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# THE EFFECT OF THE NUMBER OF DEFORMATION MODES ON THE DUCTILE-BRITTLE TRANSITION CRITERION IN SOLIDS

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## ABSTRACT

A necessary condition for the deformation of a polycrystalline solid without the formation of cracks is that the grains can undergo a general strain; for this to be possible, five independent slip systems are required. Even when more than this limiting number are available, the material can still be brittle, but recent theories of the ductile-brittle transition in such a situation have not taken into account the number of available slip systems. An analysis has therefore been undertaken of the effect of the number of slip systems on the ductile-brittle transition criterion, and the results compared with those obtained from the earlier theories. It is shown that, in some situations, the value of the fracture surface energy obtained from the transition criterion may be modified considerably.

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## INTRODUCTION

It is usually assumed[1] that a necessary condition for the
ation of a polycrystalline solid without the formation of
is that the grains can undergo a general strain. If this
in shape is to be produced by slip, then five independent
ystems are required[2]; thus as a general rule it is found
hen five independent systems are not available, then cracks
rmed during the plastic deformation of a polycrystalline
ate. However, it is well known that in some cases even if the
ystems are available, the material can be brittle and the
entred cubic structure is a very important example. Theories
ductile-brittle transition in such situations have been put
d notably by Stroh, Cottrell and Petch[3-8], particular
nce being paid to the behaviour of mild steel. These theories
take into account the number of available slip systems, as
ted by Meakin and Petch[9], who however point out that a
se in the number of systems will increase the probability of
e fracture. Accordingly as part of a general theoretical
of the way in which crystal orientation and preferred
ation affects the mechanical properties of solids, an analysis
en undertaken (Section 2) of the effect of the number of slip
s on the ductile-brittle transition criterion in solids.

## THEORETICAL ANALYSIS

### 2.1. The Lower Yield Stress Theory of Armstrong et al[10]

Theories of the ductile-brittle transition in solids are
tely related to those of the lower yield stress $\sigma$ of polycrystalline
. and it is well known that this obeys the relationship:

$$\sigma = \sigma_o + kl^{-\frac{1}{2}} \quad \text{.... ............} \quad (1)$$

l is the grain diameter, $\sigma_o$ and k being constants for a given
testing conditions. Early theories [6,11,12] of the lower
stress were based on a model, whereby the spread of plasticity
lower yield point occurred by the concentrated shear stress
of a slip band in one grain initiating slip in a neighbouring
It was assumed that in every grain the slip plane and slip
ion both made an angle of 45° with the applied tensile stress,
us the effects of the number of slip systems and the differing
ations of the crystals which form a polycrystalline aggregate
eglected. However, Armstrong et al[10] have extended the theory
case of a random polycrystalline aggregate by using an averaging
re, and Wilson and Chapman[13] and Smith and Worthington[14]
nsidered the case where some degree of preferred orientation

rmstrong et al assumed that plasticity spreads by a slip band





in one grain operating a dislocation source in the next grain at a distance r directly ahead of the band. The shear stress at such a point is:

$$(\tau - \tau_i)\,(l/4r)^{\frac{1}{2}} \quad \text{..............} \quad (2)$$

acting on a plane parallel to the original band and in a direction parallel to the original slip direction[15], where $\tau$ is the applied shear stress acting on the band and $\tau_i$ is the shear stress opposing the motion of an unlocked dislocation in the band. Since the orientation of the grains are random, on the average, yielding will propagate when (2) reaches the average shear stress for initiation of slip. If $\tau_c$ is the critical resolved shear stress for this process, then on average yield will propagate when

$$(\tau - \tau_i)\,(l/4r)^{\frac{1}{2}} = m\tau_c/2 \quad \text{......} \quad (3)$$

or

$$\tau = \tau_i + m\tau_c\, r^{\frac{1}{2}}\, l^{-\frac{1}{2}} \quad \text{......} \quad (4)$$

where m is an average orientation factor. To obtain m, we simply average $m_f$ for a collection of randomly oriented free crystals, where $m_f$ relates the axial tensile stress $\sigma_s$ applied to a single crystal to the shear stress $\tau_s$ on the most favourably oriented slip plane by $\sigma_s = m_f\tau_s$. For simplicity, we restrict ourselves to this Sachs[16] average which gives the condition for operation of a single source and do not use the Taylor[17] average, which allows for deformation on less favourably oriented planes to maintain continuity. The tensile yield stress will be $m\tau$, the average over the randomly oriented polycrystalline aggregate, i.e.

$$\sigma = m\tau_i + m^2\tau_c\, r^{\frac{1}{2}}\, l^{-\frac{1}{2}} \quad \text{......} \quad (5)$$

Hence, when interpreting experimental results by relation (1), $\sigma_o$ is associated with $m\tau_i$ and k with $m^2\tau_c r^{\frac{1}{2}}$.

If m = 2, the model becomes that considered by Petch[11] and Codd and Petch[12], where in every grain the slip plane and direction both make an angle of 45° with the applied tensile stress. Relation (5) then reduces to

$$\sigma = 2\tau_i + 4\tau_c\, r^{\frac{1}{2}}\, l^{-\frac{1}{2}} \quad \text{.......} \quad (6)$$

when, in the interpretation of experimental results by relation (1), $\sigma_o$ is associated with $2\tau_i$ and k with $4\tau_c r^{\frac{1}{2}}$. In recent years, when discussing experimental results on polycrystalline body-centred and face-centred cubic metals and alloys, most workers have used this simple interpretation of $\sigma_o$ and k when considering for example the effects of temperature, strain rate and alloy composition. However, as will be indicated later, this can lead to difficulties when considering the ductile-brittle transition criterion.

## 2.2. The Ductile Brittle Transition Criterion

Cottrell(6,7) has shown that if n edge dislocations, each of Burgers vector a, form a wedge shaped crack, then it will propagate indefinitely under an applied tensile stress p, provided that

$$pna > 2\gamma \qquad \text{.............} \qquad (7)$$

where $\gamma$ is the total energy expended in exposing unit area of the crack faces. In developing this relationship Cottrell(6) used a rather specific model of crack formation, where dislocations on intersecting slip planes at 45° to the tensile stress axis coalesced to form the wedge shaped crack. However, as indicated later by Cottrell(7) and Meakin and Petch(9), relation (7) should have general applicability irrespective of the detailed mechanism by which the crack is formed.

In determining na, Cottrell(6,7) notes that a slip band of length l subject to a shear stress $\tau$ and having a friction stress $\tau_i$ can undergo a shear displacement $(\tau - \tau_i)l/\mu$ and that this approximately determines na. Thus since $\sigma = 2\tau$ for Cottrell's model, the fracture criterion (7) becomes

$$\sigma \left\{ \frac{\sigma}{2} - \tau_i \right\} \frac{l}{\mu} > 2\gamma \qquad \text{..........} \qquad (8)$$

For the case when fracture occurs at the lower yield stress, Cottrell uses relation (6) for the value of the lower yield stress and obtains the following well-known relationship, which was derived by Petch(8) independently

$$\frac{\sigma\, k l^{\frac{1}{2}}}{4\mu} = \gamma \qquad \text{...........} \qquad (9)$$

from which $\gamma$ can be determined, knowing $\sigma$, k and l at the transition point. This is the criterion which defines the ductile-brittle transition, but by using relation (6) in deriving it Cottrell has not taken into account the differing orientations of the various grains which form the polycrystalline aggregate. To extend his theory and take this factor into account, relation (5) must be used for the lower yield stress instead of (6), when (8) then gives

$$\underset{(I)}{\frac{\sigma\, k l^{\frac{1}{2}}}{4\mu}} + \underset{(II)}{\left(1 - \frac{2}{m}\right)\frac{\sigma\sigma_o l}{4\mu}} = \gamma \qquad \text{......} \qquad (10)$$

as the modified criterion which defines the ductile-brittle transition; $\sigma_o$ and k are here, as in (9), the experimentally measured values for the lower yield parameters.





When m = 2, relation (10) reduces to (9) as expected, and it is seen that in determining the value of $\gamma$ from the modified ductile-brittle transition criterion, the expression for $\gamma$ contains a term (II) in addition to that (I) obtained by Cottrell; as indicated above, this extra term arises as a consequence of considering the effect of the differing orientations of the various grains which form the polycrystalline aggregate.

## 3. DISCUSSION

In the previous section it has been shown, by application of the averaging procedure proposed by Armstrong et al(10), how the effect of the differing orientations of the grains which form a polycrystalline aggregate (and consequently the multiplicity of deformation modes), can be incorporated into the ductile-brittle transition criterion developed by Cottrell(6,7).

So far in the present paper, it has been assumed that fracture is initiated by a slip mechanism; however, for metals and alloys this may not always be true. Although situations arise where slip definitely appears to initiate fracture (18-21), there is equally good evidence that twinning processes are responsible in some other situations(19,22,23). In the case of ionic solids, cleavage cracks often form where slip bands meet grain boundaries or where one band meets another(24-26). Assuming for the present that fracture is always nucleated by slip processes at the ductile-brittle transition point, (10) then being the governing criterion, there immediately arises the question as to what conditions must be operative for us to neglect the modifying term (II) and hence simply use the experimentally measured values of the parameters $\sigma_o$ and k in the fracture criterion.

Now from relation (10), the ratio of the two terms on the left hand side is

$$\frac{II}{I} = \left[1 - \frac{2}{m}\right]\frac{\sigma_o}{k l^{-\frac{1}{2}}} \qquad \text{............} \qquad (11)$$

For face-centred cubic metals which have 4 {111} slip planes and 3 <110> slip directions in each plane (i.e. 12 modes in all), the value of m using the Sachs(16) average is 2.2. In the case of body-centred cubic metals when slip occurs on the 6 {110} planes with 2 <111> slip directions in each plane, one expects the Sachs average to be somewhat similar. This deformation system has been observed by Hull(27) in 3% silicon iron single crystals at -196°C, but the value of m as obtained from measured values of $\sigma_o$ and $\tau_i$(28) appears to be about 3, i.e. closer towards the Taylor(17) average, which allows for deformation on less favourably oriented planes to maintain continuity. A more general situation in body-centred cubic structures is when slip occurs on any plane provided the slip direction is always <111>, as in 3% silicon iron at room temperature[27]; in such a situation as this, m should not be much greater than 2 and the value 2.4 has been obtained experimentally[28]. With such a value for m, II/I in relation (11) will be less than 20% if $\sigma_o < kl^{-\frac{1}{2}}$ and this will be the case for a conventional mild steel having a fine grain size ($l \sim 0.01$ mm) when tested around the transition temperature ($\sim -196$°C). Even for a steel with a coarse grain size ($l \sim 0.2$ mm), II/I should not be greater than about 50%. Thus in the case of mild steel, for which the earlier theories were primarily developed, the simple fracture criterion (9) is probably adequate; consideration of the second term (II) in the modified criterion (10) will only give slightly greater values of $\gamma$.

However, as seen by inspection of (11), the effect of the differing orientations of the grains which form a polycrystalline aggregate will be important (i.e. II becomes large) when $\sigma_o \gg kl^{-\frac{1}{2}}$. Such a situation exists in many of the transition metals [29,30]; $\sigma_o / kl^{-\frac{1}{2}}$ can typically be 10 or more even for a fine grain material. Then II/I in relation (11) will be almost 2 (taking m = 2.4) and can be even higher for coarse grain size material. Values of $\gamma$ obtained from a consideration of the simple relation (9) will then be far too low. In the extreme case when $\sigma_o \gg kl^{-\frac{1}{2}}$, criterion (10) reduces to

$$\frac{m(m-2)\tau_i^{\,2} l}{4\mu} = \gamma \quad \ldots\ldots\ldots\ldots\quad (12)$$

Thus low values of $\tau_i$, the lattice friction stress, l, the grain size and m (i.e. a high multiplicity of deformation modes) are all conducive to ductility.

This discussion has so far been based on slip processes being responsible for crack formation; as indicated earlier, however, it appears that in some situations cracks can be formed as a result of mechanical twinning, the detailed mechanisms having been reviewed by Hull[31]. We now examine how the ductile-brittle transition criterion is modified. There are two cases to discuss[32]:

(a) Where the critical event in the fracture process is the growth of a crack which has been nucleated by twinning processes. In such cases twins are formed at stress levels lower than the fracture stress, but these levels are not high enough for the energetic condition for crack growth to be satisfied. Such situations are characterized by the presence of mechanical twins away from the fracture surfaces.

(b) Where the critical event in the fracture process is the onset of mechanical twinning. In such cases the energetic condition for crack growth is more than satisfied at the applied stress level required for twinning. Such situations are characterized by the





absence of mechanical twins away from the fracture surfaces.

For case (a), na in relation (7) should then be $[(\sigma/2 - \tau_i(tw)]\,1/\mu$ where $\tau_i(tw)$ is the friction stress which opposes the motion of each twinning dislocation (except perhaps the leading one) from which a twin is formed. Unfortunately $\tau_i(tw)$ is a quantity about which very little is known, and thus alternatively na has been estimated by actually measuring twin thicknesses and using na $\sim$ t tan $\theta$ where t is the average twin thickness and $\theta$ is the angle of shear[31,33]. As pointed out elsewhere however[28], this procedure is not very satisfactory as one is measuring the thicknesses of twins which have had the opportunity to relax the local stresses near their tips by slip or more twinning; thus one obtains too large a value for the thickness of an unrelaxed twin, which is the important parameter in initiating fracture. Fortunately we do know, at least in the case of ferritic materials, that $\tau_i(tw)$ is smaller than $\tau_i$ for slip; thus proceeding to the extreme case and putting $\tau_i(tw)$ to be zero, the fracture criterion (7) becomes

$$\frac{\sigma^2 l}{4\mu} = \gamma \quad \ldots\ldots\ldots\ldots\ldots\ldots\quad (13)$$

and in determining $\gamma$, the experimentally observed values for $\sigma$ and l at the transition point can be used, without needing to know the value of m.

For case (b), it is necessary to examine the criterion for the onset of twinning. Experimental work[28, 34-36] suggests that some localized slip is required to form a twin; and very approximately the stress for twinning can be expressed as

$$\sigma(tw) = 2\tau_i + k(tw)l^{-\frac{1}{2}} \quad \ldots\ldots\quad (14)$$

Accordingly for fracture at the lower yield stress we have

$$2\tau_i + k(tw)l^{-\frac{1}{2}} = \sigma_o + kl^{-\frac{1}{2}} \quad \ldots\quad (15)$$

or

$$\left(1 - \frac{2}{m}\right)\sigma_o = [k(tw) - k]l^{-\frac{1}{2}} \quad \ldots\quad (16)$$

Of course in this case $\gamma$ cannot be obtained from experimental measurements at the transition point, and we reach the general conclusion that only in the case of slip induced cleavage is it necessary to have a knowledge other than the experimentally measured values of $\sigma_o$, k and l at the transition point in determining $\gamma$; in this special case m must be known.

Finally, it is worth emphasizing that throughout this paper, whether slip or twinning causes fracture, it has been assumed as indeed also in the original model[6], that all the dislocations in the slip or twin band enter the crack and assist it to grow. This may not necessarily be the case, but an analysis allowing for this effect is beyond the scope of the present paper; in this context the present analysis will give an upper bound to values of $\gamma$ obtained from the transition criterion. All one can do at this stage is to proceed empirically and let a proportion $\alpha(<1)$ of the dislocations enter the crack. Then relation (10) becomes

$$\frac{\alpha\sigma k l^{\frac{1}{2}}}{4\mu} + \alpha\left(1 - \frac{2}{m}\right)\frac{\sigma\sigma_o l}{4\mu} = \gamma \qquad \ldots\ldots\ldots \qquad (17)$$

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