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FRACTURE AT STRESS CONCENTRATIONS

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ABSTRACT

When a large structure is subject to a temperature lower than the crack-arrest temperature of the particular steel from which it is built, the problem of fracture initiation at stress concentrations becomes of paramount importance. If a steel is very brittle, then inclusions or small defects may act as the cracks from which final fracture is initiated; the present work emphasizes the importance of the distribution of defects on the behaviour of steel structures.

The theoretical analysis is based on a plastically relaxed model for a slit and the criterion for fracture initiation is the attainment of a critical local displacement. Previous results concerning the interaction between relaxed slits are briefly reviewed and some new results are described using a simple procedure. The models are particularly suitable for representing a locally bad region containing a distribution of defects Quantitative estimates are given. and the work indicates that a distribution of small defects can be as dangerous as a single large one. even though the volume fraction may be smaller in the first case. Thus one must consider the distribution as well as the size of defects when assessing the risk of a failure.

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1. INTRODUCTION

The most important characteristic of a brittle fracture of a steel structure is that a crack can propagate and cause complete failure of the structure, at a stress which is less than the design or general yield stress If instead a crack were initiated in a small highly stressed area, or in a region of poor quality material arising respectively from unsatisfactory stress relieving or welding operations and then only propagated part way into the bulk of the structure, the consequences, while very inconvenient, might not be so alarming, as it might be possible to deal with such a crack. Thus a crack must be prevented from propagating in a fast unstable manner, and a way of doing this is to operate the structure at a temperature in excess of the "crack arrest temperature" of the steel[1]. This critical temperature is one above which a crack cannot propagate by a cleavage mechanism, and although it might still propagate by a ductile process, the minimum unstable length for this mode of fracture is generally very large. Accordingly, a structure should be reasonably safe from a brittle failure if its temperature is not allowed to fall below its crack arrest temperature.

The standard method of measuring the crack arrest temperature of a material is to employ the Robertson test[2], whereby a local explosion is used to start a crack in a thick plate of the material, which is subject to a tensile stress representative of the actual operating stress in service. The plate is subject to a temperature gradient, and the temperature in that region of the plate where the crack stops is used as the crack arrest temperature. It is difficult to imagine a severer test than this, but unfortunately it is very expensive to conduct, and one usually uses some simpler test whose results correlate with those obtained from the Robertson test; such a test is the Charpy impact energy test.

If a large structure is operated at a temperature in excess of the crack arrest temperature of the steel, it has a built-in resistance to brittle fracture; however, it is well known that the service temperatures of some engineering structures are below their crack arrest temperatures, and in such circumstances prevention of fracture initiation is of paramount importance. When failure occurs just below the crack arrest temperature it frequently starts at the root of a notch of some description as a ductile fibrous fracture changing over to cleavage as the fracture progresses. At lower temperatures or if the steel is embrittled by, for example, hydrogen pick-up or strain-ageing effects, the fracture can start directly as a cleavage fracture. In either case, except perhaps in some very special circumstances, fracture initiation will be preceded by some local plastic deformation, and it is the main function of the notch to produce a sufficient localized concentration of strain to initiate a fracture. In practice, the notch may be a design fault, a welding defect, or in the case of a particularly brittle steel or at very low temperatures, a small defect or foreign inclusion in the steel itself.

Following earlier work, the criterion for fracture initiation at a notch is assumed to be the attainment of a critical displacement at the root[3]. In the lower temperature range, where unstable cleavage fracture starts directly from plastic deformation, the criterion will be both a necessary and a sufficient one for complete failure, and notch sharpness can be taken into account by assuming that the critical displacement is the product of the notch root diameter and a critical local strain. It is expected that this relation holds down to the critical notch sharpness at which a microscopic size effect appears[4]. In the higher temperature range, where unstable cleavage fracture is preceded by stable ductile fracture, this approach can still be used if we wish to be over-safe and do not want even stable ductile fractures to form However, the condition for complete failure must then be such that the critical displacement is the total displacement at the notch root needed for unstable fracture to occur.

Of course, in practice the critical displacement will be dependent on the mode of fracture initiation, and hence for example on the degree to which a plane strain situation is developed (In fact, when fracture initiation is of the cleavage type, some workers[5,6,7] have assumed that the initiation is governed by the achievement of a critical local tensile stress. However since local strain is required to achieve this stress we persevere with the displacement criterion as this allows a unified analysis to be conducted for the whole temperature range. The validity of this simplifying assumption is being examined in detail.)

If the critical displacement can be accommodated by short plastic zones compared with the specimen width, then fracture can be initiated at a stress below that required for general yield, but of course in excess of that required for local yield at the notch. Thus, in order to ascertain whether fracture occurs before or after general yield, it is necessary to determine what length of plastic zone is needed to accommodate a given plastic displacement at the notch root, and to find the fracture stress it is necessary to know what applied stress gives rise to a given local displacement.

In an earlier paper[3] a simple form of this problem was analyzed by means of the theory of distributions of dislocations. The model considered was that of an isolated plastically relaxed slit in an infinite body subject to a uniform applied stress, and since an anti-plane strain model was used, it represents a slit or notch in a semi-infinite body. The solutions were in fact applied to the case of a notch in a body of finite width, thus neglecting the stress-free nature of the surface opposite the notch in the actual physical situation. In order to overcome this objection, a later paper[8] considered the plastic relaxation at an infinite series of identical and equally-spaced coplanar slits in an infinite body; as an anti-plane strain model was again used, the solution to this problem can be applied, without objection. to the case of a notch at the surface of a body of finite width.

The main purpose of the present paper is to consider some effects of distributions of defects on the fracture characteristics of large structures. The analysis shows that it is quite simple to consider problems involving distributions of slits when these are far apart, and by extrapolation one can, in some cases, apply the results to situations where the defects are close to each other; these physically more interesting problems appear much more difficult to solve analytically. The results for the isolated slit model are presented in Section 2, as the remainder of the paper is essentially based on them. Section 3 discusses both the infinite sequence of coplanar slits and the infinite sequence of non-coplanar slits[9]. However, to represent a locally bad region containing a distribution of closely spaced defects or inclusions, it is preferable to use a model involving a finite number of slits, and with this situation in mind we consider in Section 4 the spread of plasticity between two identical coplanar slits in the interior of an infinite body. Earlier work[10,11] considered the situation when plasticity just spreads between the slits, but the present paper extends the work so as to deal with the situation where plasticity has not spread between the slits thus giving a comprehensive picture of the whole situation. The behaviour of two non-coplanar slits is also considered, and the results are used in a brief discussion of how inclusion or defect distribution affects the behaviour of large steel structures.

A general feature that emerges from the work is that results for anti-plane strain and plane strain conditions can be very different; the work thus emphasizes the need for exercising caution in using anti-plane strain models (whose behaviour it is relatively simple to analyze) as a substitute for the more frequent plane strain conditions which are met with in practice.

2. THE ISOLATED SLIT SUBJECT TO A UNIFORM APPLIED STRESS

The anti-plane strain model of an isolated relaxed slit considered by Bilby, Cottrell and Swinden, is shown in Figure 1. An infinite isotropic elastic body (shear modulus G), subject to a uniform applied shear stress $p_{23} = \sigma$ at infinity, contains a continuous distribution of long straight screw dislocations lying parallel to the x_3 axis in the x_1x_3 plane. The resistance to motion is zero for the dislocations which describe the freely slipping slit $|x_1| < c$, $x_2 = 0$, whilst those dislocations beyond $|x_1| = c$, $x_2 = 0$ represent the plastically relaxed regions at the end of the slit, and have a resistance to motion σ_1 which is representative of the shear yield stress of the steel under consideration ($\sigma_1 > \sigma$). Considering the problem as one in the theory of distributions of dislocations, and formulating the singular integral equation which expresses the requirement that the resultant shear stress on any dislocation in the distribution is zero when the system is in

equilibrium, Bilby, Cottrell and Swinden[3] showed that the extent of spread of ppasticity a is given by

$$c/a = \cos(\pi\sigma/2\sigma_1) \qquad \text{.....................} \quad (1)$$

and that the relative displacement of the positive side of the plane $x_2 = 0$ with respect to the negative side at the slit tip is

$$\Phi(c) = (4\sigma_1 c/\pi G) \ln \sec(\pi\sigma/2\sigma_1) \qquad \text{........} \quad (2)$$

It is worth emphasizing that the above results are for an anti-plane strain situation, described by a distribution of screw dislocations. For the plane strain situation (described by a distribution of edge dislocations with their Burgers vectors parallel to the x_2 axis) of a slit in a body subject to an applied shear stress $p_{12} = \sigma$, the results are identical to those given by equations (1) and (2) except that the expression for $\Phi(c)$ is larger in the plane strain case by a factor $(1 - \nu)$, where ν is Poisson's ratio.

3. INFINITE SEQUENCES OF IDENTICAL AND EQUALLY SPACED SLITS

3.1. The Infinite Series of Identical and Equally Spaced Coplanar Slits

When σ/σ_1 is small, the shear stress p_{23} at a point $(x_1,0,0)$ along the plane $x_2 = 0$ of an isolated slit and at a large distance from it will be unaffected by the plastic relaxation and the component due to the slit itself will accordingly have a magnitude $\sigma c^2/2x_1^2$. It follows[11] that for an infinite series of identical and equally spaced coplanar slits, the effective applied shear stress on any one of them will be

$$\sigma + 2\frac{\sigma}{2} \sum_{n=1}^{\infty} \frac{c^2}{(2nh)^2} = \sigma\left(1 + \frac{\pi^2 c^2}{24h^2}\right) \qquad \text{................} \quad (3)$$

if the distance between the slit centres is 2h where h >> c (Figure 2). Accordingly, the extent of spread of plasticity a in this case is given, from the use of equations (1) and (3), by

$$\frac{c}{a} = 1 - \frac{\pi^2\sigma^2}{8\sigma_1^2}\left(1 + \frac{\pi^2 c^2}{12h^2}\right) \qquad \text{...........} \quad (4)$$

and the relative displacement at a slit tip is

$$\Phi(c) = \frac{4\sigma_1 c}{\pi G}\,\frac{\pi^2\sigma^2}{8\sigma_1^2}\left(1 + \frac{\pi^2 c^2}{12h^2}\right) \qquad \text{........} \quad (5)$$

using equations (2) and (3).

Expressions (4) and (5) are in agreement with the series expansions, for small c/h and σ/σ_1, derived from the following exact expressions[8] obtained for all c/h, σ/σ_1:

$$\frac{\sin(\pi c/2h)}{\sin(\pi a/2h)} = \cos(\pi\sigma/2\sigma_1) \qquad \text{.........} \qquad (6)$$

$$\Phi(c) = \frac{4h\sigma_1 \sin\alpha}{\pi^2 G} \int_{\psi}^{\pi/2} \frac{\cos\chi}{(1 - \sin^2\alpha \sin^2\chi)^{\frac{1}{2}}} \ln\left(\frac{\sin(\chi + \psi)}{\sin(\chi - \psi)}\right) d\chi \qquad \text{.........} \qquad (7)$$

where $\alpha = \pi a/2h$ and $\psi = \pi(1 - \sigma/\sigma_1)/2$. Expression (7) was not evaluated analytically in the paper by Bilby, Cottrell, Smith and Swinden[8], except for $a = h$, i.e. when the plasticity spreads completely between the infinite sequence of coplanar slits. However, the integration has been performed[12] using the Ferranti Mercury computer. and the results agree with those obtained from equation (5).

As with the model of an isolated slit, the results for an infinite sequence of coplanar slits are similar for both anti-plane strain and plane strain situations. However, whereas the anti-plane strain results can be applied directly to the case of a slit at the surface of a body of finite width, those for the plane strain model cannot be used in a similar manner.

3.2. The Infinite Series of Identical and Equally Spaced Slits with Constant Distance of Vertical Separation

In 3.1. we examined the relaxation of stresses around coplanar slits and emphasized that anti-plane and plane strain shear models give essentially the same results. However, this is not the case when the slits are non-coplanar, for then anti-plane strain and plane strain situations can give rise to vastly different results. Each situation will be examined separately, the discussion, as in the previous section, again being based on the isolated slit results.

3.2.1. The Anti-Plane Strain Model

Consider the spread of plasticity from the infinite sequence of slits $|x_1| \leq c$, $x_2 = \pm\, nh$ ($n = 0, 1, 2$, etc.)* in an infinite body, subject to an externally applied shear stress $p_{23} = \sigma_A$ which causes the body to deform in an anti-plane strain mode (Figure 3). As in the previous work, the discontinuity of displacement along each of the planes $x_2 = \pm\, nh$ can be represented by a continuous distribution of long straight screw dislocations parallel to the x_3 axis and lying in the planes $x_2 = \pm\, nh$. Plastic relaxation around the tips of the slits is represented by screw dislocations coplanar with the slits, the resistance to motion of these dislocations being, as before, due to a friction stress $\sigma_1 > \sigma_A$ and not zero as for the dislocations which represent the slits. The effective applied shear

* Hereafter in the paper, these values of n will be assumed.

stress p_{23} from the slit along the plane $x_2 = 0$ acting on the slit on the plane $x_2 = nh$ will be $-\sigma_A c^2/2n^2h^2$. It follows that for the infinite series, the effective applied shear stress on any one of them will be

$$\sigma_A \left(1 - \frac{\pi^2 c^2}{6h^2}\right) \qquad \text{...................} \qquad (8)$$

Accordingly, the extent of spread of plasticity a_A in this case is given, from the use of equations (1) and (8), by

$$\frac{c}{a_A} = 1 - \frac{\pi^2\sigma_A^2}{8\sigma_1^2}\left(1 - \frac{\pi^2 c^2}{3h^2}\right) \qquad \text{.........} \qquad (9)$$

and the relative displacement at a slit tip is

$$\Phi_A(c) = \frac{4\sigma_1 c}{\pi G} \cdot \frac{\pi^2\sigma_A^2}{8\sigma_1^2}\left(1 - \frac{\pi^2 c^2}{3h^2}\right) \qquad \text{.....} \qquad (10)$$

using equations (2) and (8). These results are in agreement with those obtained from expanding in series form the expressions derived by a formal analysis[9].

3.2.2. The Plane Strain Model

If the screw dislocations of the model in 3.2.1. are replaced by edge dislocations with their Burgers vectors parallel to the x_1 axis and the external stress is altered to $p_{12} = \sigma_P$, the new model is that of an infinite body containing an infinite sequence of relaxed slits deforming under plane strain conditions (Figure 4). In this case, the effective applied shear stress p_{12} on the slit at $x_2 = nh$ arising from the crack along $x_2 = 0$ will be $+\,\sigma_P c^2/2n^2h^2$. It follows that for the infinite series, the effective applied shear stress on any one of them will be

$$\sigma_P \left(1 + \frac{\pi^2 c^2}{6h^2}\right) \qquad \text{.............} \qquad (11)$$

Accordingly, the extent of spread of plasticity a_P is given, from the use of equations (1) and (11), by

$$\frac{c}{a_P} = 1 - \frac{\pi^2\sigma_P^2}{8\sigma_1^2}\left(1 + \frac{\pi^2 c^2}{3h^2}\right) \qquad \text{.....} \qquad (12)$$

and the relative displacement at a slit tip is

$$\Phi_P(c) = \frac{4\sigma_1 c}{\pi G} \frac{\pi^2\sigma_P^2}{8\sigma_1^2}\left(1 + \frac{\pi^2 c^2}{3h^2}\right) \quad \ldots. \quad (13)$$

using equations (2) and (11). Again, a more formal discussion[9] of this problem gives the same results.

3.2.3. Discussion of the Infinite Sequence of Non-Coplanar Slit Results

The extent of spread of plasticity for an infinite sequence of non-coplanar relaxed slits, using the anti-plane strain model, is given by equation (9); for a given applied shear stress, the extent of spread decreases as the distance between the slits decreases. The opposite conclusion is reached for the plane strain model since equation (12) shows that as the distance between the slits decreases, the extent of spread increases.

The difference between the two types of model also manifests itself when the results are applied to the problem of fracture at stress concentrations. Thus, whereas $\Phi_A(c)$ decreases as the cracks become closer in the anti-plane strain model, $\Phi_P(c)$ increases for the plane strain model (see equations (10) and (13)). Assuming that fracture is initiated at a slit tip when the relative displacement there exceeds some critical value[1,3], it is seen that as the slits become closer, the tendency for fracture decreases for the anti-plane strain model, whereas it increases with the plane strain model. The work of this section thus emphasizes the need for exercising caution in using anti-plane strain models, as a substitute for the more frequent plane strain conditions met with in practice. A similar conclusion is reached from the models for two non-coplanar slits which are discussed in the next section, but, as mentioned earlier, no such difference arises with coplanar slit models where the results are essentially the same for both anti-plane and plane strain situations.

4. MODELS INVOLVING TWO SLITS

4.1. Two Coplanar Slits

When a steel is very brittle or when the temperature is very low, then as indicated by Cottrell[11], the critical notch size becomes so small that inclusions or very small defects may act as the cracks from which final fracture is initiated, and the results of the infinite sequence models may be applied directly. However, to represent a locally bad region containing a distribution of small defects or inclusions, it is preferable to use a model involving a finite number of slits. With this situation in mind, we considered[10,11] the spread of plasticity between two identical coplanar slits in the interior of an infinite body and concentrated on the situation when plasticity has spread between the slits. However, in the present section we shall also discuss the situation where plasticity has not spread completely between

the slits, and thus obtain a complete picture of the situation. The details of the model (Figure 5) are identical in all respects with those described for an infinite series of coplanar slits, except of course for the number of slits involved. (As indicated earlier, results for plane strain and anti-plane strain situations are essentially the same when the slits are coplanar; we need therefore only consider the anti-plane strain case.) In the earlier analysis[10,11] we calculated the stress required for plasticity to just spread between the two slits as a function of c/h, where $2c$ is the length of each slit and $2h$ is the spacing between their centres (Figure 6); as the stress is subsequently increased, the relative displacement will be greatest at an inner tip, and its value as a function of c/h for different values of σ/σ_1 is given in Figure 7. When the slits are far apart and plasticity has not spread between them, the effective applied shear stress p_{23} on each of them will be

$$\sigma\left(1 + \frac{c^2}{4h^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

Accordingly, from the use of equations (2) and (14), the relative displacement at the inner slit tip will be

$$\Phi_I = \frac{4\sigma_1 c}{\pi G} \frac{\pi^2\sigma^2}{8\sigma_1^2}\left(1 + \frac{c^2}{2h^2}\right) \quad \ldots\ldots\ldots \quad (15)$$

This result is also presented in Figure 7, thus completing the picture of the effects that can arise from two coplanar slits.

If Φ_c is the critical relative displacement that must be attained at a slit tip for fracture to be initiated, it follows from equation (2) that for a given applied stress σ, the maximum length of isolated slit that is permissible without fracture occurring is $2c^*$ where

$$c^* = \frac{\pi G\Phi_c}{4\sigma_1 \ln \sec(\pi\sigma/2\sigma_1)} \quad \ldots\ldots\ldots\ldots \quad (16)$$

However, if there are two coplanar slits each of length $2c$ and whose centres are a distance $2h$ apart with $h + c \doteq c^*$, provided that σ is large enough for plasticity to spread between the slits and enable Φ_I to exceed Φ_c, then fracture can occur at the inner tip and the slits will link up to form a single slit of total length $2c^*$; this can then propagate since the stress σ is sufficient to allow Φ_c to be attained at the new slit tips.

From equation (16) it follows directly that $\Phi_I = \Phi_c$ when

$$\frac{\pi G\Phi_I}{4c\sigma_1} = \left(1 + \frac{h}{c}\right) \ln \sec\left(\frac{\pi\sigma}{2\sigma_1}\right) \quad \ldots\ldots \quad (17)$$

This variation with c/h is shown for various values of σ/σ_1 by the dotted curves in Figure 7. For example, for $\sigma/\sigma_1 = 2/3$, if $c/h > 0.7$ the presence of the two slits will give rise to complete failure, whilst if $c/h < 0.7$ the applied stress is insufficient to initiate fracture at the inner tips and such a distribution of two slits will not lead to complete failure.

The results of the analysis will now be applied to the problem of fracture initiation at inclusions or small defects. If the defect root diameter ρ is 0.010 in. and assuming that the critical displacement Φ_c can be expressed as $\Phi_c = \rho\epsilon_{FR}$ where ϵ_{FR} is the local fracture strain, which we take to be 10%, Φ_c becomes 0.0001 in. (it would have the same value if $\rho = 0.001$ in. and $\epsilon_{FR} = 1\%$). Thus if the operating stress of the structure is $2\sigma_1/3$, $c^* = 0.1$ in. from equation (16) or the critical inclusion size is 0.2 in. ($G/\sigma_1 \sim 10^3$). However, two inclusions with the same sharpness ρ near each other but of much smaller lengths, i.e. 0.08 in., can lead to fracture provided they are sufficiently close, i.e. 0.04 in. Thus the theory has shown, in a quantitative manner, the necessity of considering the distribution as well as the size of inclusions when assessing the risk of a failure. Moreover, the work indicates that a distribution of small inclusions can be as dangerous as a large one, even though the volume fraction may be smaller in the first case.

4.2. Two Non-Coplanar Slits

The behaviour of two identical slits placed above each other will now be examined and both anti-plane and plane strain situations will be considered in turn. The results for the infinite sequence of non-coplanar slits models suggest that there may be substantial differences in the two cases, and this suggestion is confirmed.

4.2.1. The Anti-Plane Strain Model

Let the two slits be $|x_1| \leq c$, $x_2 = \pm h/2$, and the body is subject to the applied shear stress $p_{23} = \sigma_A$ which causes plasticity to spread from each slit along the planes $x_2 = \pm h/2$ (Figure 8). The effective applied shear stress p_{23} acting on each of them will be

$$\sigma_A \left(1 - \frac{c^2}{2h^2}\right) \qquad \ldots\ldots\ldots\ldots \qquad (18)$$

Thus the extent of plasticity a_A will be given, using equations (1) and (18), by

$$\frac{c}{a_A} = 1 - \frac{\pi^2\sigma_A^2}{8\sigma_1^2}\left(1 - \frac{c^2}{h^2}\right) \qquad \ldots\ldots\ldots\ldots \qquad (19)$$

and the relative displacement at a slit tip is

$$\Phi_A(c) = \frac{4\sigma_1 c}{\pi G}\,\frac{\pi^2\sigma_A^2}{8\sigma_1^2}\left(1 - \frac{c^2}{h^2}\right) \qquad \ldots\ldots\ldots \qquad (20)$$

by use of equations (2) and (18).

4.2.2. The Plane Strain Model

Let the body containing the slits $|x_1| \leq c$, $x_2 = \pm h/2$ be subject to the applied shear stress $p_{12} = \sigma_P$ so that it deforms under plane strain conditions (Figure 9). Neglecting any effect due to yawing of the slits, the effective applied shear stress p_{12} acting on each of them will be

$$\sigma_P \left(1 + \frac{c^2}{2h^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$

Thus the extent of spread of plasticity a_P will be given, using equations (1) and (21), by

$$\frac{c}{a_P} = 1 - \frac{\pi^2\sigma_P^2}{8\sigma_1^2}\left(1 + \frac{c^2}{h^2}\right) \qquad \ldots\ldots\ldots \qquad (22)$$

and the relative displacement at a slit tip is

$$\Phi_P(c) = \frac{4\sigma_1 c}{\pi G}\,\frac{\pi^2\sigma_P^2}{8\sigma_1^2}\left(1 + \frac{c^2}{h^2}\right) \qquad \ldots\ldots \qquad (23)$$

by use of equations (2) and (23).

4.3. Discussion

The results for the non-coplanar two slit models confirm that, as with the infinite sequence models for non-coplanar slits, there are appreciable differences between the anti-plane and plane strain situations. Thus, as before, the extent of spread of plasticity for a given applied stress, and the probability of fracture initiation, are greater in the plane strain case as the slits become closer, whereas they are less in the anti-plane strain case.

Therefore, when applying the results of the analysis to the problem of fracture at stress concentrations under the plane strain conditions which are frequently met with in practice, one must in fact use the relevant plane strain model. Regarding, as before, inclusions or small defects as the slits from which final fracture is initiated, equation (23) suggests that the critical size of defect can decrease by about 10% when there is a distribution such that $c/h = 0.25$; an even greater reduction is expected if the defect spacing is smaller.

Thus the work of this section clearly shows how distributions of defects can be more dangerous than isolated ones when dealing with the safety of large structures. So far it has been assumed that the steels are very brittle, thereby having low Φ_c values. For more ductile steels with their increased Φ_c values, if other stress concentrations are absent it is very likely that general yield will always occur before fracture initiation. However, if a severe stress concentration (e.g. a notch) is present and an inclusion (or inclusions) is situated near its root, then it is possible that fracture could be initiated first at the inclusion and then spread backwards towards the notch root. This means that fracture will occur at a lower applied stress than when the inclusion is absent; of course, this effect can be described by a reduced Φ_c at the base of the large stress concentration. Prevention of such reductions is compatible with the care taken in observation for inclusions at the base of stress concentrations (e.g. key-ways).

5. GENERAL DISCUSSION

In the previous sections we have examined how fracture initiation at a slit is affected by the presence of other slits, and have applied the results to the problem of fracture arising from the presence of inclusions or other similar defects. Particular attention has been given to the situation when the slits are far apart, so that the applied stress on each can be regarded as being essentially uniform; it is then relatively simple to use the results for an isolated slit to discuss problems involving distributions of widely spaced slits. A similar approach can be used for the case of a small isolated slit in a finite sized body, whose surface is a large distance from the slit; the effect of body size can then be determined, and as an example we have examined the problem of a relaxed slit in a cylinder (Smith, unpublished work). Of course, one particular example, that of a slit in a finite body, has already been discussed in Section 3.1.

The approach indicated in the present paper relies on the attainment of a critical relative displacement for fracture initiation, and for the case when the applied stress and the plastic zone size are both small, the results are in accord[3,4,11] with the "fracture mechanics" approach pioneered by Orowan[13] and Irwin[14], where the basis is to relate the change in strain energy with the work required for crack extension. For example, equation (2) reduces to

$$\sigma = \left(\frac{2}{\pi} \cdot \frac{G\sigma_1\Phi_c}{c}\right)^{\frac{1}{2}} \qquad \dots\dots\dots\dots\dots\dots \qquad (24)$$

if Φ_c is the critical relative displacement required for fracture. This result is equivalent to that obtained using the fracture mechanics approach with $\gamma = \sigma_1\Phi_c$ as the work required to extend a slit by a unit amount. Thus in principle the analysis of the present paper is equivalent to that of solving the elastic problem for the relevant slit model, and modifying the solution to allow for small amounts of plastic relaxation at the slit tips; we have in fact demonstrated how these solutions can be obtained for distributions of slits that are widely spaced.

In principle γ can be determined by measuring the fracture strengths of specimens containing sharp notches of various depths, provided that they break before general yield; one can then estimate the maximum size of defect that can be tolerated in a given structure, and this is the standard practice adopted when designing against brittle fracture when using the "fracture mechanics" approach. However, this procedure is only valid for high strength materials that have a low value of γ; for one finds that with the normal mild steels used in engineering structures, general yield and gross deformation always precede fracture if typical laboratory size test specimens are used, and it is not possible to reproduce the full weakening effect of a deep notch in a large body, when fracture will occur before general yield. The conventional fracture mechanics approach breaks down in such a situation, and to predict the behaviour of large structures from tests on small specimens, one must then measure the value of the critical local displacement[11] and then find γ from the relationship $\gamma = \sigma_1\Phi_c$. (Such a concept of measuring a critical local displacement, or crack opening displacement, has also been suggested by Wells[15].)

A basic principle throughout this paper is that high relative displacements, sufficient to initiate fracture, can be produced at the roots of stress concentrations. The severity of the concentration and the shape of the associated plastic region play a major role in this respect[11,16] and one way in which the deformation can be concentrated is by the presence of fatigue cracks. If a fatigue crack exists at the root of a blunt notch then Φ_c will be lowered because the effective gauge length over which the local fracture strain is measured will be reduced; of course there may ultimately be a limit to the effective sharpness of a crack, for as indicated in the Introduction, the processes that cause fracture initiation need a certain minimum volume in which to operate[1]. Thus at low temperatures for a mild steel, when fracture occurs before general yield, the fracture stress is markedly decreased by the presence of a fatigue crack at the base of a machined notch[17]. At higher temperatures, where fracture occurs after general yield with typical laboratory size specimens, the detrimental effect of a fatigue crack is reflected in a lower reduction in area as compared with notched specimens without fatigue cracks; with much larger structures that would normally break before general yield, this last effect would be expected to be reflected by a lower fracture stress. Thus since a fatigue crack can be more serious than a notch of the severity usually accomplished by standard machining techniques, it follows that it is necessary to sharpen a notch by fatigue cracking in order to provide the most severe stress concentration; this procedure is adopted by Irwin and his colleagues in the United States. Moreover, there is a danger that when a structure is operated under conditions where fatigue might initiate a crack, then the probability of it failing in a brittle manner will increase.

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6. ACKNOWLEDGMENTS

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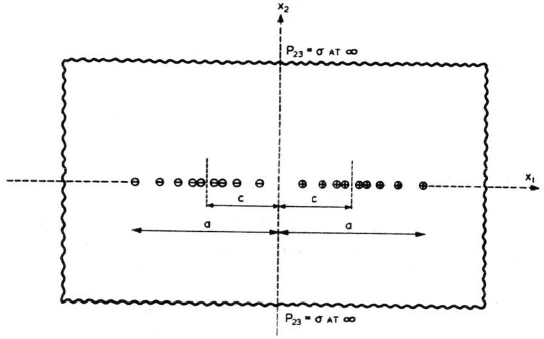

Figure 1. An isolated plastically relaxed slit in an infinite body subject to an applied shear stress $p_{23} = \sigma$. The plastic relaxation is described by a distribution of screw dislocations parallel to the x_3 axis; the slit, which is infinitely long in the x_3 direction, is of length 2c in the x_1 direction, and the extent of spread of plasticity is a. ⊕, positive screw dislocations; ⊖ negative screw dislocations.

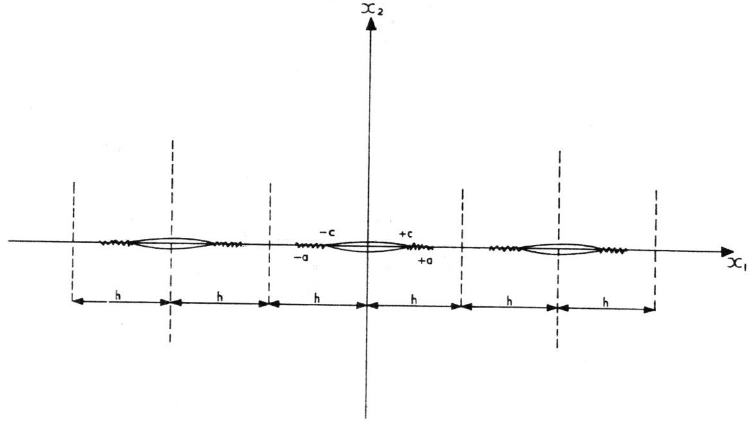

Figure 2. An infinite series of identical and equally spaced coplanar slits in an infinite body subject to an applied shear stress $p_{23} = \sigma$. The slits are infinitely long in the x_3 direction, are of length 2c in the x_1 direction, and have their centres a distance 2h apart. ∿∿ indicates plastically relaxed regions which extend to a distance a from each slit centre.

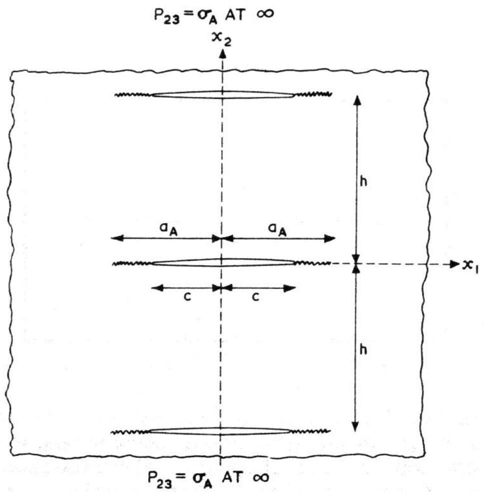

Figure 3. An infinite series of identical and equally spaced non-coplanar slits in an infinite body subject to an applied shear stress $p_{23} = \sigma_A$ (anti-plane strain deformation). The slits are infinitely long in the x_3 direction, are of length 2c in the x_1 direction, and have their centres a distance h apart. ～～ indicates plastically relaxed regions which extend to a distance a_A from each slit centre.

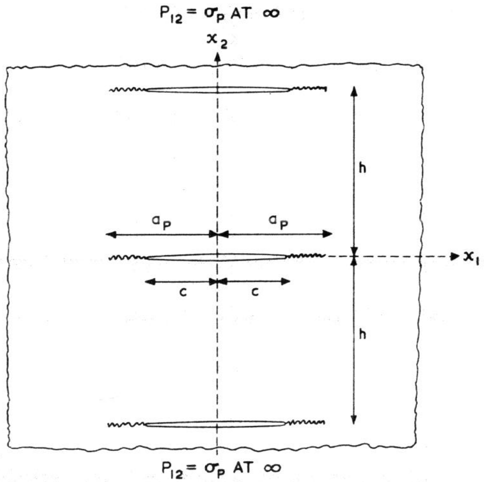

Figure 4. An infinite series of identical and equally spaced non-coplanar slits in an infinite body subject to an applied shear stress $p_{12} = \sigma_P$ (plane strain deformation). The slits are infinitely long in the x_3 direction, are of length 2c in the x_1 direction, and have their centres a distance h apart. ～～ indicates plastically relaxed regions which extend to a distance a_P from each slit centre.

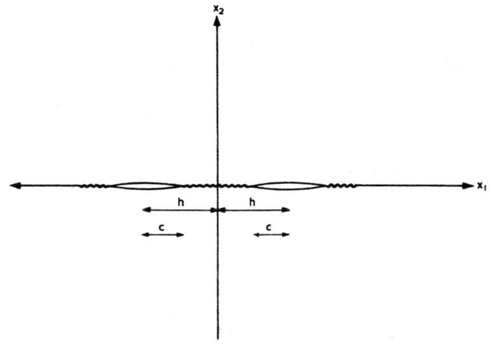

Figure 5. Two identical coplanar slits in an infinite body subject to an applied shear stress $p_{23} = \sigma$. The slits are infinitely long in the x_3 direction are of length 2c in the x_1 direction, and have their centres a distance 2h apart. ～～ indicates plastically relaxed regions.

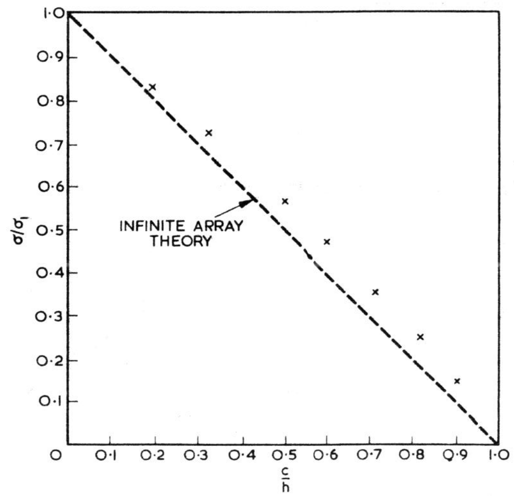

Figure 6. The stresses (indicated by ×) required for plasticity to just spread between two slits for different values of c/h. 2c = slit length, 2h = distance between slit centres, σ = applied shear stress and σ_1 = shear yield stress. The results are compared with the corresponding ones obtained from the infinite array theory (------------).

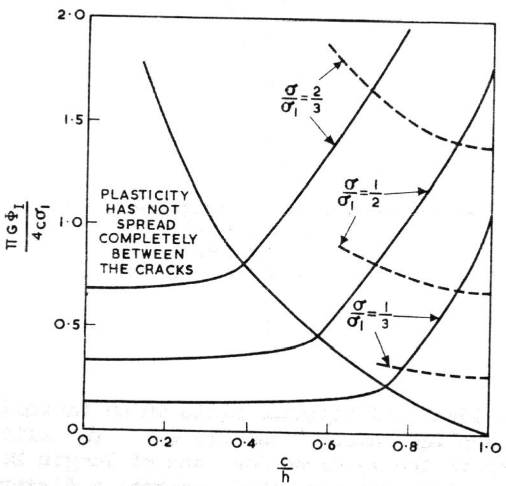

Figure 7. The displacement Φ_I at an inner tip of a slit (for the two coplanar slits model) for different values of the applied stress σ (full curves). The dashed curves illustrate equation (17). σ_1 = shear yield stress, 2c = slit length. 2h = distance between slit centres and G = shear modulus.

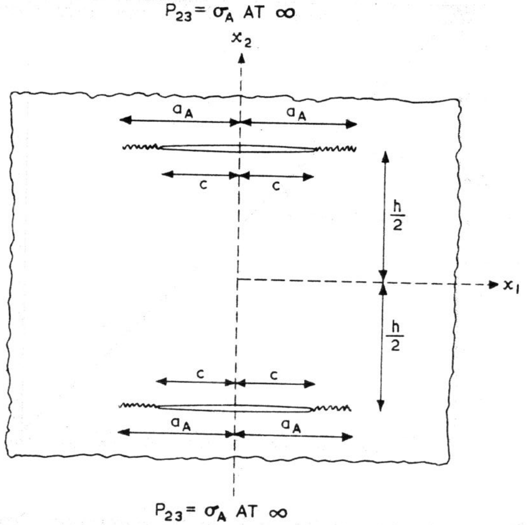

Figure 8. Two identical non-coplanar slits in an infinite body subject to an applied shear stress $p_{23} = \sigma_A$ (anti-plane strain deformation). The slits are infinitely long in the x_3 direction are of length 2c in the x_1 direction, and have their centres a distance h apart. ~~~~ indicates plastically relaxed regions which extend to a distance a_A from each slit centre.

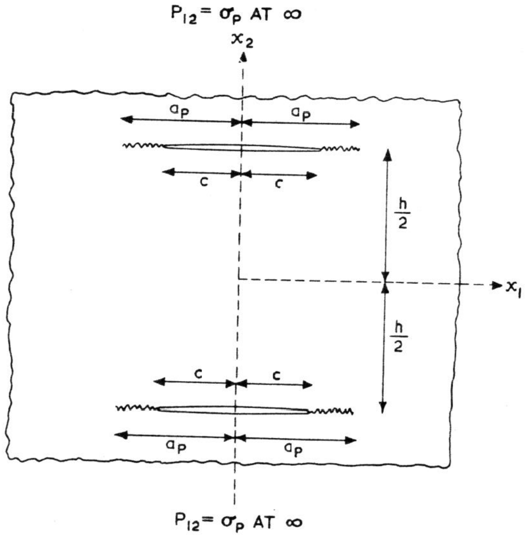

Figure 9. Two identical non-coplanar slits in an infinite body subject to an applied shear stress $p_{12} = \sigma_P$ (plane strain deformation). The slits are infinitely long in the x_3 direction, are of length 2c in the x_1 direction, and have their centres a distance h apart. ~~~~ indicates plastically relaxed regions which extend to a distance a_P from each slit centre.