A-11

INTERACTION BETWEEN ELASTIC CRACKS, DISLOCATION CRACKS AND SLIP BANDS

Takeo YOKOBORI*, Masahiro ICHIKAWA** and Masao OHASHI***

ABSTRACT

In this article the interaction between elastic cracks, dislocation cracks, slip bands and each of them were studied for the following cases: (1) Interaction of two collinear asymmetrical elastic cracks, (2) Interaction between an elastic crack and a slip band on the same plane, (3) Interaction of two collinear dislocation cracks in the case of the Bullough-Gilman type cracks and in the case of the Cottrell type cracks, (4) Interaction between an elastic crack and a dislocation crack of the Cottrell type on the same plane. From comparison of the results, characteristics of interaction in each case will be clarified.

§1. INTRODUCTION

It is reasonable to consider that fracture of a solid is caused, on the microscopic scale at least, by interaction of defects such as microcracks, inclusions, slip bands and twins. Therefore it is important to study the effect of interaction between these defects on the fracture strength. Many works of this kind have been carried out for cases of the elastic cracks, and a summary of those was given in a paper by Barenblatt[1]. Also, several cases of the relaxed cracks (the elastic-plastic cracks) were studied recently[2)-6)]. The interaction of parallel slip bands was discussed by Stroh[7]. The interaction of parallel elastic cracks[18] and the interaction of an infinite row of collinear dislocation cracks[19] have been studied based on the concept of continuous distribution of infinitesimal dislocations.

In the present article, the following cases will be studied. (1) Interaction of two collinear asymmetrical elastic cracks, (2) Interaction between an elastic crack and a slip band on the same plane, (3) Interaction of two collinear dislocation cracks in the case of the Bullough-Gilman type cracks[8)9)] and in the case of the Cottrell type cracks[10], (4) Interaction between an elastic crack and a dislocation crack of the Cottrell type on the same plane. The method was based on the concept of continuous distribution

* Director of the Research Institute for Strength and Fracture of Materials, Tohoku University, Sendai, JAPAN

, * Post-graduate, Tohoku University, Sendai, JAPAN

of infinitesimal dislocations[11)12)13)]. As a fracture criterion, the energy criterion[14)] was used.

§2. INTERACTION OF TWO COLLINEAR ASYMMETRICAL ELASTIC CRACKS

Consider an infinite elastic solid with two collinear asymmetrical elastic cracks subject to an applied stress $\sigma_y = \sigma$ at infinity as shown in Fig.2.1. Let's call the right crack with length ℓ (= a - b) the first crack and the left crack with length r(= c - d) the second crack. It is supposed that $\ell \geqq r$. t(= b - c) is the distance between the inside tips. These cracks can be simulated by an appropriate continuous distribution of infinitesimal dislocations as depicted in Fig. 2.1. Let f(x) be the distribution function of dislocations (dislocation density) with the convention that f(x) is positive in regions where the dislocations are positive and vice versa. From the requirement for equilibrium of each dislocation, f(x) must satisfy the following condition for all points x in the regions (d,c) and (b,a):

$$A\int_D \frac{f(\xi)}{\xi - x}\,d\xi + \sigma = 0, \qquad (2.1)$$

where $A = G\lambda/[2\pi(1-\nu)]$, G is the shear modulus, ν is Poisson's ratio and λ is the Burgers vector of a unit dislocation. D is the regions (d,c) and (b,a). The boundary conditions are that f(x) is unbounded at x = a,b,c and d. Eq.(2.1) may be solved by a method due to Muskhelishvili[15)] as:

$$f(x) = \frac{\sigma}{\pi^2 A}\,\frac{1}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}\int_D \frac{\sqrt{(\xi-a)(\xi-b)(\xi-c)(\xi-d)}}{\xi - x}\,d\xi + \frac{Q_1(x)}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}, \qquad (2.2)$$

where $Q_1(x)$ is an unfixed polynominal of degree not greater than unity. $Q_1(x)$ is determined from the supplementary condition that crack openings are zero at the crack tips, i.e.,

$$\int_b^a f(x)\,dx = 0 \qquad (2.3)$$

$$\int_d^c f(x)\,dx = 0\,. \qquad (2.4)$$

From Eqs.(2.2), (2.3) and (2.4), $Q_1(x)$ is determined as:

$$Q_1(x) = -\frac{\sigma i}{\pi A}\left[\frac{(a-b-c+d)^2}{8} - \frac{(a-c)(b-d)}{2}\cdot\frac{K(k)-E(k)}{K(k)}\right], \qquad (2.5)$$

where $i^2 = -1$. Thus, f(x) is:

$$f(x) = \mp\frac{\sigma}{\pi A}\cdot\frac{1}{\sqrt{-(x-a)(x-b)(x-c)(x-d)}} \times\left[x^2 - \frac{a+b+c+d}{2}x + \frac{ad+bc}{2} + \frac{(a-c)(b-d)}{2}\,\frac{K(k)-E(k)}{K(k)}\right], \qquad (2.6)$$

where the negative and positive signs are for the region (b,a) and for the region (d,c) respectively. K(k) and E(k) are the complete elliptic integrals of the first and the second kinds with the modulus $k = \sqrt{(a-b)(c-d)/[(a-c)(b-d)]}$ respectively.

Next, the stress σ_y on the y = 0 plane is calculated for $c < x < b$ as:

$$\sigma_y = A\int_D \frac{f(\xi)}{\xi - x}\,d\xi + \sigma = \frac{-\sigma}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}\left[x^2 - \frac{a+b+c+d}{2}x + \frac{ad+bc}{2} + \frac{(a-c)(b-d)}{2}\cdot\frac{K(k)-E(k)}{K(k)}\right]. \qquad (2.7)$$

Let N(b) and N(c) be the stress intensity factors at the tips x = b and x = c respectively. These are calculated from Eq.(2.7) as:

$$N(b) = \frac{\sigma}{2}\sqrt{\frac{b-d}{(a-b)(b-c)}}\left[(a-b)-(a-c)\frac{K(k)-E(k)}{K(k)}\right] \qquad (2.8)$$

$$N(c) = \frac{\sigma}{2}\sqrt{\frac{a-c}{(b-c)(c-d)}}\left[(c-d)-(b-d)\frac{K(k)-E(k)}{K(k)}\right]. \qquad (2.9)$$

The calculated values of the ratio N(c)/N(b) are shown in Fig.2.2 as a function of ℓ, r and t. From Fig. 2.2 it is seen that $N(b) \geqq N(c)$. Therefore it is concluded that crack propagation is generally initiated by the first crack.

Now, let σ_2' be the stress required for propagation of the first crack, which occurs when the following condition is satisfied:

$$-\frac{\pi(1-\nu)}{G}N^2(b) + 2\gamma = 0. \qquad (2.10)$$

From Eqs. (2.8) and (2.10), σ_2' is obtained as:

$$\sigma_2' = \left[\frac{8G\gamma}{\pi(1-\nu)}\cdot\frac{\ell t}{r+t}\right]^{\frac{1}{2}} \Bigg/ \left[\ell - (\ell+t)\frac{K(k)-E(k)}{K(k)}\right]. \qquad (2.11)$$

When $\ell = r$, Eq.(2.11) reduces to the result for two collinear symmetrical elastic cracks derived by Wilmore[16)]. If the second crack does not exist, σ_2' reduces to σ_1 which is obtained by $t \to \infty$ in Eq. (2.11) as:

$$\sigma_1 = \sqrt{\frac{8G\gamma}{\pi(1-\nu)\ell}}\,. \qquad (2.12)$$

When the applied stress reaches σ_2', two cracks join as a result of propagation of the first crack. Let σ_2'' be the stress required for propagation of the joined crack. σ_2'' is obtained by replacing ℓ with (ℓ+ r + t) in Eq. (2.12) as:

$$\sigma_2'' = \sqrt{\frac{8G\gamma}{\pi(1-\nu)(\ell+r+t)}} \quad (2.13)$$

Now, σ_2, the fracture stress of the solid is determined by the larger of σ_2' and σ_2''. The ratio σ_2/σ_1 is calculated by Eqs. (2.11), (2.12) and (2.13). This ratio gives a measure of amount by which the solid with the first crack only is weakened by appearance of the second crack. The results are shown in Fig. 2.3.

From Fig. 2.3 it is seen that the effect of the second crack length is remarkable when the distance between the inside tips is small. Next, in the case of two collinear symmetrical cracks, if the distance between the inside tips exceeds the crack length, the change in the fracture strength due to the interaction becomes at most 5% and in this sense the interaction may be neglected.[16] Let's consider this point for the case of two collinear asymmetrical cracks. Using Fig. 2.3 we obtain the result of Fig. 2.4 which shows the critical distance with which the change in the fracture strength due to interaction is at most 5% (that is, $\sigma_2/\sigma_1 \geqq 0.95$). From Fig. 2.4 it is concluded that the interaction may be neglected in the above mentioned sense if the distance between the inside tips exceeds the second crack length. Next, when the distance between the inside tips is smaller than a certain value, σ_2'' becomes larger than σ_2' and therefore the two cracks can be regarded as one joined crack as far as the fracture strength is concerned. This critical distance is about 10% of the first crack length nearly independent of the second crack length as Fig. 2.3 shows.

§3. INTERACTION BETWEEN AN ELASTIC CRACK AND A SLIP BAND ON THE SAME PLANE

Consider an infinite elastic solid with an elastic crack of length ℓ (=a-b) subject to both the normal stress σ and the shear stress τ at infinity as shown in Fig. 3.1. Suppose that under action of τ, slip occurs on the plane y = 0 in the region $d < x < c$ and dislocations pile up against an obstacle at x=c. Let r(=c-d) be the length of the slip band and t(=b-c) be the distance between the piled-up end and the crack tip nearer to the piled-up end. This system can be simulated approximately by superposing two distributions of infinitesimal dislocations (a) and (b) which are shown in Fig. 3.1. Let f(x) and g(x) be the distribution functions of (a) and (b) respectively. From the requirement for equilibrium of each dislocation, these must satisfy the following equations:

$$-A\int_D \frac{f(\xi)}{\xi - x}\, d\xi + \tau = 0 \quad (3.1)$$

$$A\int_b^a \frac{g(\xi)}{\xi - x}\, d\xi + \sigma = 0, \quad (3.2)$$

where D is the regions (d, c) and (b, a). The boundary conditions are that f(x) is unbounded at x=a, b and c and bounded at x=d, and that g(x) is unbounded at x=a and b. With these and the following supplementary conditions:

$$\int_b^a f(x)\, dx = 0 \quad (3.3)$$

$$\int_b^a g(x)\, dx = 0, \quad (3.4)$$

f(x) and g(x) can be obtained respectively as:

$$f(x) = \pm\frac{\tau}{\pi A}\sqrt{\frac{-(x-d)}{(x-a)(x-b)(x-c)}}\Big[x - \frac{a+b+c-d}{2} + \frac{1}{2}\,\frac{(b-d)[(a-d)K(k)-(a-c)E(k)]}{(c-d)K(k)+(b-c)\Pi(\rho,k)}\Big] \quad (3.5)$$

$$g(x) = -\frac{\sigma}{\pi A}\,\frac{x-\frac{a+b}{2}}{\sqrt{-(x-a)(x-b)}}, \quad (3.6)$$

where the positive and negative signs are for the region (b, a) and for the region (d, c) respectively. $\Pi(\rho, k)$ is the complete elliptic integral of the third kind with the modulus $k = \sqrt{(a-b)(c-d)/[(a-c)(b-d)]}$ and the parameter $\rho = -(a-b)/(a-c)$. Now, let n be the number of the piled-up dislocations in the slip band. n is calculated as:

$$n = \int_d^c f(x)\,dx = \frac{\tau}{\pi A}\sqrt{\frac{b-d}{a-c}}\,[(a-d)K(k)-(a-c)E(k)] \times \Big[1 - \frac{(b-d)K(k)-(b-c)\Pi(\rho',k)}{(c-d)K(k)+(b-c)\Pi(\rho,k)}\Big], \quad (3.7)$$

where $\rho' = -(c-d)/(b-d)$. If the crack does not exist, n reduces to n_o, which is obtained by $(b - c) \to \infty$ in Eq. (4.7) as:

$$n_o = \frac{\tau}{2A}(c-d), \quad (3.8)$$

which is identical with the result for an isolated slip band derived by Head and Louat[12]. The calculated values of the ratio n/n_o by Eqs. (3.7) and (3.8) are shown in Fig. 3.2 as a function of ℓ, r and t. From Fig. 3.2, it is seen the number of piled-up dislocations increases due to the presence of the near-by crack, rapidly increasing as the distance between the leading dislocation and the crack tip decreases.

Next, we consider the effect of the interaction on the fracture strength assuming that fracture is initiated by the crack, not by the slip band. Since the stress concentration is larger at the tip x=b than at the tip x=a due to interaction, crack propagation is initiated at the tip x=b. From Eqs. (3.5) and (3.6), τ_{xy} and σ_y on the y=0 plane are calculated for $c < x < b$ as:

$$\tau_{xy} = A\int_D \frac{f(\xi)}{x-\xi}\,d\xi + \tau$$

$$= -\tau\sqrt{\frac{x-d}{(x-a)(x-b)(x-c)}}\left[x - \frac{a+b+c-d}{2} + \frac{b-d}{2}\cdot\frac{(a-d)K(k)-(a-c)E(k)}{(c-d)K(k)+(b-c)\Pi(\rho,k)}\right] \qquad (3.9)$$

$$\sigma_y = A\int_b^a \frac{g(\xi)}{\xi - x}\,d\xi + \sigma = -\sigma\,\frac{x-\frac{a+b}{2}}{\sqrt{(x-a)(x-b)}}\,. \qquad (3.10)$$

Thus, N_τ and N_σ, the stress intensity factors for τ_{xy} and σ_y at the tip x=b are:

$$N_\tau = \tau\sqrt{\frac{b-d}{(a-b)(b-c)}}\left[\frac{a-b+c-d}{2} - \frac{b-d}{2}\cdot\frac{(a-d)K(k)-(a-c)E(k)}{(c-d)K(k)+(b-c)\Pi(\rho,k)}\right] \qquad (3.11)$$

$$N_\sigma = \frac{\sigma}{2}\sqrt{a-b}\,. \qquad (3.12)$$

Assuming crack propagation occurs in the direction of the crack axis, the condition for propagation is:

$$-\frac{\pi(1-\nu)}{G}\left(N_\sigma^2 + N_\tau^2\right) + 2\gamma = 0. \qquad (3.13)$$

Now, consider the case of uni-axial tension as shown in Fig. 3.3. As the angle between the tensile axis and the positive x direction, $\pi/4$ is chosen since the applied shear stress component τ on the y=0 plane is greatest and therefore the interaction between the crack and the slip band is most remarkable in this case. Let σ_2' be the applied uni-axial stress required for crack propagation. Noting $\sigma = \tau$ =(applied uni-axial stress) x $\frac{1}{2}$ we obtain σ_2' from Eqs. (3.11), (3,12) and (3,13) as:

$$\sigma_2' = \sqrt{\frac{32G\gamma}{\pi(1-\nu)}}\left[(a-b) + \frac{b-d}{(a-b)(b-c)}\left\{(a-b+c-d) - (b-d)\cdot\frac{(a-d)K(k)-(a-c)E(k)}{(c-d)K(k)+(b-c)\Pi(\rho,k)}\right\}^2\right]^{-\frac{1}{2}}. \qquad (3.14)$$

When the applied stress reaches σ_2', crack propagation is initiated. Subsequent propagation is possible under the stress σ_2', until the crack tip arrives at the point x=c. But circumstances become somewhat different thereafter. At present it is not possible to obtain an exact value of the stress σ_2'' required for propagation to continue. However, both the upper and the lower limit of σ_2'' can be found. σ_2'' is considered to be smaller than the stress required for propagation of a crack with length (ℓ+ t) and larger than the stress required for propagation of a crack with length (ℓ + r + t). That is,

$$\sqrt{\frac{8G\gamma}{\pi(1-\nu)(\ell+r+t)}} \leq \sigma_2'' \leq \sqrt{\frac{8G\gamma}{\pi(1-\nu)(\ell+t)}}\,. \qquad (3.15)$$

The fracture stress σ_2^* is determined as the larger of σ_2' and σ_2''. If the slip band does not exist, σ_2^* reduces to σ_1^* which is obtained by t $\to\infty$ in Eq. (3.14) as:

$$\sigma_1^* = \sqrt{\frac{8G\gamma}{\pi(1-\nu)\ell}}\,. \qquad (3.16)$$

The calculated values of the ratio σ_2^*/σ_1^* by Eqs. (3.14), (3.15) and (3.16) are shown in Fig. 3.4. In the region where $\sigma_2'' > \sigma_2'$, the results are shown by a hatched area bounded by the upper and the lower limit of σ_2''. From Fig. 3.4 it is found that the fracture strength decreases due to the interaction.

Next, let's compare the effect of interaction in the present case with that in the case of two collinear elastic cracks. For this purpose, we consider the interaction of two collinear elastic cracks in the case when the uni-axial tensile stress is applied in the direction making $\pi/4$ with the x axis in Fig. 2.1 as in Fig. 3.3. The shear stress component has equivalent effect to the normal stress component on crack propagation as far as the energy criterion is concerned. Therefore, the applied uni-axial stress required for propagation of the first crack in this case is equal to $\sqrt{2}\,\sigma_2'$, where σ_2' is that given by Eq. (2.11). Similarly, the applied uni-axial stress required for propagation of the isolated first crack and that required for propagation of the joined crack in this case are given by $\sqrt{2}\,\sigma_1$ and $\sqrt{2}\,\sigma_2''$ respectively, where σ_1 and σ_2'' are those given by Eqs. (2.12) and (2.13) respectively. Therefore, the ratio of the fracture stress of a solid with two collinear cracks to that of a solid with the first crack only in the case when the tensile axis makes $\pi/4$ with the crack axis is equal to the corresponding ratio in the case when the tensile axis is normal to the crack axis, i.e., the case treated in § 2. Thus, direct comparison of σ_2^*/σ_1^* in Fig. 3.4 and σ_2/σ_1 in Fig. 2.3 is justified. The comparison is made in Fig. 3.5. From Fig. 3.5 it is found that the effect of interaction is larger in the case of an elastic crack and a slip band on the same plane than in the case of two collinear elastic cracks except when the distance t is small.

§4. INTERACTION OF TWO COLLINEAR DISLOCATION CRACKS

4.1 The case of the Bullough-Gilman type dislocation cracks

Case I The case in which crack expansion occurs at the inside tips

Consider an infinite elastic solid with two collinear dislocation cracks of the Bullough-Gilman type [8)9)] subject to an applied stress $\sigma_y = \sigma$ at infinity as shown in Fig. 4.1.

It is assumed in the present case that crack expansion occurs at the inside tips only. These dislocation cracks are simulated by superposing two distributions of infinitesimal dislocations, (a) and (b), which are depicted in Fig. 4.1[17]. It is assumed that n, the number of slip dislocations contained in each crack in the distribution (a) does not change during deformation. Let f(x) and g(x) be the distribution functions of (a) and (b) respectively. From the requirement for equilibrium of each dislocation, these must satisfy the following equations:

$$-A\int_D \frac{f(\xi)}{\xi - x}\,d\xi = 0 \tag{4.1}$$

$$A\int_D \frac{g(\xi)}{\xi - x}\,d\xi + \sigma = 0, \tag{4.2}$$

where D is the regions (-b, -a) and (a, b). The boundary conditions are that both f(x) and g(x) are unbounded at x=±a and ±b. From these and the following supplementary conditions:

$$-\int_{-b}^{-a} f(x)\,dx = \int_a^b f(x)\,dx = n \tag{4.3}$$

$$-\int_{-b}^{-a} g(x)\,dx = \int_a^b g(x)\,dx = 0, \tag{4.4}$$

f(x) and g(x) are obtained as:

$$f(x) = \pm\frac{nb}{K'(a/b)}\frac{1}{\sqrt{(b^2-x^2)(x^2-a^2)}} \tag{4.5}$$

$$g(x) = \pm\frac{\sigma}{\pi A}\left\{-\frac{x^2}{\sqrt{(b^2-x^2)(x^2-a^2)}} + \frac{E'(a/b)}{K'(a/b)}\frac{b^2}{\sqrt{(b^2-x^2)(x^2-a^2)}}\right\}, \tag{4.6}$$

where the positive and negative signs are for the region (a, b) and for the region (-b, -a) respectively. K'(a/b) and E'(a/b) are the complete elliptic integrals of the first and the second kinds with the modulus $\sqrt{1-(a/b)^2}$, respectively.

Next, the stresses τ_{xy} and σ_y on the y=0 plane are calculated for $-a < x < a$ from Eqs. (4.5) and (4.6) as:

$$\tau_{xy} = -\frac{Gn\lambda}{2(1-\nu)}\frac{b}{K'(a/b)}\frac{1}{\sqrt{(b^2-x^2)(a^2-x^2)}} \tag{4.7}$$

$$\sigma_y = \frac{\sigma}{\sqrt{(b^2-x^2)(a^2-x^2)}}\left\{-x^2 + b^2\frac{E'(a/b)}{K'(a/b)}\right\}. \tag{4.8}$$

Therefore, N_τ and N_σ, the stress intensity factors for τ_{xy} and σ_y at the tips x=±a are:

$$N_\tau = -\frac{Gn\lambda}{2(1-\nu)\sqrt{2a(b^2-a^2)}}\frac{b}{K'(a/b)}\frac{1}{} \tag{4.9}$$

$$N_\sigma = \frac{\sigma b^2}{\sqrt{2a(b^2-a^2)}}\left\{\frac{E'(a/b)}{K'(a/b)} - \frac{a^2}{b^2}\right\}. \tag{4.10}$$

From Eqs. (4.9) and (4.10), the rate of change in energy E of the system accompanying expansion at the tips x=±a is:

$$\frac{dE}{dc} = -\frac{\pi(1-\nu)}{G}(N_\sigma^2 + N_\tau^2) + 2\gamma$$
$$= -A'\frac{Gn^2\lambda^2}{4\pi(1-\nu)c} - B'\frac{\pi(1-\nu)\sigma^2 c}{4G} + 2\gamma, \tag{4.11}$$

where $c = b - a$, $t = c/b$

$$A' = \frac{\pi^2}{4}\frac{1}{(1-t)(1-\frac{t}{2})}\frac{1}{K'^2(1-t)}, \text{ and } B' = \frac{1}{t^2(1-t)(1-\frac{t}{2})}\left\{\frac{E'(1-t)}{K'(1-t)} - (1-t)^2\right\}^2.$$

When t→∞, Eq. (4.11) reduces to the corresponding equation for an isolated dislocation crack of the Bullough-Gilman type derived by Bullough[17]. Now, E has stationary values at such values of c that fulfil the condition:

$$-A'\frac{Gn^2\lambda^2}{4\pi(1-\nu)c} - B'\frac{\pi(1-\nu)\sigma^2 c}{4G} + 2\gamma = 0. \tag{4.12}$$

Eq. (4.12) for c has generally two real roots. The smaller one corresponds to a minimum in E and,therefore,is the stable equilibrium length. It is easily found that the stable equilibrium length is a monotonously increasing function of σ. Therefore, as σ is increased from zero continuously, each crack increases its length, taking always a stable equilibrium length for a current value of σ. When σ exceeds a certain critical value σ'_{2B}, Eq. (4.12) has no real roots for c. Physically this means that the cracks can expand without further increase in the applied stress if $\sigma > \sigma'_{2B}$. Therefore, σ'_{2B} is the stress required for unstable expansion of the cracks.

In order to find σ'_{2B}, we proceed as follows. Eq. (4.12) is rewritten as:

$$\sigma = \sigma_{1B}\left[2\left(\frac{B^*}{c^*}\right) - \left(\frac{A^*}{c^*}\right)^2\right]^{\frac{1}{2}}, \tag{4.13}$$

where $\sigma_{1B} = 4\gamma/(n\lambda)$, $c^* = c/c_o$, $c_o = Gn^2\lambda^2/[4\pi(1-\nu)\gamma]$, $A^* = \sqrt{A'/B'}$ and $B^* = 1/B'$. σ_{1B} is the stress required for unstable expansion of an isolated dislocation cracks of the Bullough-Gilman type, which is equal to the fracture stress of a solid in the case of an isolated crack[17]. c_o is the critical length of this isolated crack, i.e., the length at the onset of unstable expansion[17]. Now, the right hand side of Eq. (4.13) has a maximum at a certain value of c^* for a given value of b. When σ exceeds this maximum, Eq. (4.13) has no real roots for c. Therefore, σ'_{2B} is found as the maximum of the right hand side of Eq. (4.13), i.e.,

$$\sigma'_{2B} = \sigma_{1B}\left[2\left(\frac{B^*}{c^*}\right) - \left(\frac{A^*}{c^*}\right)^2\right]^{\frac{1}{2}}_{max} \equiv \sigma_{1B}\varphi\left(\frac{b}{c_0}\right). \qquad (4.14)$$

The calculated values of the ratio σ'_{2B}/σ_{1B} by Eq. (4.14) are plotted against b/c_0 in Fig. 4.2. From Fig. 4.2 the following are seen: (i) the stress required for unstable expansion of two collinear dislocation cracks of the Bullough-Gilman type is smaller than that for an isolated dislocation crack of the same type, and decreases as the distance between the outside tips of the cracks decreases. (ii) When this distance is smaller than about 2.6 c_0, the unstable expansion occurs under no applied stress. In the case of elastic cracks[16], the similar trend as (i) is seen, but the second trend is not found. The second trend is due to energy of slip dislocations contained in the cracks.

Now, when σ reaches σ'_{2B}, two dislocation cracks join. Let σ''_{2B} be the stress required for propagation of the joined crack, which is given as:

$$\sigma''_{2B} = \sqrt{\frac{4G\gamma}{\pi(1-\nu)b}} = \sigma_{1B}\left(\frac{b}{c_0}\right)^{-\frac{1}{2}}. \qquad (4.15)$$

Then, σ_{2B}, the fracture stress of the bulk solid is obtained as the larger of σ'_{2B} and σ''_{2B} from Eqs. (4.14) and (4.15) as:

$$\sigma_{2B} = \begin{cases} \sigma_{1B}\left(\frac{b}{c_0}\right)^{-\frac{1}{2}} & if\ \frac{b}{c_0} \lesssim 1.9 \\ \sigma_{1B}\varphi\left(\frac{b}{c_0}\right) & if\ \frac{b}{c_0} \gtrsim 1.9\ . \end{cases} \qquad (4.16)$$

The calculated values of the ratio σ_{2B}/σ_{1B} by Eq. (4.16) are plotted against b/c_0 in Fig.4.2. From Fig.4.2 it is seen that the effect of interaction on the fracture strength in this case is different from that in the case of two collinear elastic cracks in that the fracture strength increases due to the interaction in the former case when the distance between two cracks is small, whereas the fracture strength of a solid with two collinear elastic cracks never exceeds the fracture strength of a solid with an isolated elastic crack.

Case II The case in which crack expansion occurs at the outside tips

We consider two collinear dislocation cracks of the Bullough-Gilman type which are the same as shown in Fig. 4.1. In the present case, however, it is supposed that crack expansion is prevented at the inside tips by obstacles and it occurs at the outside tips. Calculating the

stress intensity factors at the tips x=±b, the rate of change in energy accompanying expansion can be obtained in the similar way as in the case I as:

$$\frac{dE}{dc} = -M'\frac{Gn^2\lambda^2}{4\pi(1-\nu)c} - N'\frac{\pi(1-\nu)\sigma^2 c}{4G} + 2\gamma, \qquad (4.17)$$

where $S = \frac{c}{a}$, $M' = \frac{\pi^2}{4}\frac{1+S}{1+\frac{S}{2}}\frac{1}{K'^2\left(\frac{1}{1+S}\right)}$, and $N' = \frac{(1+S)^3}{S^2(1+\frac{S}{2})}\left\{1 - \frac{E'\left(\frac{1}{1+S}\right)}{K'\left(\frac{1}{1+S}\right)}\right\}^2$.

Let σ^*_{2B} be the stress required for unstable expansion, which, in the present case, is regarded as the fracture stress of the bulk solid. σ^*_{2B} is obtained as:

$$\sigma^*_{2B} = \sigma_{1B}\left[2\left(\frac{N^*}{c^*}\right) - \left(\frac{M^*}{c^*}\right)^2\right]^{\frac{1}{2}}_{max}, \qquad (4.18)$$

where $M^* = \sqrt{M'/N'}$, $N^* = 1/N'$. The calculated values of the ratio $\sigma^*_{2B}/\sigma_{1B}$ by Eq. (4.18) are plotted against a/c_0 in Fig. 4.3. From Fig.4.3 it is seen that the fracture stress of a solid with two collinear dislocation cracks of the Bullough-Gilman type is greater than the fracture stress of a solid with an isolated dislocation crack of the same type and increases as the distance between the inside tips decreases, when expansion at the inside tips is prevented. The contrary trend is seen in the case of elastic cracks if expansion at the inside tips is prevented. Moreover, effect of interaction on the fracture stress is far more remarkable in the case of dislocation cracks of the Bullough-Gilman type than in the case of elastic cracks. In fact, when the distance between the inside tips approaches to zero, the fracture stress becomes infinitely large in the former case whereas the fracture stress decreases only by a factor of $\sqrt{2}$ compared with the fracture stress of a solid with an isolated crack in the latter case. This is due to slip dislocations contained in the dislocation cracks.

4.2 The case of the Cottrell type dislocation cracks

Case I The case in which crack expansion occurs at the inside tips

Consider an infinite elastic solid with two collinear dislocation cracks of the Cottrell type subject to an applied stress $\sigma_y = \sigma$ at infinity as shown in Fig. 4.4.

These cracks are simulated by superposing two distributions of infinitesimal dislocations, (a) and (b), which are depicted in Fig. 4.4[17]. It is assumed that n the number of dislocations of each crack in the distribution (a) is constant during deformation. By the similar treatment as in § 4.1, it is found that the distribution functions of (a) and (b) are given by f(x) in Eq. (4.5) and g(x) in Eq. (4.6), respectively. Let N'_σ and N_σ be the stress intensity factors at the tips x=±a due to the distributions (a) and (b) respectively. N'_σ may be calculated from Eq. (4.5) as:

$$N'_\sigma = \frac{G n\lambda}{2(1-\nu)}\,\frac{b}{\sqrt{2a(b^2-a^2)}}\,\frac{1}{K'(a/b)}\,. \tag{4.19}$$

N_σ has been already given by Eq. (4.10). Thus, the rate of change in energy accompanying expansion at the tips $x=\pm a$ can be obtained as:

$$\begin{aligned}\frac{dE}{dc} &= -\frac{\pi(1-\nu)}{G}(N'_\sigma + N_\sigma)^2 + 2\gamma \\ &= -A'\frac{G n^2\lambda^2}{4\pi(1-\nu)c} - B'\frac{\pi(1-\nu)\sigma^2 c}{4G} - \sqrt{A'B'}\,\frac{n\lambda\sigma}{2} + 2\gamma,\end{aligned} \tag{4.20}$$

where A', B' and c are the same as defined in § 4.1. Eq. (4.20) reduces to the corresponding equation for an isolated crack of the Cottrell type derived by Bullough[17]. Let σ'_{2c} be the stress required for unstable expansion. By the quite similar treatment as in § 4.1, σ'_{2c} is obtained as:

$$\sigma'_{2c} = \sigma_{1c}\cdot 2\left[\sqrt{\frac{2B^*}{C^*}} - \frac{A^*}{C^*}\right]_{max} \equiv \sigma_{1c}\,\psi\!\left(\frac{b}{c_o}\right), \tag{4.21}$$

where c_o, c^*, A^* and B^* are the same as defined in § 4.1 and $\sigma_{1c}=2\gamma/n\lambda$. σ_{1c} is the stress required for unstable expansion of an isolated dislocation crack of the Cottrell type, which is equal to the fracture stress of a bulk solid in the case of an isolated crack[17]. The calculated values of σ'_{2c}/σ_{1c} by Eq. (4.21) are plotted against b/c_o in Fig. 4.2. From Fig. 4.2 it is found that the effect of interaction of two collinear cracks on the stress required for unstable expansion in the case of the Cottrell type cracks is similar to that in the case of the Bullough-Gilman type cracks of the case I.

σ_{2c}, the fracture stress of the bulk solid is given by the similar reasoning as in § 4.1 as:

$$\sigma_{2c} = \begin{cases} 2\sigma_{1c}\left(\frac{b}{c_o}\right)^{-\frac{1}{2}} & \text{if } \frac{b}{c_o} \lesssim 5.2 \\ \sigma_{1c}\,\psi\!\left(\frac{b}{c_o}\right) & \text{if } \frac{b}{c_o} \gtrsim 5.2\,. \end{cases} \tag{4.22}$$

The calculated values of the ratio σ_{2c}/σ_{1c} are plotted in Fig. 4.2. From Fig. 4.2 it is found that the general trend of the effect of interaction of two collinear dislocation cracks on the fracture stress in the case of the Cottrell type cracks is similar to that in the case of the Bullough-Gilman type cracks. However the degree of effect is considerably greater in the former case than in the latter case, when the distance between two cracks is so small that the propagation stress of the joined crack determines the fracture stress. This is due to that the propagation stress of an isolated dislocation crack of the Cottrell type is smaller than that of the Bullough-Gilman type crack by a factor of 2.

Case II The case in which crack expansion occurs at the outside tips

In the present case, we consider two collinear dislocation cracks of the Cottrell type as shown in Fig. 4.5. Calculating the stress intensity factors at the tips $x=\pm b$, the rate of change in energy accompanying crack expansion is calculated in the similar way as in the Case I as:

$$\frac{dE}{dc} = -M'\frac{G n^2\lambda^2}{4\pi(1-\nu)c} - N'\frac{\pi(1-\nu)\sigma^2 c}{4G} - \sqrt{M'N'}\,\frac{n\lambda\sigma}{2} + 2\gamma, \tag{4.23}$$

where M' and N' are the same as defined in § 4.1. The stress required for unstable expansion, σ^*_{2c}, which is regarded as the fracture stress of the bulk solid in this case is obtained as:

$$\sigma^*_{2c} = \sigma_{1c}\cdot 2\left[\sqrt{\frac{2N^*}{C^*}} - \frac{M^*}{C^*}\right]_{max}. \tag{4.24}$$

The calculated values of $\sigma^*_{2c}/\sigma_{1c}$ by Eq. (4.24) are plotted against a/c_o in Fig. 4.3. The general trend of the variation of $\sigma^*_{2c}/\sigma_{1c}$ with a/c_o is similar to that of $\sigma^*_{2B}/\sigma_{1B}$ in § 4.1.

§5. INTERACTION BETWEEN AN ELASTIC CRACK AND A DISLOCATION CRACK ON THE SAME PLANE

Consider an infinite elastic solid with a dislocation crack of the Cottrell type and an elastic crack on the same plane subject to an applied stress $\sigma_y = \sigma$ at infinity as shown in Fig. 5.1. Let $\ell(=a-b)$ be the length of the elastic crack, $r(=c-d)$ be that of the dislocation crack, and $t(=b-c)$ be the distance between the inside tips. Here, r and t depend on the applied stress, whereas $s=r+t$ is constant. This system can be simulated by superposing two distributions (a) and (b) of infinitesimal dislocations as shown in Fig. 5.1. It is assumed that n, the number of dislocations of the Cottrell type crack in the distribution (b) is constant during deformation. Let f(x) be the distribution function of the superposed distribution, (a) + (b). From the requirement for equilibrium of each dislocation, f(x) must satisfy the following equation:

$$A\int_D \frac{f(\xi)}{\xi - x}\,d\xi + \sigma = 0, \tag{5.1}$$

where D is the regions (d, c) and (b, a). The boundary conditions are that f(x) is unbounded at x=a, b, c and d. With these and the following supplementary conditions:

$$\int_b^a f(x)\,dx = 0 \tag{5.2}$$

$$-\int_d^c f(x)\,dx = n, \tag{5.3}$$

Eq.(5.1) can be solved as:

$$f(x)=\frac{\mp}{\sqrt{-(x-a)(x-b)(x-c)(x-d)}}\left[\frac{\sigma}{\pi A}\left\{x^2-\frac{a+b+c+d}{2}x+\frac{ad+bc}{2}+\frac{(a-c)(b-d)}{2}\frac{K(k)-E(k)}{K(k)}\right\}+\frac{n}{2}\frac{\sqrt{(a-c)(b-d)}}{(b-c)K(k)}\cdot\frac{(x-c)K(k)-(b-c)\Pi(\rho,k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}\right], \tag{5.4}$$

where the negative and positive signs are for the region (b, a) and for the region (d, c) respectively. k, ρ and ρ' are the same as in § 3. Since the stress concentration is considered greater at the tip x=b than at the tip x=a, propagation of the elastic crack will be initiated at the tip x=b. The stress σ_y on the y=0 plane is calculated from Eq. (5.4) for $c<x<b$ as:

$$\sigma_y=\frac{-1}{\sqrt{(x-a)(x-b)(x-c)(x-d)}}\left[\sigma\left\{x^2-\frac{a+b+c+d}{2}x+\frac{ad+bc}{2}+\frac{(a-c)(b-d)}{2}\cdot\frac{K(k)-E(k)}{K(k)}\right\}+An\frac{\sqrt{(a-c)(b-d)}}{b-c}\cdot\frac{\pi/2}{K(k)}\cdot\frac{(x-c)K(k)-(b-c)\Pi(\rho,k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}\right]. \tag{5.5}$$

From Eq.(5.5) N(b) and N(c), the stress intensity factors at the tips x=b and x=c, are;

$$N(b)=\frac{\sigma}{2}\sqrt{\frac{b-d}{(a-b)(b-c)}}\left[(a-b)-(a-c)\frac{K(k)-E(k)}{K(k)}\right]+An\sqrt{\frac{a-c}{(a-b)(b-c)}}\cdot\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)-K(k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)} \tag{5.6}$$

$$N(c)=\frac{\sigma}{2}\sqrt{\frac{a-c}{(b-c)(c-d)}}\left[(c-d)-(b-d)\frac{K(k)-E(k)}{K(k)}\right]+An\sqrt{\frac{b-d}{(b-c)(c-d)}}\cdot\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}. \tag{5.7}$$

Now, the stable equilibrium length of the dislocation crack for a given value of σ is found by solving the equation, $-[\pi(1-\nu)/G]N^2(c)+2\gamma=0$, that is,

$$\frac{\frac{\ell}{r_c}+\frac{t}{r_c}}{\frac{t}{r_c}}\left[\frac{r}{r_c}-\left(\frac{r}{r_c}+\frac{t}{r_c}\right)\frac{K(k)-E(k)}{K(k)}\right]^2-\left(\frac{\sigma}{\sigma_{1c}}\right)^{-2}\left[4\left(\frac{r}{r_c}\right)-\frac{\frac{r}{r_c}+\frac{t}{r_c}}{\frac{t}{r_c}}\left\{\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}\right\}^2\right]$$
$$+2\left(\frac{\sigma}{\sigma_{1c}}\right)^{-1}\frac{\sqrt{\left(\frac{r}{r_c}+\frac{t}{r_c}\right)\left(\frac{\ell}{r_c}+\frac{t}{r_c}\right)}}{\frac{t}{r_c}}\left[\frac{r}{r_c}-\left(\frac{r}{r_c}+\frac{t}{r_c}\right)\frac{K(k)-E(k)}{K(k)}\right]\times\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}=0, \tag{5.8}$$

with respect to r. Here $t=s-r$, $\sigma_{1c}=2\gamma/(n\lambda)$ and $r_c=Gn^2\lambda^2/[2\pi(1-\nu)\gamma]$. σ_{1c} is the stress required for unstable expansion of an isolated dislocation crack of the Cottrell type and r_c is the critical length of it.[17] Provided that the dislocation crack initiates unstable expansion before the elastic crack does, σ_c', the stress required for unstable expansion of the dislocation crack in the present case is obtained from the condition that Eq.(5.8) for r has no real roots for the values of σ larger than σ_c. On the other hand, the condition for propagation of the elastic crack is given as:

$$-\frac{\pi(1-\nu)}{G}N^2(b)+2\gamma=0,$$

that is,

$$\frac{\frac{r}{r_c}+\frac{t}{r_c}}{\frac{t}{r_c}}\left[\frac{\ell}{r_c}-\left(\frac{\ell}{r_c}+\frac{t}{r_c}\right)\frac{K(k)-E(k)}{K(k)}\right]^2-\left(\frac{\sigma}{\sigma_{1c}}\right)^{-2}\left[4\left(\frac{\ell}{r_c}\right)-\frac{\frac{\ell}{r_c}+\frac{t}{r_c}}{\frac{t}{r_c}}\left\{\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)-K(k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}\right\}^2\right]$$
$$+2\left(\frac{\sigma}{\sigma_{1c}}\right)^{-1}\frac{\sqrt{\left(\frac{r}{r_c}+\frac{t}{r_c}\right)\left(\frac{\ell}{r_c}+\frac{t}{r_c}\right)}}{\frac{t}{r_c}}\left[\frac{\ell}{r_c}-\left(\frac{\ell}{r_c}+\frac{t}{r_c}\right)\frac{K(k)-E(k)}{K(k)}\right]\times\frac{\pi/2}{K(k)}\cdot\frac{\Pi(\rho,k)-K(k)}{\Pi(\rho,k)+\Pi(\rho',k)-K(k)}=0. \tag{5.9}$$

Provided that the elastic crack initiates unstable expansion before the dislocation crack does, σ_E, the stress required for propagation of the elastic crack is obtained by solving the simultaneous equations of Eqs.(5.8) and (5.9) with respect to r and σ. Let r_c' be the critical length of the dislocation crack in the present case and σ_E' be σ_E for $r=r_c'$. Then, (i) if $\sigma_c'<\sigma_E'$, crack propagation is initiated by the dislocation crack, (ii) if $\sigma_c'>\sigma_E'$, it is initiated by the elastic crack, and (iii) if $\sigma_c'=\sigma_E'$, it is initiated by both the cracks at the same time. It is to be noticed that the stress required for crack propagation in the case when $\sigma_c'>\sigma_E'$ is not given by σ_E' but by the root of the simultaneous equations of Eqs.(5.8) and (5.9), which is necessarily smaller than σ_E.

Now, we consider the case when each crack has the same propagation stress when isolated. By treating such a case, it will be clarified which of the dislocation crack and the elastic crack suffers larger effect of interaction. Since the propagation stress of the dislocation crack of the Cottrell type is $2\gamma/n\lambda\,(=\sqrt{2G\gamma/[\pi(1-\nu)r_c]})$ and that of the elastic crack is $\sqrt{8G\gamma/[\pi(1-\nu)\ell]}$ when each crack is isolated, the above mentioned condition yields the relation $\ell=4r_c$.

Substituting this relation into Eqs.(5.8) and (5.9), σ_c' and σ_E' were calculated as a function of s. The results showed that σ_c' is smaller than σ_E' independent of values of s. This implies that propagation is initiated by the dislocation crack in this case and that the dislocation crack suffers larger effect of interaction than the elastic crack. The calculated values of σ_c' are plotted in Fig.5.2 in terms of the ratio σ_c'/σ_{1c}. It is seen that σ_c' decreases as s decreases and that the dislocation crack propagates under no applied stress if $s \lesssim 0.5\, r_c$.

Now, when the applied stress σ reaches σ_c', two cracks join. The joined crack is a dislocation crack of the Cottrell type with length $\ell +$ s $(=4r_c + s)$ in the present case. In Fig.5.3 the relation between E, energy of a solid with an isolated dislocation crack of the Cottrell type and its length r is illustrated schematically. For $\sigma = \sigma_{1c}$, E has a stationary value at $r=r_c$. For $\sigma = \sigma_c'$, E has a maximum at $r=r_a$ and a minimum at $r=r_b$, since $\sigma_c' < \sigma_{1c}$. Now, if the length of the joined crack, $\ell + s$ is larger than r_a, the joined crack continues to propagate under σ held at σ_c', since E decreases as propagation proceeds. However if $\ell + s$ is smaller than r_a, propagation does not occur without further increase in σ. Thus when the length of the joined crack, $\ell + s$ is smaller than r_a, σ should be increased to the value σ_c'' for which E has a maximum at $r = \ell + s$, so that propagation may become possible. That is, for $\sigma = \sigma_c''$, E decreases as propagation proceeds. Notice that $\ell + s$ cannot be smaller than r_b, because $r_b < r_c$ and $\ell + s > 4r_c$. σ_c'' is obtained by $t \to \infty$ and $r \to \ell + s$ in Eq. (5.8) as:

$$\sigma_c'' = \sigma_{1c}\,\frac{2\sqrt{\dfrac{\ell+s}{r_c}} - 1}{\dfrac{\ell+s}{r_c}} \qquad (5.10)$$

Since Eq.(5.10) holds good when $\ell + s > r_a$ as well as when $\ell + s < r_a$, it follows that σ_2^{**}, the fracture stress of the bulk solid is always determined by the larger of σ_c' and σ_c'' whether $\ell + s$ is smaller than r_a or not. The calculated values of σ_2^{**} by Eqs.(5.8) and (5.10) are shown in Fig.5.2 in terms of the ratio $\sigma_2^{**}/\sigma_{1c}$. It is seen that the general trend of the effect of interaction on the fracture stress in the present case is similar as in the case of two collinear elastic cracks.

§6. CONCLUSIONS

The effect of interaction between elastic cracks, dislocation cracks and slip bands on the fracture strength of a solid was studied for various cases. Comparing the results for each case, the following conclusions were obtained:

(1) In the case of two collinear asymmetrical elastic cracks, crack

propagation is initiated by the larger crack.

(2) The effect of interaction of two collinear asymmetrical elastic cracks on the fracture strength of a solid is considerably dependent upon the length of the second crack when the distance between the inside tips is small.

(3) The interaction of two collinear asymmetrical elastic cracks may be neglected when the distance between the inside tips exceeds the second crack length, in the sense that the change in the fracture strength due to interaction is at most 5%.

(4) The critical distance between inside tips of two collinear asymmetrical elastic cracks with which two cracks can be regarded as one joined crack as far as the fracture strength is concerned, is about 10% of the length of the first crack nearly independent of the length of the second crack.

(5) The effect of interaction between an elastic crack and a slip band on the same plane upon the fracture strength of a solid is greater than that of two collinear elastic cracks except when the distance between the crack and the slip band is small, if the length of the slip band is equal to that of the second crack and if the distance between the piled-up end and the crack tip nearer to the piled-up end is equal to the distance between the inside tips of two cracks.

(6) The number of piled-up dislocations increases due to interaction between the slip band and an elastic crack on the same plane.

(7) The effect of interaction of two collinear dislocation cracks on the fracture strength of a solid in the case of the Bullough-Gilman type cracks is similar to that in the case of the Cottrell type cracks except when the distance between two cracks is small.

(8) In the case in which crack expansion occurs at the inside tips the effect of interaction of two collinear dislocation cracks (both in the case of the Bullough-Gilman type cracks and in the case of the Cottrell type cracks) on the fracture strength of a solid is different from that of two collinear elastic cracks in that the fracture strength increases due to the interaction in the case of dislocation cracks when the distance between two cracks is small, whereas the fracture strength of a solid with two collinear elastic cracks never exceeds that of a solid with an isolated elastic crack.

(9) In the case in which crack expansion occurs at the inside tips, two collinear dislocation cracks (both in the case of the Bullough-Gilman type cracks and in the case of the Cottrell type cracks) join under no applied stress when the distance between two cracks is small. The similar phenomenon exists in the case of an elastic crack and a dislocation crack of the Cottrell type on the same plane.

(10) In the case in which crack expansion occurs at the outside tips, the fracture strength of a solid with two collinear dislocation cracks (both in the case of the Bullough-Gilman type cracks and in the case of the Cottrell type cracks) becomes greater than the fracture strength of a solid with an isolated dislocation crack due to the interaction.

(11) The effect of interaction between an elastic crack and a dislocation crack of the Cottrell type on the same plane upon the fracture strength is similar to that of two collinear elastic cracks.

(12) In the case of an elastic crack and a dislocation crack of the Cottrell type on the same plane, crack propagation is initiated by the latter, if each crack has the same propagation stress when isolated.

REFERENCES

1) G.I. Barenblatt, Advance in Appl. Mech., Vol.7, 1962, P.55.
2) B.A. Bilby, A.H. Cottrell and K.H. Swinden, Proc. Roy. Soc. A., Vol.272, 1963. P.304.
3) B.A. Bilby, A.H. Cottrell, E. Smith and K.H. Swinden, Proc. Roy. Soc. A., Vol.279, 1964, P.1.
4) E. Smith, Proc. Roy. Soc. A., Vol.282, 1964, P.422.
5) E. Smith, Int. J. Eng. Sci., Vol.2, 1964, P.379.
6) E. Smith, Proc. Roy. Soc. A., Vol.285, 1965, P.46.
7) A.N. Stroh, Proc. Roy. Soc. A., Vol.232, 1955, P.548.
8) A. Deruyttere and G.B. Greenough, J. Inst. Met., Vol.84, 1956, P.337.
9) J.J. Gilman, Trans.Amer. Inst. Min. Eng., Vol.212, 1958, P.783.
10) A.H. Cottrell, Trans. Amer. Inst. Min. Eng., Vol.212, 1958, P.192.
11) G. Leibfried, Z. Phys., Bd.130, 1951, s.214.
12) A.K. Head and N. Louat, Aust. J. Phys., Vol.8, 1955, P.1.
13) R. Priestner and N. Louat, Acta Met., Vol.11, 1963 ,P.195.
14) A.A. Griffith, Phil. Trans. Roy. Soc., Vol.221, 1920, P.163.
15) N.I. Muskhelishvili, Singular Integral Equations, N.V.P., Noordhoff, Groningen, 1953.
16) T.J. Wilmore, Quart. J. Mech. Appl. Math., Vol.2, 1949, P.53.
17) R.Bullough, Phil. Mag., Vol.9, 1964, P.917.
18) M. Ichikawa, M. Ohashi and T. Yokobori, Rep. Res. Inst. Strength and Fracture of Materials, Vol.1, No.1, 1965, P.1.
19) T. Yokobori and M. Ichikawa, Int. J. Fracture Mech., Vol.1, No.3, 1965, P.217.

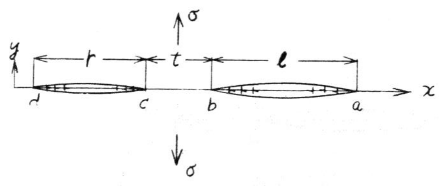


Fig.2.1. Schematic illustration of two collinear asymmetrical elastic cracks.

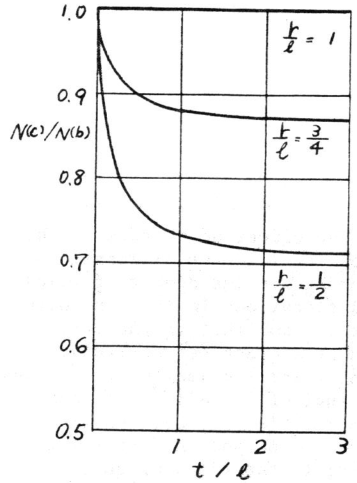


Fig.2.2. The ratio of N(c), the stress intensity factor at the inside tip of the second crack to N(b), that of the first crack. ℓ and r are the length of the first crack and that of the second crack respectively, and t is the distance between the inside tips.

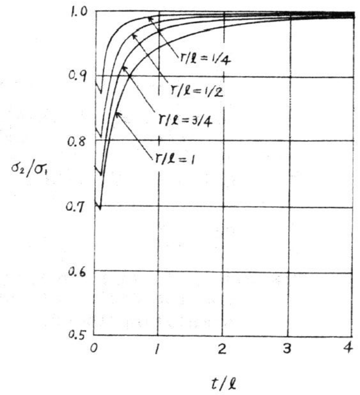


Fig.2.3. The dependence of the ratio of the fracture stress σ_2 of a solid with two collinear cracks to the fracture stress σ_1 of a solid with the first crack only, σ_2/σ_1, upon the length of the first crack ℓ, that of the second crack r, and the distance between the inside tips t.

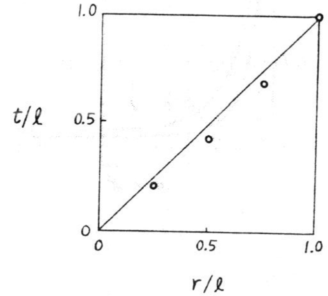


Fig.2.4. The critical distance between the inside tips with which the change in the fracture strength due to interaction is at most 5%. o expresses the critical distance.

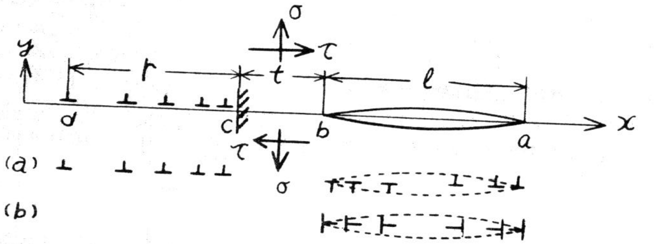


Fig.3.1. A slip band and an elastic crack on the same plane, and the distributions of infinitesimal dislocations simulating those.

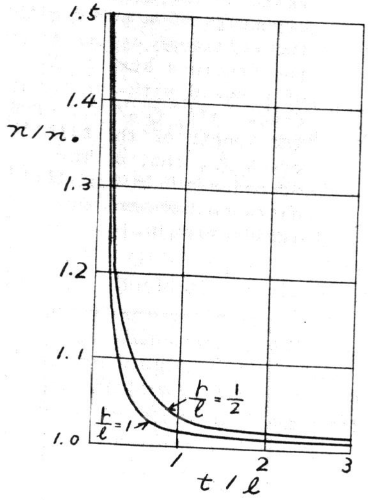


Fig.3.2. The effect of a crack on the number of piled-up dislocations. n and n_0 are the number of piled-up dislocations in the case with a crack, and that in the case without a crack respectively. ℓ and r are the length of the crack and that of the slip band, and t is the distance between the piled-up end and the crack tip nearer to the piled-up end.

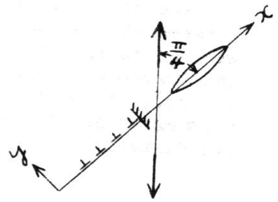
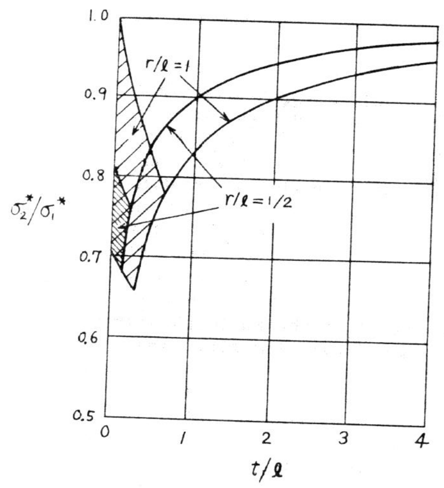


Fig.3.3. The case of uni-axial tension. The tensile axis makes $\pi/4$ with the y=0 plane.



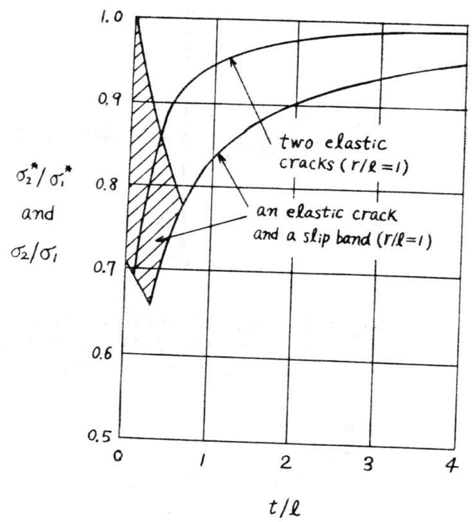


Fig.3.4. The effect of a slip band on the fracture stress of a solid with a crack. σ_2^* is the fracture stress in the case with a slip band and σ_1^* is that in the case without a slip band. ℓ and r are the length of the crack and that of the slip band respectively, and t is the distance between the piled-up end and the crack tip nearer to the piled-up end. In the region where t/ℓ is small, the results are shown by the hatched area bounded by the upper and the lower limits of the ratio σ_2^*/σ_1^*.

Fig.3.5. Comparison of the interaction between an elastic crack and a slip band on the same plane with the interaction of two collinear elastic cracks. σ_2^* is the fracture strength of a solid with a crack (length ℓ) and a slip band (length r) with a distance t. σ_2 is the fracture strength of a solid with two collinear elastic cracks (length ℓ and r) with a distance t. $\sigma_1^*(=\sigma_1)$ is the fracture strength of a solid with an isolated crack (length ℓ).

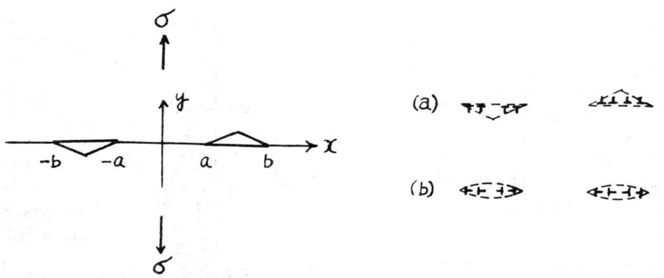


Fig.4.1. Two collinear dislocation cracks of the Bullough-Gilman type and the distributions of infinitesimal dislocations simulating the cracks.

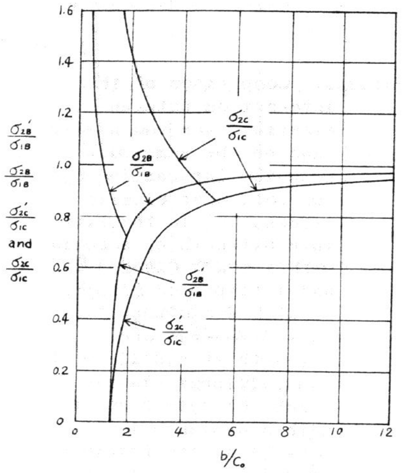


Fig.4.2. The effect of interaction of two collinear dislocation cracks on the stress required for unstable crack expansion and on the fracture stress of a solid in the case when crack expansion occurs at the inside tips. σ_{1B} and σ_{1C} are the fracture stresses of a solid with an isolated dislocation crack of the Bullough-Gilman type and that for the Cottrell type one respectively. σ_{2B}' is the stress required for unstable expansion of two collinear dislocation cracks of the Bullough-Gilman type, and σ_{2C}' is that for the Cottrell type ones. σ_{2B} is the fracture stress of a solid with two collinear dislocation cracks of the Bullough-Gilman type, and σ_{2C} is that for the Cottrell type ones. b is a half of the distance between the outside tips, and c_o is the critical length, that is, the length at the onset of unstable expansion, of an isolated dislocation crack of the Bullough-Gilman type.

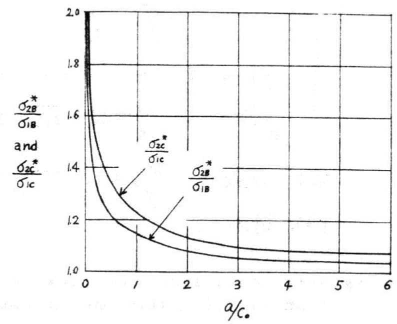


Fig.4.3. The effect of interaction of two collinear dislocation cracks on the fracture stress in the case when crack expansion occurs at the outside tips. σ_{1B} and σ_{1C} are the fracture stresses of a solid with an isolated dislocation crack of the Bullough-Gilman type and that for the Cottrell type one, respectively. σ_{2B}^* is the fracture stress of a solid with two collinear dislocation cracks of the Bullough-Gilman type, and σ_{2C}^* is that for the Cottrell type ones. a is a half of the distance between the inside tips and c_o is the critical length of an isolated dislocation crack of the Bullough-Gilman type.

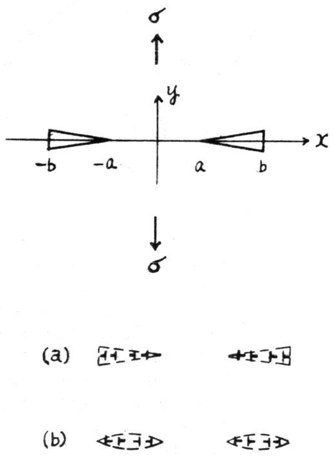


Fig.4.4. Two collinear dislocation cracks of the Cottrell type and the distributions of infinitesimal dislocations simulating the cracks.

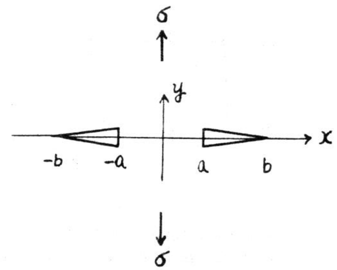


Fig.4.5. Two collinear dislocation cracks of the Cottrell type.

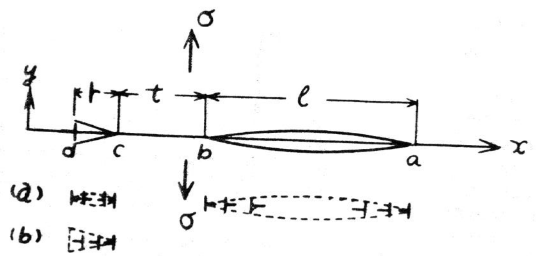


Fig.5.1. A dislocation crack of the Cottrell type and an elastic crack on the same plane.

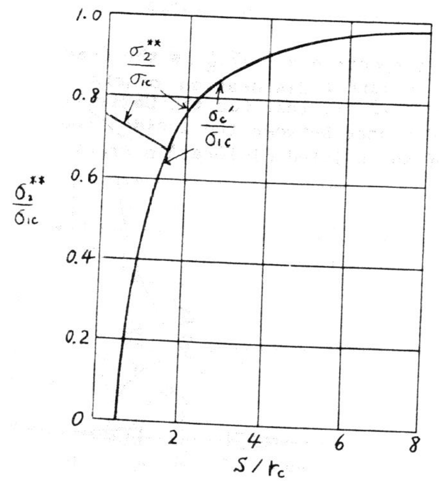


Fig.5.2. Effect of interaction on the stress required for crack propagation and on the fracture stress. σ_{1c} is the propagation stress of an isolated dislocation crack of the Cottrell type, which is equal to that of an isolated elastic crack in this case. σ'_c is the stress required for unstable expansion of the dislocation crack. σ_2^{**} is the fracture stress of the bulk solid. s is the distance between the outside tip of the dislocation crack and the inside tip of the elastic crack. r_c is the critical length of an isolated dislocation crack of the Cottrell type.

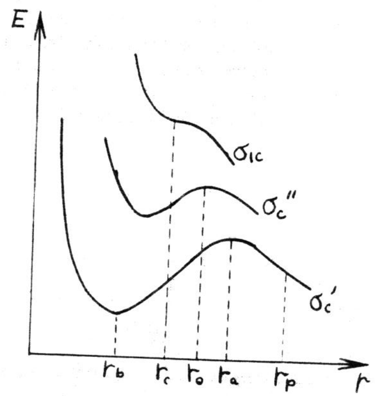


Fig.5.3. Schematic illustration of the relation between E, energy of a solid with an isolated dislocation crack of the Cottrell type and its length r. When the joined crack has such length larger than r_a as r_p, it propagates under σ'_c. When the joined crack has such length r_a smaller than r_a, it cannot propagate until σ is increased to σ''_c.