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The Propagation of Brittle Fracture

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ABSTRACT

The isochromatic strain patterns on the surfaces of mild steel plates containing moving brittle fractures have been determined by using strain birefringent coatings and high speed photographic techniques. The major part of these patterns may be described by a formula for the isochromatic stress field near a stationary crack in an elastic medium. The variations from this formula are caused by shock waves in the specimen, by 'inertia effects' due to the high velocity of the crack, and by yielding near the crack.

The inertia effects cause a region of biaxial tension ahead of the crack on the plate surface, as predicted by theoretical analyses of the stress field near moving cracks. A crack model which predicts the extent of the plastic zone near the crack tip has been proposed. The predictions from this model and other theories are compared with experimental estimates of the plastic zone depth.

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1. INTRODUCTION

The thermodynamic condition for semi-brittle fracture in mild steel depends upon two effects, the mechanical instability, determined by the energy balance around the crack, and the plastic instability (onset of local yielding) which determines the major part of the energy absorbed in crack propagation. Since both of these effects are governed by the state of stress near the crack tip, there have been many attempts to determine the stress state both theoretically and experimentally. Recently particular effort has been directed to finding the size of the yielded zone and the plastic displacement at the tip of a stationary crack or notch, in order to help solve the problem of brittle fracture initiation (1).

The alternative approach, typified by the work of Robertson (2), accepts that initiation is likely to occur somewhere in a structure, and is more concerned with the problem of propagation, and of how a running crack may be stopped from propagating. The use of photoelastic coatings with high speed photography (3, 4), has given useful information about the stress field around a propagating brittle crack.

In this paper, these earlier experimental results are considered in some detail, and new theoretical estimates are presented for the depth of the zone of local yielding around the crack. Finally the depth of yielding in a typical specimen, measured in a number of ways, is compared with the theoretical estimates and with those of other workers.

2. THE STRESS FIELD AROUND A RUNNING CRACK

An isochromatic pattern typical of those obtained with photoelastic coatings is shown in Fig. 1. The major part of this pattern may be described by a formula (derived from Westergaard's results (5)) for the isochromatic stress field near a stationary crack in an elastic material. Predictions from this formula, superposed upon experimentally obtained isochromatic patterns, are shown in Fig. 2. While the agreement is good, there are at least three ways in which the measured stress field differs from that predicted by the stationary elastic crack theory:

1. Shock waves ahead of the crack.

Typical results obtained by van Elst (3) show isochromatic patterns basically similar to those obtained in the current work, but with complex stress patterns behind the crack, and shock waves ahead of the crack. In our research (4) the only recorded shock waves appeared to have been initiated by the bolt gun used to start the brittle fracture.

2. Biaxial tensile stress region.

All the isochromatic patterns near moving cracks obtained in this research show a region ahead of the crack where the shear stress is lower than predicted by stationary crack theory. The theoretical solutions for stress fields about moving cracks (6, 7) all suggest that, as the crack speed increases to relativistic values, inertia effects become important, and the stress field changes from the static configuration; the faster the crack speed the greater is this change. The theoretical work of Yoshiki, Kanazawa and Itagaki (8) predicts that for high crack velocities the tensile stress normal to the crack, σ_y, will have a minimum value a short distance ahead of the crack tip. The lattice model of Gaus (9) forecasts that when strain energy is released by crack formation, a tensile stress pulse is generated and propagates away from the crack tip, to create a zone of biaxial tension ahead of the crack.

The wire resistance strain gauge results reported by Rolfe and Hall (10) and Cargill (11) indicate a lowering of shear strain ahead of the crack. The effect is seen only when the gauges are situated close to the fracture. Wells and Post (12) interpreted their results from strain birefringent models to show that the dynamic stress distributions in the vicinity of a crack approximated to the static distributions about slits. However, their results do show that the shear stress just ahead of the crack is slightly lower in the dynamic case than in the static case. The theoretical results of Akita and Ikeda (7) and Yoshiki, Kanazawa and Itagaki (8) are compared with the experimental results of Wells and Post (12), and Cargill (11), in Fig. 3, where σ_y/σ is plotted against x/a for various crack velocities. σ_y is the vertical stress in line ahead of the crack, σ is the applied stress, x is the distance ahead of the crack, and a is half the crack length. This figure shows that although all the results indicate a lowering of vertical stress ahead of the crack, the effect is largest for the strain gauge results from steel. The stress σ_y cannot be obtained from isochromatic recordings, but the shear strains obtained from strain gauge measurements are everywhere in agreement with the isochromatic patterns obtained in this research.

3. Plastic zones near the crack.

As a brittle crack crosses a steel plate plastic flow occurs near the fractured surface. The measured stress fields deviate from the theoretical stress fields close to the crack, within the zone where yielding has occurred (4).

3. THE SIZE OF THE PLASTIC ZONE

3.1. Theoretical Estimates

In this section a crack model will be proposed to enable predictions of the plastic zone depth to be made near a moving crack in mild steel. Consider a crack moving through a steel plate with "elastic yield zones" at its tips, in which the redistribution of stress due to yielding is neglected. Assume that the dynamic stress field is similar to the static case; this will be true unless the crack velocity is very high. The variation of the shear stress acting upon a series of small elements, δv, ahead of the crack, Fig. 4, may be calculated as the crack passes them. This has been done and is shown in Fig. 5; these curves give the variation of τ/σ with x/a along lines of constant y/a. If the crack is travelling at constant velocity v, the x/a axis may be given in terms of time t as x/a multiplied by a/v; the curves in Fig. 5 then become the stress histories of the elements of material, assuming that the half-crack length a is large compared to x and y, so that as the crack grows from $2a$ to $2a + 2x$ the stress field is unaltered. These stress histories show that, as the crack passes, the elements of material are subjected to a stress pulse; the closer the elements are to the crack the greater the pulse is in magnitude and the shorter it is in time.

There are now two ways in which the plastic zone depth can be found for this crack model, a graphical method (13), and an analytical method akin to that used by Hahn et al. (14).

In the graphical method the duration of the pulse in Fig. 5 is taken as the period of time that the stress is greater than three-quarters of its maximum value and the stress is assumed to remain at the maximum value for this time. Graphs of stress versus time of loading may be drawn, using the above approximation, one point being obtained from each curve of Fig. 5; Figs. 6 and 7 show the effects of varying crack velocity v, and varying applied stress σ, respectively. On these graphs the lower values of time correspond to smaller values of y; thus the closer a crack passes to an element of material, the larger is the stress upon it, and the shorter the duration of this stress. In mild steel the onset of plastic yielding is delayed, so that a given stress will be supported for a finite time before yielding occurs. Krafft and Sullivan (15), have measured the delay times before yielding at various stresses for a mild steel similar to that used in our experimental work reported later. Fig. 8 shows their results for supported stress versus delay time plotted upon Fig. 7; the vertical lines of Fig. 8 show values of constant y; each of these lines is obtained from one of the curves of Fig. 5. A 10 inch crack and a crack speed of 5×10^4 inches per second are chosen to illustrate the technique; obviously similar curves can be obtained for any desired crack length and velocity. Only those elements of material which are subjected to a stress pulse which lies above the Krafft and Sullivan line on Fig. 8 will yield. Thus for example on this model a crack of half length 10 inches, with each end extending at 5×10^4 inches per second, under an applied stress of 8 tons per square inch, will have a yield zone extending approximately 0.13 inches above and below it, at its tips. From a series of graphs similar to Fig. 8, but for varying crack length, Fig. 9 has been plotted; this shows the variation of the plastic zone depth with σ and a. The effect of varying crack velocity on Fig. 9 is shown in Fig. 10.

The analytical method for finding the depth of the plastic zone in our model makes use of the work of Hahn et al. (14). They derived a formula to enable the onset of yielding to be calculated for any arbitrary loading rate for which the variation of stress with time is known:

$$\sigma_o^{\,m} (0.5bfC)^{-1} \log_e \left(1 + \frac{fC\varepsilon p}{\rho_o'}\right) = \int_o^t \sigma^m \, dt$$

where;

σ_o is the stress corresponding to unit dislocation velocity,
m is a velocity parameter,
f is the fraction of dislocations contributing to yielding,
C is the dislocation multiplication parameter,
ρ_o' is the number of initially mobile dislocations,
εp is the value of the plastic strain corresponding to σ_Y, taken as 0.1%,
and b is the Burgers vector.

This equation may be used to predict the onset of yielding for the stress histories of material off the crack axis in our model in the following form:-

$$\left(\frac{\sigma_Y}{2\sigma}\right)^m \left(\frac{u}{a}\right) \left(\frac{\rho_o' + fC\varepsilon p}{\dot{\varepsilon}pfC}\right) \log_e \left(1 + \frac{fC\varepsilon p}{\rho_o'}\right) = \int_o^{x \gg a} \left(\frac{\tau}{\sigma}\right)^m d\left(\frac{x}{a}\right)$$

where u is the crack velocity and $\dot{\varepsilon}p$ is the plastic strain rate employed when σ_Y is measured. Let this integral be = I.

The variation of I with y/a has been computed and is shown in Fig. 11, for a value of m = 13. Using typical values for mild steel for yielding, $I = 3.6 \times 10^4$. Thus from Fig. 11 yielding extends to y/a = .015.

Errors in this model will be caused by neglecting the redistribution of stresses caused by plastic flow and by the high velocity of brittle crack propagation. These errors will be particularly serious for large rapidly spreading cracks. In the above model yielding is assumed to occur when the shear stress in the x-y plane satisfies certain conditions, and this shear stress might not be the largest when three dimensions are considered. This should not affect the depth of the yielded zone, but it will affect the distance ahead of the crack in which yielding occurs. In this model only central cracks in infinite plates have been considered, and no allowance for finite specimen width or end effects has been made.

3.2. Experimental Estimates

The depth of yielding below a brittle fracture surface, typical of those obtained in this research, was measured to test the validity of the above model. The depth of the plastic zone was determined by means of a technique similar to that used by Orowan (16). Successive layers of steel were etched from the fracture surface, and back reflection X-ray photographs were taken of the surface between each etch. The depth of material removed was measured by a point ended micrometer. Fig. 12 shows the resulting X-ray photographs, the effect of plastic flow being a blurring of the spots on the X-ray film. This technique put a lower limit on the depth of plastic flow as 0.075 inches. Because of the roughness of the fracture surface accurate fixing of the plane of the fracture surface was impossible, but this technique showed that plasticity extended at least 0.075 inches below the bottom of a valley in the chevron pattern.

The depth of the plastic zone was estimated in another way by comparing the theoretical and experimental isochromatic patterns about a crack (4). This comparison suggested a depth of 0.15 inches. This will be an overestimate because the birefringent coating measures surface strains, and even though in this specimen the shear lips were very small, there will probably have been more yielding on the plate surface than in the body of the specimen.

4. DISCUSSION

The most important feature of this paper is the model which has been developed to account for the depth of the plastic zones around a brittle crack. The amount of yielding determines the value of the effective surface energy in the thermodynamic criterion for fracture, and in semi-brittle materials this is far larger than the true surface energy. In this discussion the experimental plastic zone depths will be compared with the predictions from a number of theories and models. The experimental estimates suggested a depth of about 0.1 inches for a crack of half-length a = 8", propagating under an applied stress of 11.2 t.s.i., at a speed of about 2,000 ft/sec.

Approximate values for the extent of yielding near moving cracks can be obtained by substituting the dynamic yield stress for the static yield stress, in formulae for the plastic zone size near stationary cracks. A dynamic yield stress of 60 t.s.i. is taken as typical of mild steel subjected to the strain rates which occur close to a brittle crack. The simplest estimate for plastic zone depth is made by assuming that the steel will yield wherever the shear stress upon it is greater than the dynamic yield stress in shear. This method gives a value for the depth of 0.19 inches. On the other hand Irwin (17) has defined a plastic zone correction factor, based upon the distance from the crack tip along its axis where the maximum principal stress equalled the tensile yield stress. The distance ahead of the crack where this condition is fulfilled is 0.14 inches. If this yield criterion is applied all around the crack, the plastic zone depth perpendicular to the axis of the crack would be 0.21 inches. Similarly Dixon's (18) approach predicts a kidney shaped plastic zone at the crack tip, the depth of the plastic zone being 0.20 inches, whereas Dugdale's (19) analysis forecasts yielding 0.48 inches ahead of the crack tip, the yielded zone being thin and tapered to a fine point. Goodier and Field (20) combined the dynamic analysis of Craggs (6) with Dugdale's (19) crack model, and, neglecting non-singular terms in the stress analysis, found that the length of plastic zone ahead of the crack was unaffected by crack velocity. In order to use this analysis to predict the plastic zone size near a brittle crack, the pressurized segment opening the crack is assumed to act upon the whole crack length, so that the shear stress field near the crack is equivalent to that of a crack under biaxial tension. Applying this analysis to our typical specimen, the plastic zone extends 0.2 inches ahead of the crack tip.

The application of dislocation dynamics to the problem of yielding at high rates of strain makes it possible to predict plastic zone depths without assuming a unique value for the dynamic yield stress as we have done so far. In this way Hahn, Gilbert and Reid (14) have been able to calculate the onset of yielding for any arbitrary loading rate for which the variation of stress with time is known. Their calculated delay times agree well with the experimental results of Krafft and Sullivan (15). They adopted a crack model with a circular plastic zone which supports no load, and assumed that yielding in advance of the crack is analogous to yielding in uniaxial tension and is not retarded by plastic constraint.

On this model the plastic zone should have a radius of 0.8 inches in our typical specimen. The assumption of a circular plastic zone need not be made if elements off the crack axis are considered in the way outlined i Section 3.1 of this paper. Our model forecasts a plastic zone depth of 0.28 inches when solved graphically, and a depth of 0.12 inches when solved analytically. No estimate was made of the length of the zone ahead of the crack in this case.

All the above estimates are listed in Table 1. All of the theoretical models forecast too large a value for the plastic zone depth probably because none of them make any allowance for the degeneration of the stress field at high crack velocities. The model which gives the most reasonable result is the analytically solved model described in thi paper.

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Table 1

Comparison of Experimentally Determined and Theoretically Calculated Plastic Zone Depths about a Running Brittle Fracture

Method of Estimation	Over or under Estimate	Plastic Zone Depth inches	Length inches
X-ray results	Under	0.075	
Breakdown of elastic analysis	Over	0.15	
Theoretical predictions			
Simple shear model		0.19	0.12
Irwin's model (17)		0.21	0.14
Dixon's model (18)		0.20	0.12
Dugdale's model (19)) slim tapering			0.48
Goodier and Field's model) yield zone			0.20
Hahn et alii model (14)		0.8	0.8
Model proposed in this paper			
Graphical method (13)		0.28	
Analytical method		0.12	

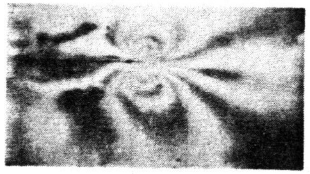


Fig. 1. Typical isochromatic pattern about a brittle crack moving through a mild steel plate under a stress of 11.2 t.s.i. The photoelastic coating is 2mm. thick. The area shown is 2 inches x $3^1/4$ inches.

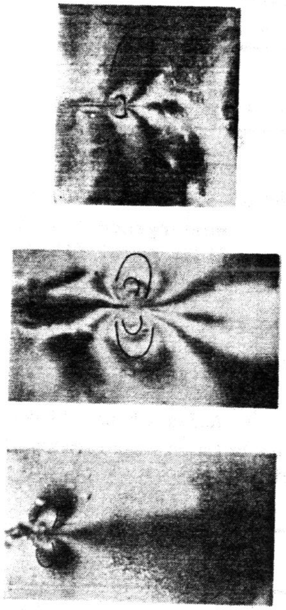
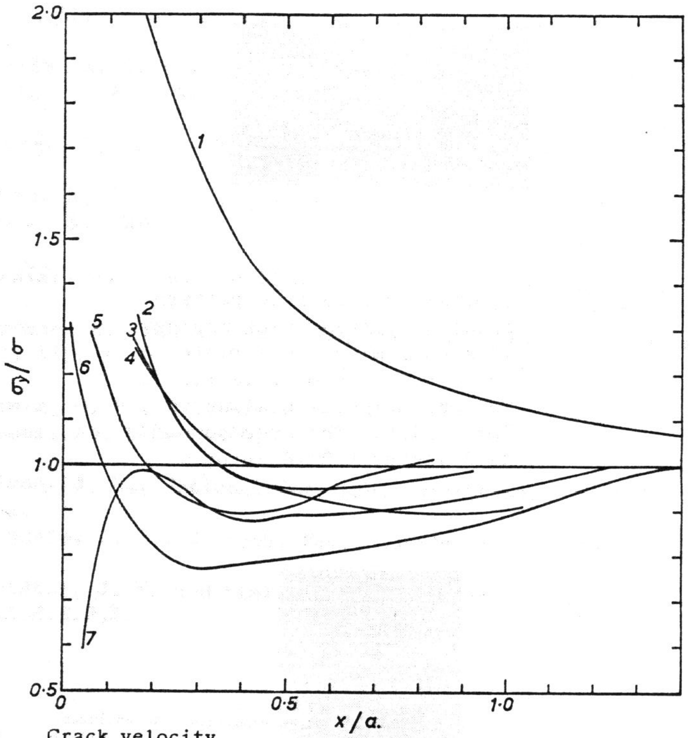


Fig. 2. Theoretical isochromatic lines superposed upon typical experimental isochromatic patterns.



Curve Number	Crack velocity / Velocity of Sound	Source	Method
1	0	Westergaard (1939)	Theory
2	0.3	Akita et al (1959)(a)	Theory
3	0.3	Yoshiki et al (1961)	Theory
4	0.35	Wells and Post (1958)	Birefringent resin
5	0.35	Yoshiki et al (1961)	Theory
6	0.35	Cargill (1963)	Strain gauges on Steel plates
7	0.4	Akita et al (1959)(a)	Theory

Fig. 3. Comparison between experimentally and theoretically determined values of the vertical stress ahead of moving cracks.

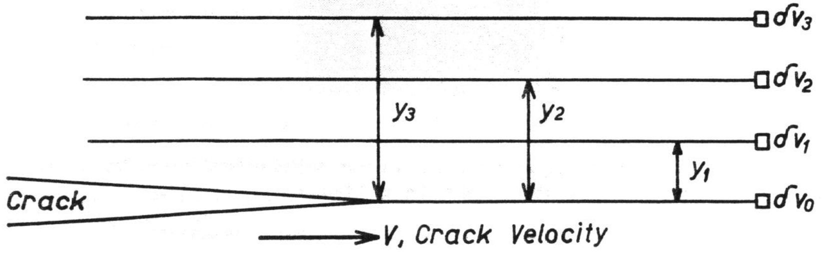


Fig. 4. Series of small elements ahead of a moving crack.

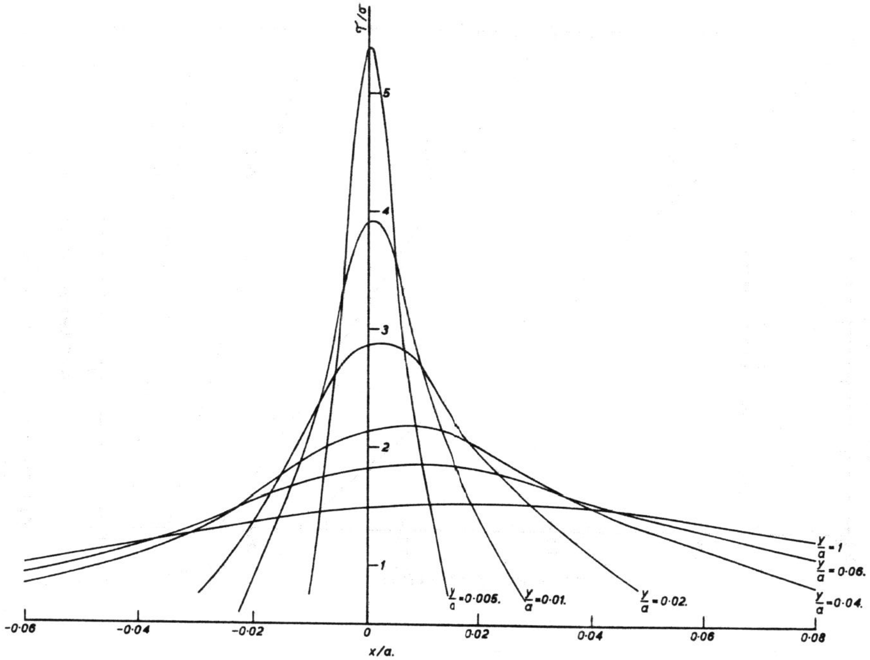


Fig.5. The variation of maximum shear stress along lines of constant y near crack tip.

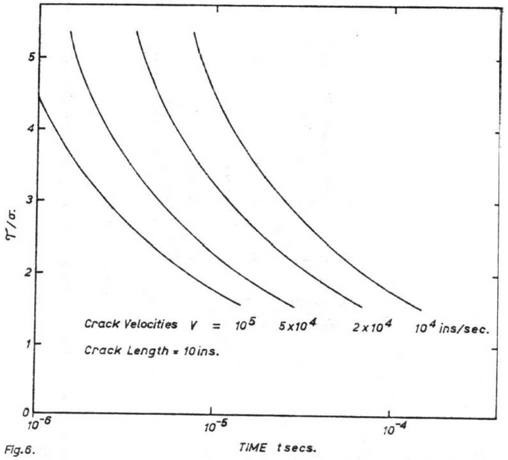


Fig.6.
The variation of the magnitude of the shear stress pulse on elements of material near a crack with the duration of the pulse for various crack velocities

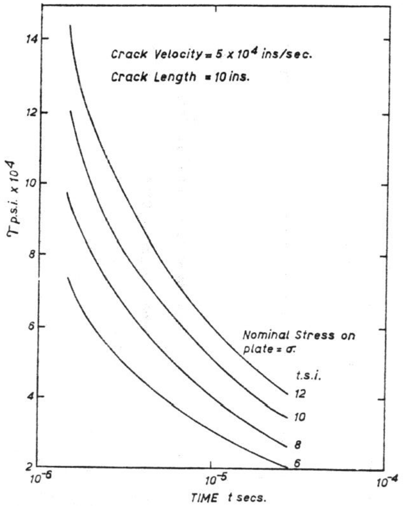


Fig 7.
The variation of the magnitude of the shear stress pulse on elements of material near the crack with the duration of pulse for values of applied stress.

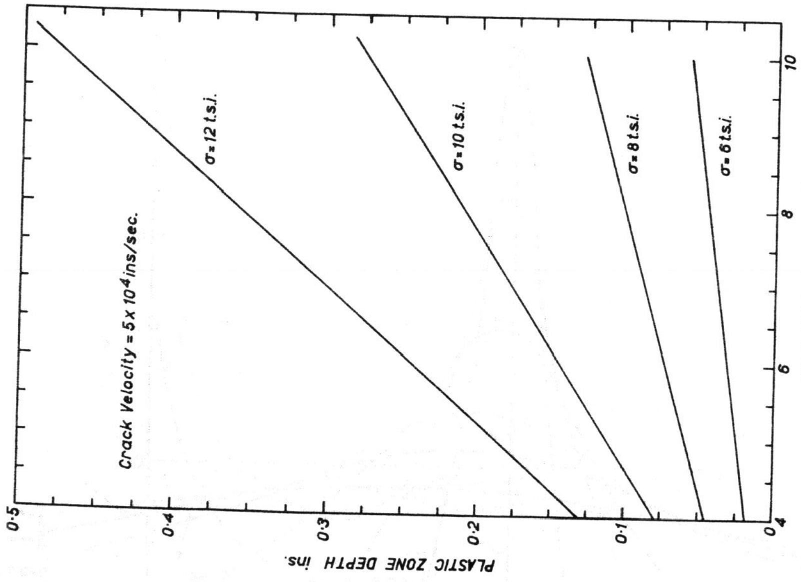


Fig.9. The variation of the plastic zone depth with applied stress and crack length.

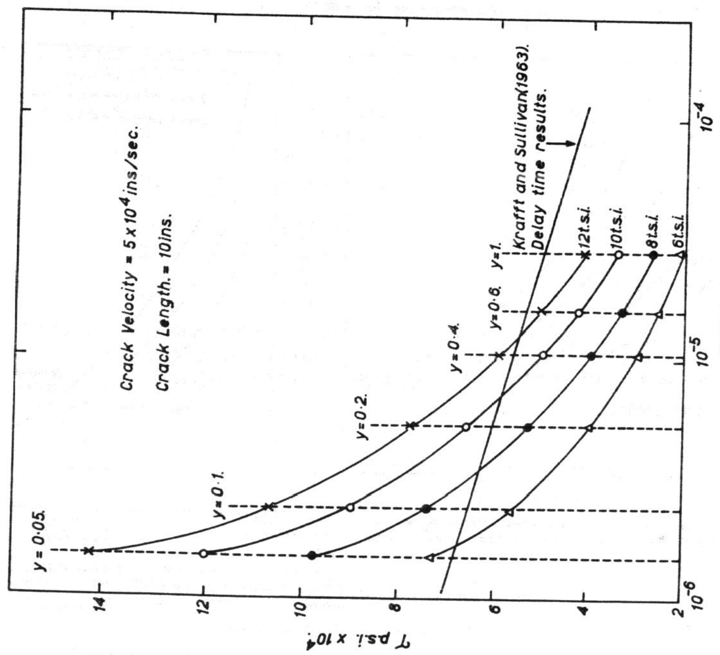


Fig. 8. Krafft and Sullivan's (1963) delay time results superposed on Fig. 7.

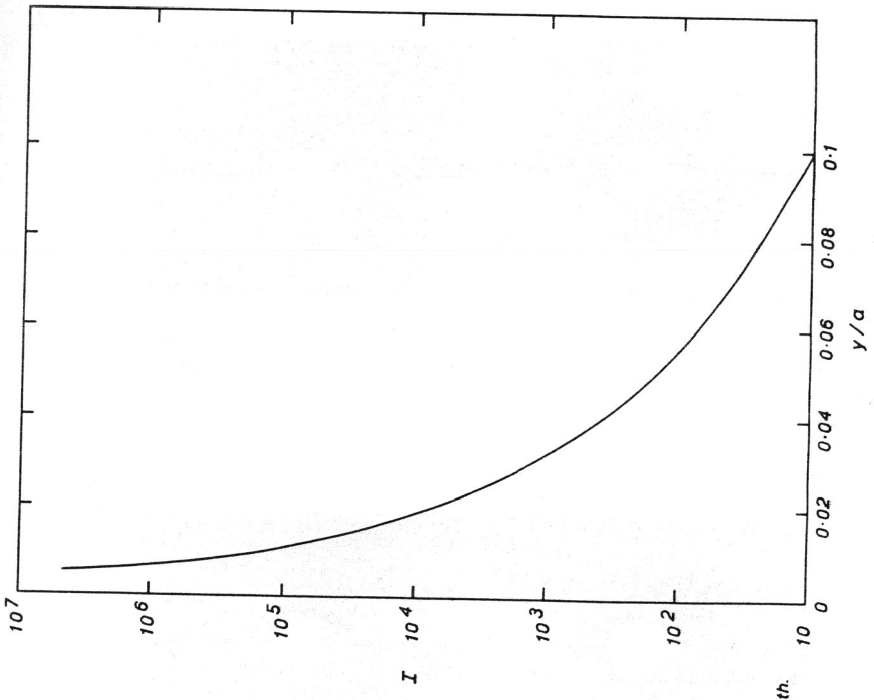


Fig. 11. The variation of I with perpendicular distance from the crack.

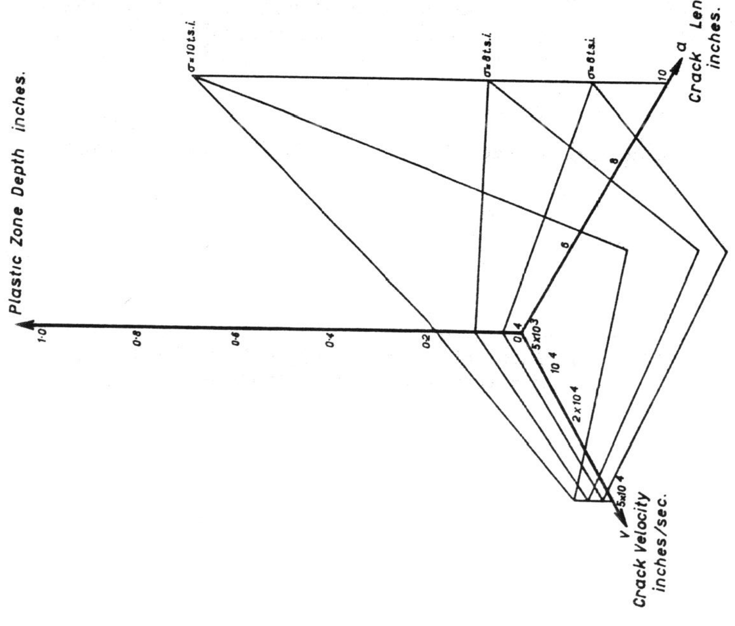
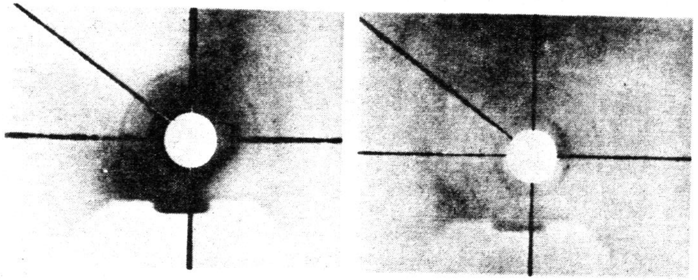
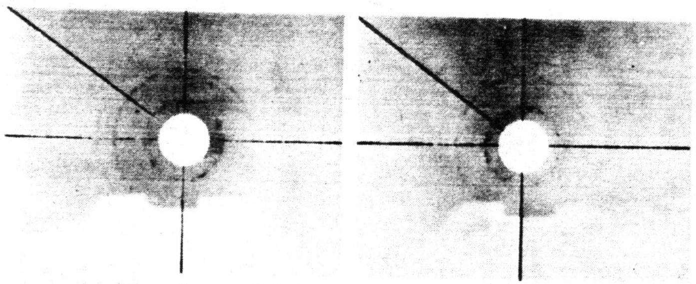


Fig. 10. The variation of plastic zone depth with crack velocity, applied stress, and crack length



Fracture surface

0.03 inches removed



0.06 inches removed

0.075 inches removed

Fig. 12. Back reflection X-ray photographs of the fracture and sections beneath it