A-9 RATE OF GROWTH OF FATIGUE CRACKS CALCULATED FROM THE THEORY OF INFINITESIMAL DISLOCATIONS DISTRIBUTED ON A PLANE

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ABSTRACT

The Bilby Cottrell and Swinden crack theory is applied to the problem of the growth of fatigue cracks. It is found that if a total critical displacement is adopted as the fracture criterion, the theory leads to observed growth laws.

INTRODUCTION

The purpose of this paper is to apply the theory of continuously distributed infinitesimal dislocations on a plane to the problem of the rate of growth of fatigue cracks. The concept of infinitesimal dislocations appears to have been originated by Eshelby (5), who used it in a calculation of the "width" of a discrete dislocation in a periodic lattice. The first detailed development of continuously distributed infinitesimal dislocations was given by Leibfried (9, 10). Head and Louat (7) later pointed out that Muskhelishvili's work (14, 15, 16) contains the general solution to the problem of a continuous distribution of dislocations on a plane. This fact also was noted and used by Leonov and Shvaiko (12). The general solution was used by Bilby, Cottrell and Swinden (1) and by Bilby, Cottrell, Swinden and Smith (2) to investigate theoretically the plastic yielding in front of sharp notches in metal specimens subjected to shear stresses. We ourselves (19) employed the general theory in considering slippage on earthquake faults.

The problem of the opening of cracks by tensile stresses also can be treated by infinitesimal dislocation theory. Such studies have been carried out by Priestner and Louat (17), and by Bullough (3). These investigators considered only cracks in material which could support infinite stresses in the region immediately ahead of a crack. Bilby, Cottrell and Swinden (1) pointed out that the results of their analysis of a freely slipping crack in a material of finite shear strength also constitute the solution to the problem of a crack opened by tension in a material with a finite tensile strength. Their solution is equivalent to one found by Dugdale (4) by another method.

We wish to show in this paper that the crack theory of Bilby, Cottrell and Swinden (1) (called the BCS theory hereafter) can be developed further to lead to the crack growth law found from fatigue studies.

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REVIEW OF THE BILBY, COTTRELL AND SWINDEN SOLUTION TO THE FREELY SLIPPING CRACK PROBLEM

Consider the freely slipping crack of Figure 1a which is acted upon by an applied shear stress S. The crack extends an infinite distance perpendicular to the plane of the figure. It is assumed that the crack itself cannot support a shear stress and that the material in the plane of the crack beyond the ends of the crack ($x = \pm a$) can support only a finite shear stress σ.

The slip produced on the plane of the crack by an applied shear stress S can be described by a distribution function B(x) of infinitesimal dislocations (Figure 1b). The distribution function B(x) is so defined that $B(x)\delta x$ represents the total length of the Burgers vectors of all the dislocations lying between x and $x + \delta x$. In Figure 1 the distribution function extends beyond the edges of the crack out to $x = \pm b$ because the material there can support only a finite shear strength. If the material containing the crack could support infinite stresses the dislocation distribution would not extend beyond the edges of the crack.

If a dislocation distribution occurs only in one region on a plane, such as in the case in Figure 1 where the dislocations are confined to the region $-b \le x \le b$, the distribution function B(x) is given by the equation

$$B(x) = (2\alpha/\pi\mu)(b^2-x^2)^{\frac{1}{2}} \int_{-b}^{b} \frac{\tau(x')dx'}{(x'-x)(b^2-x'^2)^{\frac{1}{2}}} \quad (1)$$

where μ is the shear modulus, $\alpha = 1$ if the dislocations are screw dislocations but $\alpha = 1-\nu$, where ν is Poisson's ratio, if the dislocations are edge dislocations, and τ is the stress on the plane of the crack which arises from the dislocations themselves. For the freely slipping crack problem obviously τ must equal -S in the region $-a < x < a$ and must equal σ-S in the region $a < |x| \le b$.

The distance b must have a value such that the following equation is satisfied

$$\int_{-b}^{b} \frac{\tau(x')dx'}{(b^2-x'^2)^{\frac{1}{2}}} = 0 \quad (2)$$

The solution of equations (1) and (2) found by Bilby, Cottrell and Swinden for the freely slipping crack is

$$B(x) = \sigma(2\alpha/\pi\mu)\log\left|\frac{x(b^2-a^2)^{\frac{1}{2}} + a(b^2-x^2)^{\frac{1}{2}}}{x(b^2-a^2)^{\frac{1}{2}} - a(b^2-x^2)^{\frac{1}{2}}}\right| \text{(for } |x| \le b) \quad (3)$$

$B(x) = 0$ (for $|x| \ge b$)

and b is given by

$$b = a/\cos(\pi S/2\sigma) \quad (4)$$

If σ approaches ∞ this solution reduces to

$$B(x) = (2\alpha Sx/\mu)/(a^2 - x^2)^{\frac{1}{2}} \quad (5)$$

and $b = a$.

CRACK UNDER TENSION

A crack which is opened up by a tensile stress T also can be considered equivalent to a distribution of infinitesimal dislocations (1, 3). The infinitesimal dislocations are edge dislocations and their Burgers vector are perpendicular rather than parallel to the plane of the crack. Equations (1) through (5) thus still apply if $\alpha = 1 - \nu$ and the tensile stress T replaces the shear stress S.

RATE OF GROWTH OF FATIGUE CRACKS

Bilby, Cottrell and Swinden showed that their theory applied to the propagation of cracks leads to the same equation as the Orowan-Irwin modification of the Griffith criterion for the stress required to propagat[e] a crack in a material which can partially deform plastically. This equation is:

$$\text{Stress} \approx (E\gamma/a)^{\frac{1}{2}} \quad (6)$$

where E is a modulus, a is the half-width of the crack, and γ is a constant. In the Griffith theory γ is equal to the surface energy of the solid and in the Orowan-Irwin theory γ is identified with the plastic work which is dissipated in opening up the crack. Bilby et. al. obtained a quantity γ from their fracture criterion, which states that the displacement

$$D(a) \equiv \int_a^b B(x)dx$$

at the tip ($x = a$) of the crack cannot exceed some critical value, say D*. They find that when $T \ll \sigma$, $\gamma = \sigma D^*$, where σ is the finite stress defined previously. Physically, the constant γ of the BCS theory has nothing in common with the constant γ of the Orowan-Irwin theory. We will show in a later section that another constant γ can be obtained from the BCS theory that does arise from a dissipation of energy similar to the plastic energy dissipation of the Orowan-Irwin theory. However the magnitude of this constant is always so small that the BCS theory cannot be turned into an Orowan-Irwin type theory of fracture. This failure encourages us to retai[n] the critical displacement criterion adopted by Bilby et. al.

For cracks cyclically loaded it would be natural to generalize the BCS static fracture criterion to the following: a crack will increase its length whenever the sum of all the cyclic displacements at the crack tip exceeds the critical value D*, and a growing crack will stop extending itself only when the total displacement at its tip is less than D*. (By the term "sum" is meant the sum of the absolute values of the displacements.)

The dislocation distribution around a crack subjected to cyclic stressing is found easily (19) if the half-width a of the crack remains constant. Suppose we let $B(x, T, b_0)$ and $b_0(T)$ represent the dislocation distribution B(x) and the distance b given by Equations (3) and (4) for the initial (tensile) stress cycle. Let T_m represent the maximum tensile stress which is applied. On reverse loading (T decreasing in value) the dislocation distribution is equal to $B_0(x, T_m, b_0(T_m)) + B_1(x, T, b_1(T))$, where

$$B_1(x,T,b_1(T)) = -(4\alpha\sigma/\pi\mu) \log \left| \frac{x(b_1{}^2-a^2)^{\frac{1}{2}} + a(b_1{}^2-x^2)^{\frac{1}{2}}}{x(b_1{}^2-a^2)^{\frac{1}{2}} - a(b_1{}^2-x^2)^{\frac{1}{2}}} \right| \qquad (7)$$

$$(\text{for } |x| \le b_1)$$

$$B_1(x,T,b_1(T)) = 0 \qquad (\text{for } |x| \ge b_1)$$

and

$$b_1(T) = a/\cos(\pi\{T_m - T\}/4\sigma) \qquad (8)$$

Upon stressing the sample again in the original direction (T increasing) the dislocation distribution becomes $B_0(x, T_m, b_0(T_m)) + B_1(x, T^*, b_1(T^*)) + B_2(x, T, b_2(T))$, where $T^* = 0$ if the cyclic stressing is done between the limits of $0 \le T \le T_m$.[2]

Here

$$B_2(x,T,b_2(T)) = (4\alpha\sigma/\pi\mu)\log \left| \frac{x(b_2{}^2-a^2)^{\frac{1}{2}} + a(b_2{}^2-x^2)^{\frac{1}{2}}}{x(b_2{}^2-a^2)^{\frac{1}{2}} - a(b_2{}^2-x^2)^{\frac{1}{2}}} \right| \qquad (9)$$

$$(\text{for } |x| \le b_2)$$

and

$$b_2(T) = a/\cos(\pi\{T - T^*\}/4\sigma \qquad (10)$$

[2] If the crack is stressed in push-pull testing where the stress is cycled between limits $-T_m < T < T_m$ the analysis becomes rather complex. The total dislocation distribution at $T = -T_m$ cannot be merely the negative of the distribution at $T = T_m$ because at no point can the distance the crack gaps open be permitted to have a negative value. Obviously the analysis of a stress cycle taken between $0 \le T \le T_m$ will approximate that for the stress taken between $-T_m \le T \le T_m$ since the major portion of the crack will be closed up during that part of the cycle where $T < 0$.

Repetition of the stress cycle gives further dislocations distribution functions B_3, B_4, . . . B_{2n}, B_{2n+1}, . . . which are the same as those given in Equations (7) and (9).

The displacement D(x) at the position x arising from an asymmetric dislocation distribution function B(x) which extends out to the distance b is simply $\int_x^b B(x)dx$. The sum $\sum_{2n} |d(x)|$ of the absolute values of all the displacements produced at the end ($T = T_m$) of n cycles at point x is found through integration of Equations (3), (7) and (9). This sum is

$$D^\dagger(x) = \sum_{2n} |D(x)| = \frac{2\alpha\sigma}{\pi\mu} \left\{ a \log \left| \frac{(b_0^2 - a^2)^{\frac{1}{2}} + (b_0^2 - x^2)^{\frac{1}{2}}}{(b_0^2 - a^2)^{\frac{1}{2}} - (b_0^2 - x^2)^{\frac{1}{2}}} \right| \right.$$

$$- x \log \left| \frac{x(b_0^2 - a^2)^{\frac{1}{2}} + a(b_0^2 - x^2)^{\frac{1}{2}}}{x(b_0^2 - a^2)^{\frac{1}{2}} - a(b_0^2 - x^2)^{\frac{1}{2}}} \right|$$

$$+ 4gna \log \left| \frac{(b_1^2 - a^2)^{\frac{1}{2}} + (b_1^2 - x^2)^{\frac{1}{2}}}{(b_1^2 - a^2)^{\frac{1}{2}} - (b_1^2 - x^2)^{\frac{1}{2}}} \right|$$

$$\left. - 4gnx \log \left| \frac{x(b_1^2 - a^2)^{\frac{1}{2}} + a(b_1^2 - a^2)^{\frac{1}{2}}}{x(b_1^2 - a^2)^{\frac{1}{2}} - a(b_1^2 - a^2)^{\frac{1}{2}}} \right| \right\} \qquad (11)$$

where $b_0 = b_0(T_m)$, $b_1 = b_1(T^*)$, and $g = 0$ for $|x| \ge b_1$ and $g = 1$ for $|x| \le b_1$.

At the crack tip (x = a) the displacement sum $D^\dagger(a)$ is

$$D^\dagger(a) = \frac{4\alpha\sigma a}{\pi\mu} \left[\log (b_0/a) + 4n \log (b_1/a) \right] \qquad (12)$$

When $T \ll \sigma$ this equation can be reduced, with the aid of Equations (4) and (8), to the following

$$D^\dagger(a) = \frac{4\alpha\sigma a}{\pi\mu} \left[\left(\frac{\pi T_m}{2\sigma} \right)^2 + 4n \left(\frac{\pi(T_m - T^*)}{4\sigma} \right)^2 \right] \qquad (13)$$

For the case of $n = 0$ (which was considered by Bilby, Cottrell and Swinden) and $D^{\dagger}(a)$ set equal to the critical displacement D^*, Equation (13) reduces to

$$T_m = (\mu\sigma D^*/\pi\alpha a)^{\frac{1}{2}} \tag{14}$$

This is the fracture criterion given by Equation (6). A stress satisfying this equation would cause the crack to expand catastrophically.

Suppose $D^{\dagger}(a)$ is set equal to D^* and T_m is so small that n must be a large number for Equation (13) to be satisfied. In this case a crack will begin to increase its length when n is equal to

$$n \simeq \frac{\mu D^*\sigma}{\pi\alpha a T_m^{\ 2}} \tag{15}$$

for $T^* = 0$

The crack will not fail catastrophically after this incubation number of cycles. Rather it will increase its length until the value of $D^{\dagger}$ at the tip is less than D^*. Thereafter the crack will grow in length during each stress cycle until either it is of a length that Equation (6) is valid or it is comparable to the specimen dimensions or to the spacing between cracks. At that critical length it will fail catastrophically.

Consider now the rate at which the crack grows sometime after growth has been nucleated. Assume that $T \ll \sigma$. Let a represent the half-width of the crack at the beginning of each stress cycle (when $T = T_m$) $a + \delta a$ the half-width at the end of each cycle (when again $T = T_m$). The dislocation distribution at the beginning and end of each cycle will be given by Equation (3). The actual displacement $D(x)$ at the beginning of the cycle is

$$D(x) = \frac{2\alpha\sigma}{\pi\mu}\left\{ a \log\left|\frac{(b_0^{\ 2} - a^2)^{\frac{1}{2}} + (b_0^{\ 2} - x^2)^{\frac{1}{2}}}{(b_0^{\ 2} - a^2)^{\frac{1}{2}} - (b_0^{\ 2} - x^2)^{\frac{1}{2}}}\right| - x \log\left|\frac{x(b_0^{\ 2} - a^2)^{\frac{1}{2}} + a(b_0^{\ 2} - x^2)^{\frac{1}{2}}}{x(b_0^{\ 2} - a^2)^{\frac{1}{2}} - a(b_0^{\ 2} - x^2)^{\frac{1}{2}}}\right|\right\} \tag{16}$$

At the end of the cycle the displacement is given by the same equation except that a is replaced by $a + \delta a$. Figure 2 shows $D(x)$ in the region of $a < x < b_0$ when $T_m \ll \sigma$ and $D(x)$ when the crack has extended to $a + \delta a$ (and b_0 has changed to $b_0 + \delta b_0$, where $\delta b_0 \simeq \delta a$).

The extension δa per cycle in the length of the crack can be estimated in the following manner. The number m of cycles required for the crack to advance a distance $b_0 - a$ is approximately $(b_0 - a)/\delta a$. Since $b_0 - a \simeq a(\pi T_m/2\sigma)^2$ when $T_m \ll \sigma$, the number m is approximately equal to (when m

is large)

$$m \simeq \frac{a}{\delta a}\left(\frac{\pi T_m}{2\sigma}\right)^2 \tag{17}$$

The total displacement $D^{\dagger\dagger}$ at the point b_0 of Figure 2 which occurs between the time the dislocation distribution first has a non-zero value at b_0 (curve 1) and the time the crack grows until the tip is actually at the point b_0 of Figure 2 (curve 3) is approximately equal to the sum

$$D^{\dagger\dagger} \simeq \frac{8\alpha\sigma g}{\pi\mu}\sum_{k=0}^{m-1}\left\{ a^* \log\left|\frac{(b_I^{*2} - a^{*2})^{\frac{1}{2}} + (b_I^{*2} - x^{*2})^{\frac{1}{2}}}{(b_I^{*2} - a^{*2}) - (b_I^{*2} - x^{*2})^{\frac{1}{2}}}\right| - x^* \log\left|\frac{x^*(b_I^{*2} - a^{*2})^{\frac{1}{2}} + a^*(b_I^{*2} - x^{*2})^{\frac{1}{2}}}{x^*(b_I^{*2} - a^{*2})^{\frac{1}{2}} - a^*(b_I^{*2} - x^{*2})^{\frac{1}{2}}}\right|\right\} \tag{18}$$

where $g = 0$ for $|x^*| > b_I^*$ and $g = 1$ for $|x^*| < b_I^*$. In this equation $a^* = a + k\delta a$; $b_I^* = a + a(\pi T_m/4\sigma)^2 + k\delta a = a + k\delta a + m\delta a/4$; and $x^* = b_0 - k\delta a = a + m\delta a - k\delta a$. By setting $D^{\dagger\dagger}$ equal to the critical displacement D^* in Equation (18) it is possible to obtain values for m and the crack extension δa.

The summation given by Equation (18) can be approximated by the integral

$$D^{\dagger\dagger} \approx \frac{1}{\delta a}\int_a^{b_1} D^{\dagger}(x)\,dx \tag{19}$$

where $D^{\dagger}(x)$ is given by Equation (11) with n set equal to 1 and T^* equal to 0 and the terms containing b_0 are ignored. Upon integration Equation (19) becomes the following when $D^{\dagger\dagger} = D^*$

$$\delta a = \frac{8\alpha\sigma a}{D^*\pi\mu}\left[(b_1^{\ 2} - a^2)^{\frac{1}{2}}\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{a}{b_1}\right)\right) - 2a \log\,(b_1/a)\right] \tag{20a}$$

When $T_m \ll \sigma$ this equation reduces to

$$\delta a = \left(\frac{\alpha\sigma a^2}{3D^*\pi\mu}\right)\left(\frac{\pi T_m}{2\sigma}\right)^4 \approx (0.4)\,\frac{a^2 T_m^{\ 4}}{\gamma\mu\sigma^2} \tag{20b}$$

where $\gamma = \sigma D^*$ is the constant previously defined by Bilby, Cottrell and Swinden.

Equation (20b) predicts[3] that the increase δa in the length of the crack per stress cycle is proportional to $(aT_m^2)^2$. Fatigue data (18) do follow such a rate of growth law at low stress amplitudes. Equation (20a) predicts a more rapidly increasing growth rate at higher stresses, a behavior which is also observed.

In order to compare the magnitude of the growth rate predicted by Equation (20b) with experimental data it is necessary to know the values of γ and σ for the metal tested. Knott and Cottrell (9) found that $\gamma \cong 2 \times 10^8$ ergs/cm^2 for iron at a temperature at which it is ductile. This value is about the same as that for steel and is of the same magnitude as the value of γ found for aluminum alloys (8, Table 2, p. 566).

If the applied stress were much lower than the critical resolved shear stress it would be reasonable to set σ equal to the critical resolved shear stress. However, fatigue tests usually are carried out above the yield stress. Under these conditions it is reasonable to set σ equal to the ultimate strength of the metal where the ultimate strength is determined from compression rather than tensile tests in order to avoid the uncertainties caused by necking). Consider the results of McEvily and Boettner (18) on the rate of crack growth in copper. They found for a tensile stress T_m of 6.4×10^8 dynes/cm^2 and a crack halfwidth of $a = 0.19$ cm that the crack growth rate was $\delta a \cong 3.8 \times 10^{-7}$ cm/cycle (their lower left datum point in their Figure 9a). If these values of T_m and a are substituted into Equation (20b) along with the values $\gamma = 2 \times 10^8$ ergs/cm^2, $\sigma = 4.3 \times 10^9$ dynes/cm (obtained from Seitz, (18), Figure 47, p. 73), and $\mu = 4.3 \times 10^{11}$ dynes/cm^2, the predicted crack growth rate is found to be $\delta a = 15 \times 10^{-7}$ cm/cycle. Equation (20b) thus leads to not unreasonable values of the fatigue crack growth rate.

FAILURE OF ENERGY CRITERION FOR A BCS CRACK

We wish to prove a negative result in this section, namely, that in the BCS theory the energy dissipated by "plastic" work in the region ahead of a crack usually is not large enough to prevent the crack from failing catastrophically. Therefore energy is not a good fracture criterion.

The energy dissipated that will be calculated for the BCS crack is identical to that calculated by Goodier and Field (6) using the Dugdale (4) theory.[4] Goodier and Field, however, did not calculate the lowering of the elastic energy caused by the presence of the crack and therefore were unable to notice that although the energy dissipated can be large, it almost never is large enough to stop a crack from propagating catastrophically.

Suppose a BCS crack is made in a block of material which is stressed under a shear stress S (or a tensile stress T). The crack is made by cutting along the plane of the crack from $-b < x < b$, placing on the cut surface external stresses which have the initial value S, and then decreasing these applied stresses until they are equal to 0 in the region $-a < x < a$, and equal to $\sigma - S$ in the regions $a < |x| \leq b$. The cut surfaces are then rejoined and the externally applied stresses can be removed without further change in the displacements. The work W which must be done on the cut surfaces in order to create the crack is equal to minus the integral of one-half the product of the displacement across the crack and the final value of the externally applied stresses. The integration extends over the length of the crack from $-b$ to b. Hence

$$W = -\int_0^b S^* \left[\int_x^b B(x)dx \right] dx \tag{21a}$$

where S^* is equal to S for $0 \leq x < a$ and is equal to $\sigma - S$ for $a < x \leq b$. The substitution of Equation (3) into this equation gives

$$W = -\frac{4\alpha\sigma^2 a^2}{\pi\mu} \log (b/a) \tag{21b}$$

When $S \ll \sigma$ this equation reduces to

$$W = -\frac{\alpha\pi S^2 a^2}{2\mu} - \frac{\alpha\sigma^2 a^2}{3\pi\mu}\left(\frac{\pi S}{2\sigma}\right)^4 \tag{21c}$$

The energy dissipated, E_d, is simply equal to the yielding or friction stress σ times the displacement, integrated over the crack. Therefore

$$E_d = 2\int_a^b \sigma \left[\int_x^b B(x)dx \right] dx \tag{22a}$$

which is

$$E_d = \frac{4\alpha\sigma^2 a^2}{\pi\mu} \left[\frac{\pi}{2}\frac{S}{\sigma}\left(\frac{b^2}{a^2} - 1\right)^{\frac{1}{2}} - 2 \log (b/a) \right] \tag{22b}$$

When $S \ll \sigma$ this equation reduces to

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[3] Since the writing of this paper, we have learned (J.R.Rice, personal communication) that a similar crack propagation law was derived in another manner by Rice in the unpublished Leigh University report: "Fatigue Crack Growth Model: Some General Comments and Preliminary Study of the Rigid Strip Model" (1962). We wish to thank Professor Rice for bringing this work to our attention.

[4] There apparently is a misprint in Equation (6) of Goodier and Field's paper. The second term on the right hand side of their equation should be multiplied by 2.

$$E_d = \frac{2\alpha\sigma^2 a^2}{3\pi\mu}\left(\frac{\pi S}{2\sigma}\right)^4 \tag{22c}$$

It can be verified that $E_d - W$ is equal to $S\int_{-b}^{b} xB(x)dx$. This last expression is the work done by the applied stress S at the outer surface of the block when the crack is made. Here it is assumed that the outer surface is free to move.

If the outer surface is held constant in grips which do not move when the crack is made, the quantity W given by Equation (21) is approximately equal to the reduction in the elastic energy of the block. It is not exactly equal to the elastic energy reduction because the stress S decreases slightly as the crack is made. However, if the applied stress is small these two energies, when expressed in a power series in S, are identical in the lowest order term (both equal to $\alpha\pi S^2 a^2/2\mu$). They differ only in the higher order terms.

Since b/a does not depend on a it can be seen that both dW/da and dE_d/da are proportional to a. Therefore it is impossible to derive a crack criterion equation such as Equation (6) from Equations (21) and (22). Moreover, except when $S \approx \sigma$, $|dE_d/da| < |dW/da|$ and thus according to Equations (21) and (22) a crack should almost always fail catastrophically even though, as Goodier and Field calculated, Equation (22) can lead to rather large values of γ. (Of course, for very small cracks the surface tension of the crack becomes important and the Griffith theory will be valid.) It would appear that energy is not a good criterion to use in the BCS theory for fracture investigations.

SUMMARY

It has been shown that the Bilby, Cottrell and Swinden crack theory can be applied to the problem of the growth of fatigue cracks. If a fracture criterion of critical displacement is adopted it is found that the theory does lead to the observed laws of rate of growth. It is shown that an energy criterion is not satisfactory for predicting fracture with the BCS theory despite the fact that large values of energy dissipation were calculated by Goodier and Field.

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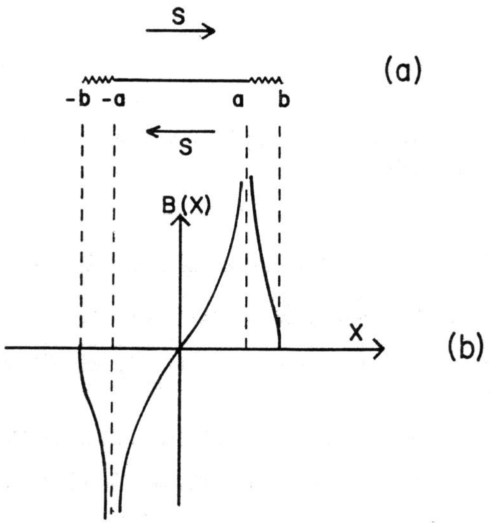

Figure 1 (a) Crack under shear stress S. Crack extends from -a to a. Slipped zone ahead of crack extends from $a \le |x| \le b$. (b) Dislocation distribution around the crack.

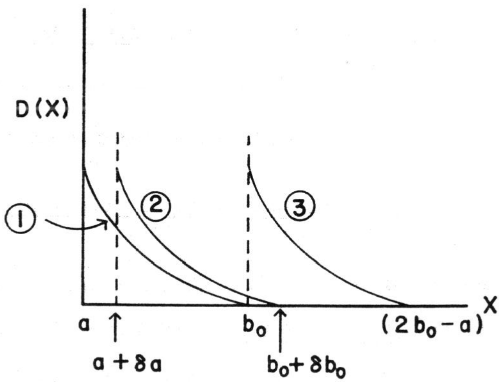

Figure 2 Displacement in deformed zone ahead of crack tip. Curve 1: when tip is at a; curve 2: after one cycle when crack has grown by amount δa; curve 3: after m cycles when crack has grown to half-width b_0.