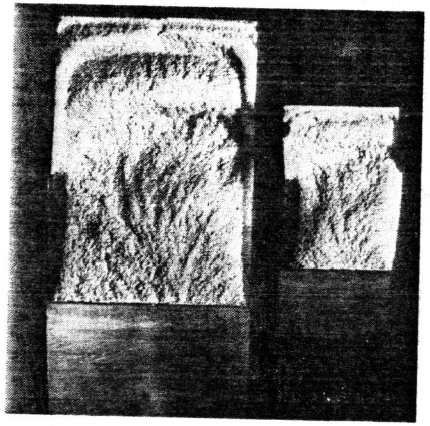

Figure 3a
Fracture Surface of Notched Geometrically Similar Specimens (Courtesy of Dr. Gene Schaeffer)

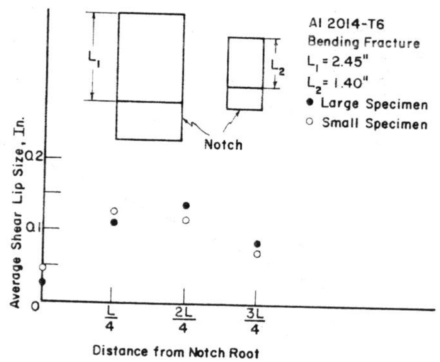

Figure 3b
Shear Lip Sizes of Geometrically Similar Specim...

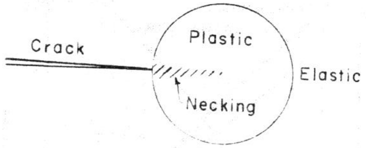

Figure 4 Necking at a Crack Tip (After McClintock, Ref. 36)

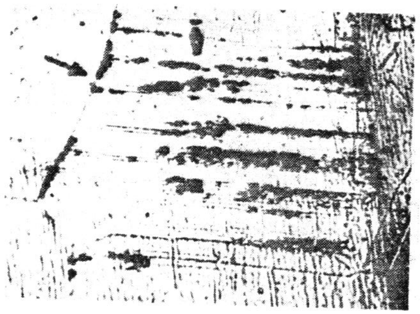

Figure 5 Pores on Slip Planes, at and Away from Grain Boundaries (Wood, Ref. 10)

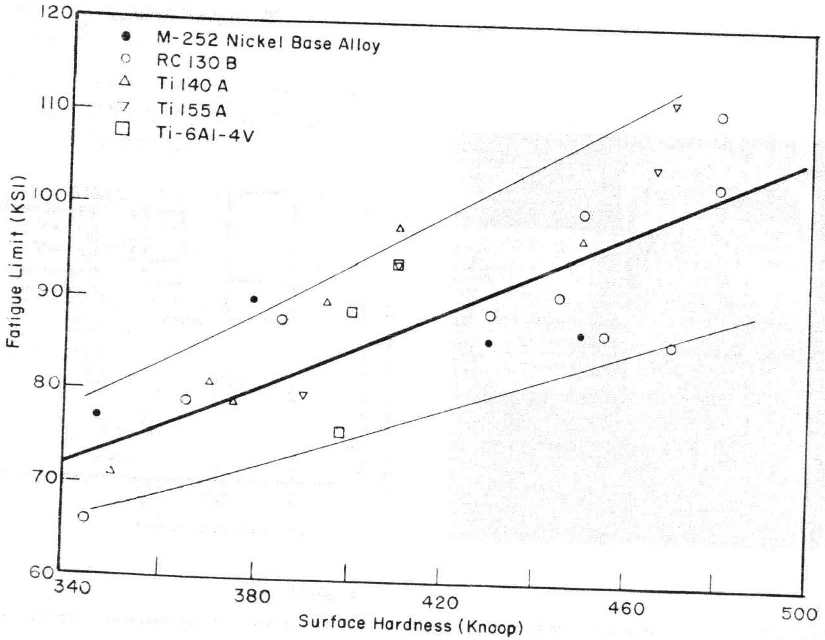

Figure 6 Fatigue Limit and Surface Hardness