2. Theoretical Aspects of Fracture

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ABSTRACT

A general outline is given of some recent views on fracture in metals. The brittle fracture of steel at low temperatures is attributed to the coalescence of glide dislocations to form microcracks, the growth of which require a stress higher than that for coalescence. The start of a tensile cup-and-cone failure in a ductile metal is discussed in terms of the growth of plastic cavities, generally nucleated at inclusions, and a comparison is made between the processes of shear fracture and adhesion occurring between sliding metallic surfaces. Evidence is given that shear stress is needed for the formation of voids at grain boundaries in creep experiments, and normal stress is needed across the boundaries for their growth, which is thought to occur by vacancy creep. Recent work on the origin of fatigue fractures is briefly summarized.

Elastic Cracks

We know from many experiments that the breaking strengths of fibers do not fall far short of the ideal value expected from simple estimates of atomic forces:

$$p_m \simeq \sqrt{\frac{E\gamma}{a}} \simeq \frac{E}{10} \tag{1}$$

Here E is Young's modulus, γ is the specific energy of the fractured surfaces, and a is the atomic spacing. Glass fibers are already exploited commercially for their high strengths, and perhaps metallic and nonmetallic whiskers will eventually be similarly exploited.

We also know, however, that large samples are generally far weaker, in proportion, than these fibers. Metals show the least discrepancy, and it is often possible to obtain breaking strengths from these that are within

an order of magnitude of the ideal value. For example, hard-drawn spring steel wire breaks at about $E/60$. Similarly, although single crystals of pure copper begin to flow plastically at about $E/10{,}000$, it is found that when they are pulled in liquid helium, they withstand stresses of up to $E/100$ before breaking; and ordinary polycrystalline copper, in the neck of a tensile specimen, breaks at about $E/100$ if the temperature is not too high.

Brittle solids show the largest discrepancies. In these, the fracture approaches most nearly to that ideally simple process in which the atomic bonds are stretched elastically to their limit. At the tip of a crack in such a material, bonds exist in every stage of elongation and fracture. As the crack grows, each of the bonds in its path takes up the strain that previously belonged to its predecessor, and the work done in stretching and breaking these bonds becomes the surface energy γ of the fractured faces. This work has to be supplied by the applied forces or by the elastic energy of the system, which leads of course directly to Griffith's formula,

$$p \simeq \sqrt{\frac{E\gamma}{c}} \tag{2}$$

for the tensile stress p to propagate a crack of length c.

Griffith's theory is very familiar, but there are two points that we might usefully make about it at this stage. First, it deals with *elastic cracks*, and, since these by their very nature have atomic bonds at their tips in every stage of elongation and fracture, there is no question of having to fulfill any criterion of stress concentration at such a tip, in addition to the Griffith criterion, in order to propagate the crack. At the end of an excavated *notch*, on the other hand, the bonds are not necessarily strained to their limit, and fracture cannot occur unless the over-all tensile stress p satisfies a condition of the type

$$p \simeq p_m \sqrt{\frac{\rho}{c}} \tag{3}$$

where ρ is the radius of the root. Using Eq. 1, we can change Eq. 3 into the same form as Eq. 2, but with an apparent surface energy (or elastic constant) that is ρ/a times the real value. In his own experiments on the strength of glass, Griffith [1] was careful to use elastic cracks, and in fact, the values of $E\gamma$ that he obtained by fitting Eq. 2 to his results agreed closely with the directly measured values of E and γ.

Second, because we shall want later to connect dislocations with fracture, it is useful to rewrite Griffith's formula as

$$pna \simeq 2\gamma \tag{4}$$

where na ($\simeq cp/E$) is the maximum displacement between the faces of the crack and can be interpreted formally as a "pile-up" of n "edge dislocations," each of Burgers vector a, as shown in Fig. 1*a*. Although these

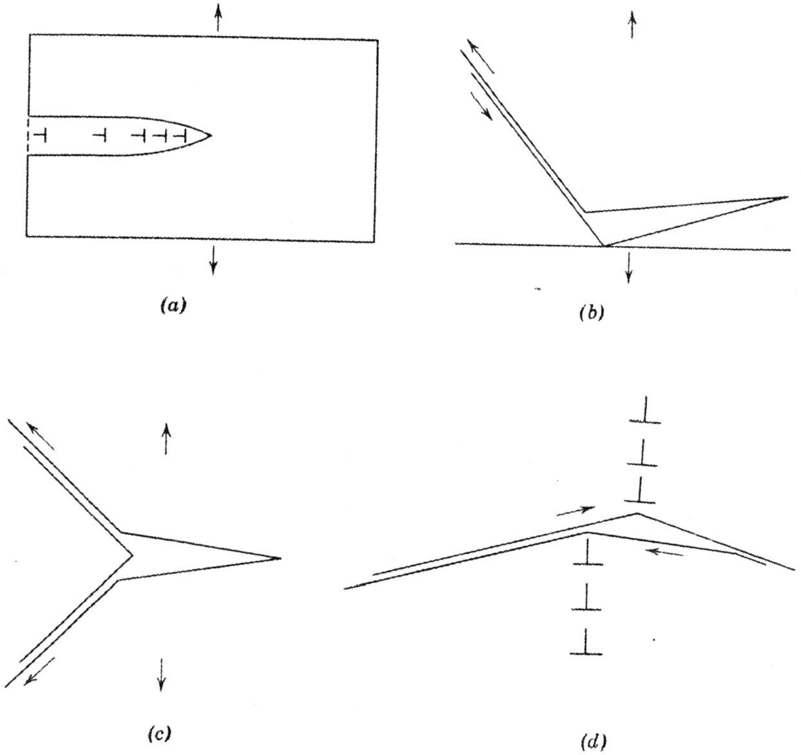

Fig. 1. Dislocations and cracks. (*a*) Elastic crack regarded as a pile-up of edge dislocations. (*b*) Crack formed from a pile-up against a boundary. (*c*) Crack resulting from shear on two bands. (*d*) Crack formed at a tilt boundary.

"dislocations" are centered in the space of the crack itself, their Burgers vector is definable by a line integral,

$$na = \int \frac{\partial \boldsymbol{u}}{\partial l}\, dl \tag{5}$$

of the elastic displacement vector $\boldsymbol{u}$ along a circuit through the medium. This circuit starts at a point on one face of the crack, passes around the tip, and ends at the corresponding point on the other face. When these cavity dislocations "glide" towards the tip of the crack (in the direction perpendicular to their Burgers vector if the crack is perpendicular to the applied tensile stress p), the work done on them by this stress is pna per unit

of area swept out by the advancing dislocation front, and this work becomes the surface energy of the crack faces that are opened up by their movement.

Crystalline Solids

Most of the experiments on Griffith's theory have been on noncrystalline materials. Cold glasses and polymers are particularly suitable for such experiments because at low temperatures the thermally activated atomic movements by which they flow can hardly occur.

By contrast, in crystalline solids there is always the possibility that plastic deformation by the glide of crystal dislocations will occur. This possibility decreases as we move away from close-packed metallic crystals and move toward low temperatures and rapid loading, but it can hardly ever be entirely disregarded. Rather than pursue the difficult question of how absolutely brittle fractures in crystals, if they occur, can be proved as such, it seems more useful to examine those fractures which, although generally classed as brittle, have in fact arisen out of processes of plastic deformation. We know, for instance, that when a notched bar of mild steel is loaded at a temperature just below the notch-brittle transition point, the first thing that happens is plastic deformation in the root of the notch. The crack forms later. Even during crack propagation, much plastic deformation occurs nearby, and the measured effective surface energy of the fracture (about 10^6 ergs/cm^2, as compared with about 2000 ergs/cm^2 for the true surface energy of iron) is much too large to be explained except as plastic work.[2,3] In fact, the appearance of the fracture shows that the propagation often occurs by the separate nucleation and coalescence of small cracks in a plastically deformed zone in front of the main crack.[4,5] At low temperatures (for example, $-196°$C), the work of fracture is smaller, although even here there are signs of plastic deformation along the fracture, and it is known that in many cases the fracture occurs at the moment of plastic yielding.[6,7] However, a recent paper by Allen gives evidence of fracture before plastic yielding.[8]

The general picture we have to examine then, for this particular class of fracture, is one in which a number of glide dislocations in a glide system, such as a slip or deformation twin band, become converted at some place in the crystal into cavity dislocations, which then spread and multiply in the form of a growing crack. This picture is basic to most recent theoretical treatments of semibrittle fractures in crystalline materials.

Formation of Cracks by Plastic Glide

How does the conversion from glide dislocations to cavity dislocations take place? Examination of the theoretical models for this and of actual crack nuclei in various materials shows that there are many ways to fracture.

The simplest and earliest suggested process is that of a pile-up in a slip band of edge dislocations against a grain boundary. The applied stress pushes the dislocations together, and a crack forms underneath their coalesced half planes (Fig. 1*b*). Stokes, Johnston, and Li [9] have observed this process in magnesium oxide crystals. Variants of the process involve twin interfaces and surface films as the barriers to the dislocations.[10,11]

There are also processes in which dislocations in two intersecting bands glide together, coalesce along the line of intersection, and thereby change into cavity dislocations (Fig. 1*c*). A theoretical model of such a process was first suggested for b.c.c. metals such as iron, in which the coalescence reaction,

$$\frac{a}{2}[\bar{1}\bar{1}1]+\frac{a}{2}[111]\rightarrow a[001] \qquad (6)$$

is particularly favorable.[12] Examples in practice have not yet been seen in iron, although a very similar process has been seen in magnesium oxide by Washburn and Parker.[13] Cracks have also been seen to be nucleated at the junctions of intersecting deformation twin bands in certain crystals; metals that have shown this are molybdenum,[14] zinc,[15] and silicon-iron.[16]

Another process, which is important for crystals with layer structures that slip and cleave on the same plane, is illustrated in Fig. 1*d*. Here a slip band crosses a tilt boundary, thus displacing its sides, and the discontinuity of displacement that occurs there resulting from the change of slip direction produces a crack on the slip plane. Friedel [17] and Stroh [18] have developed the theory of this process, and Gilman [19] has observed it in zinc.

Mechanics of Fracture Caused by Plastic Glide

One important point about all the processes described above is that, when the dislocations run together, the concentration of elastic energy that they bring with them must not be dispersed by plastic flow in the nearby material if a crack is to be formed. In layer crystals which do not slip easily except on the basal plane, this condition may be satisfied even when the dislocations are brought together gradually under an increasing applied stress, and the brittleness of some hexagonal metals may be due

to this effect. In cubic crystals with equally good slip systems in several directions, however, another explanation is necessary. As Mott [20] and Stroh [21] have pointed out, what is needed in this case is a *yield drop*, so that a large avalanche of dislocations is formed suddenly. These dislocations may then be able to run together and form a crack, and this crack may be able to spread, all in a time too short to start nearby dislocation sources working. In the theory of Mott and Stroh, the possibility of this happening depends sensitively upon temperature. Above a critical temperature, the delay time to set off other sources becomes so short that more slip bands are always nucleated in preference to cracks, and the material is therefore ductile.

The two main causes of yield drops are impurity yield points and deformation twinning. The part played by twinning in the brittle fracture of a metal such as iron is still controversial and needs to be clarified. Examples of cracks nucleated by twins are known.[14–16] Equally, examples are known in which twins were not formed before fracture.[22–24] The nucleation of a twin is likely to be about as difficult as the nucleation of a crack, so that slip bands able to nucleate the latter may also nucleate the former.

There is much evidence that brittleness in b.c.c. transition metals is connected with impurity yield points. Steel of high nitrogen content is notably susceptible to strain age embrittlement. Iron has been made ductile in tension (90% reduction of area) at the temperature of liquid helium by zone refinement.[25] Small amounts of plastic working at room temperature, which overcome the yield point and unlock some dislocations, improve the ductility of steel at low temperatures.[11,26,27] Niobium and tantalum, which generally have smaller yield drops than steel, are relatively ductile, even at low temperatures where their yield strengths are high. Olds and Rengstorff [28] have shown that the brittle temperature of molybdenum is raised considerably by the addition of the first 0.001% nitrogen or of the first 0.003% carbon, in each case an amount too small to produce detectable second phases in the microstructure. (By contrast, the strong embrittling effect of oxygen in molybdenum appears to be due to the formation of weak oxide films along the grain boundaries.) The experiments of Wain, Henderson, and Johnstone [29] and of Smith and Seybolt [30] on chromium have established the fact that small amounts of nitrogen greatly increase the brittleness of this metal; and small amounts of plastic deformation at about 400°C strikingly improve the ductility of this metal at room temperature.[31]

We must consider the criterion for producing fracture by the processes described above. These all have a common basis. A number of glide dislocations run together and, in so doing, they cause the applied stress acting on them to do some work. If fracture is to occur, this work must be at least sufficient to supply the surface energy of the new faces created. It follows that the general criterion of fracture in all cases is expressible by a formula of the type of Eq. 4, that is, by equating the product of applied stress and total Burgers vector of the dislocations to a surface-energy term.

The total Burgers vector na is readily expressible in the form

$$na \simeq \frac{(\sigma - \sigma_i)}{\mu} d \tag{7}$$

where d is the length of the band containing the dislocations, μ is the shear modulus, and $\sigma - \sigma_i$ is the effective shear stress acting on the glide band of the dislocations. (The effective shear stress is equal to the actual shear stress σ minus the "friction stress" σ_i, which results from intersecting dislocations, lattice defects, impurities, or the Peierls-Nabarro force opposing the motion of an unlocked dislocation.) When dealing with fractures that occur at or near the yield shear stress σ_y of the material, we may write $\sigma = \sigma_y$ and interpret $2d$ as approximately the grain diameter in a polycrystal or as the specimen diameter in a single crystal. In polycrystals we can then use the formula

$$\sigma_y = \sigma_i + k_y d^{-1/2} \tag{8}$$

for the lower yield stress [32–34] to obtain the Burgers vector

$$na \simeq \frac{k_y d^{1/2}}{\mu} \tag{9}$$

in terms of $k_y d^{-1/2}$, which is a measure of the yield drop on a glide band when the dislocation sources in that band become unlocked. Here $k_y = \sigma_d l^{1/2}$, where σ_d is the unpinning stress for the temperature and time concerned, and l is the spacing of dislocation sources. Values of k_y have been determined for some metals.[12]

The proper value to take for p, the applied stress which has to be multiplied by na, raises a more difficult question. There are three stages in the process of fracture: (*a*) the creation of unlocked glide dislocations, (*b*) the conversion of glide dislocations into cavity dislocations, and (*c*) the growth of the crack formed by these cavity dislocations. Now until stage (*b*) is passed, the only dislocations present are glide dislocations, and the only stress to which they respond is the effective shear stress $\sigma - \sigma_i$. Hence, if (*b*) is a more difficult stage than (*c*) and requires a greater applied stress, the fracture criterion is concerned with (*b*), and p must then be identified with $\sigma - \sigma_i$. On the other hand, if (*c*) is the more difficult stage, the whole of the applied stress, not merely $\sigma - \sigma_i$, becomes available to do work. This distinction becomes important when

we have to compare the effects of applied stress systems that differ in their hydrostatic components.

It is difficult to decide between (*b*) and (*c*) on purely theoretical grounds, because (*b*) depends critically upon conditions at the centers of dislocations. Stroh [21] came to the conclusion that, for the process of Fig. 1*b*, (*b*) was harder than (*c*); in other words, once the crack was nucleated, it would automatically spread. On the other hand, the coalescence stage can be expected to be appreciably easier in the process of Fig. 1*c*, since the dislocations in the two bands approach one another at an angle, and, for b.c.c. metals at least, the first dislocations in each band are attracted together. The process of Fig. 1*d* is appreciably different from the other two, since the dislocations that coalesce in this case are the residual dislocations left behind owing to the change in Burgers vectors of the glide dislocations as they pass through the tilt boundary. Stroh [18] has concluded that for this process (*c*) is the critical stage, and he has deduced variations in the fracture stress and ductility with crystal orientation that are in good agreement with observations on zinc crystals.[35]

Some of the consequences of choosing (*b*) or (*c*) as the critical stage of fracture are very different and can be put to the test of experiment. If we consider (*b*) as the critical stage, then $p = \sigma - \sigma_i$, and for fracture at the yield point where $\sigma - \sigma_i = \sigma_y - \sigma_i = k_y d^{-1/2}$, Eqs. 4 and 9 then give

$$k_y = \alpha\sqrt{\mu\gamma}, \qquad \alpha \simeq 1 \tag{10}$$

as the criterion of fracture. Since k_y decreases rapidly with increase in temperature, owing to the effect of thermal fluctuations in reducing the value of σ_d, this criterion can account for the influence of temperature (and rate of straining) upon brittleness. It cannot account, however, for several other effects that are known to be important, in steel at least. The applied stress has disappeared from the criterion altogether, so that fracture is expected always to occur at the yield point in a certain range of temperature and strain rate, whatever type of applied stress system is used to reach it. Yet we know that a notched thick plate of steel remains brittle to a higher temperature than a similarly notched thin plate, and, from the appearance of a ductile rim around such brittle fracture, that the interior of the material where hydrostatic stresses can be supported has a higher brittle temperature than the material near the free surface where such stresses cannot exist. We also know that unnotched samples of mild steel are appreciably more brittle at the temperature of liquid air when tested in tension rather than in torsion.[36,37]

Again, Eq. 10 does not admit any effect of grain size and friction stress on brittleness and is thus at variance with the well-known facts that the most ductile steels are soft and fine-grained, while the most brittle ones are hard and coarse-grained. Attempts have been made to bring grain size into the theory through subsidiary effects, but they have not been entirely convincing.

Recent experiments on a mild steel hardened to various degrees by neutron irradiation have also shown the inadequacy of Eq. 10.[38,39] By irradiating samples of different grain sizes, it was proved that the radiation hardening was confined entirely to an increase in σ_i and that k_y remained constant to within 5%. In spite of this, the ductility of unnotched fine-grained samples at −196°C decreased from 40% reduction of area (unirradiated) to zero (irradiated to 9×10^{19} epithermal nvt), and the brittle temperature of notched specimens rose by 65°C.

Such difficulties as these with Eq. 10 have led recently to the view that stage (c), the growth of the crack, may be the critical stage for fracture.* In this view, the conversion of glide dislocations into cavity dislocations is easier than stage (c), so that the important comparison is that between the stresses required for stages (a) and (c). When the yield stress exceeds the growth stress of the cracks produced by the yield process, the material is brittle; conversely, it is ductile. In the latter case, provided the yield stress exceeds the conversion stress, small cracks form at the yield point but do not grow beyond a size of order of the grain diameter. Such nonpropagating microcracks have been observed to form at temperatures just above the brittle point in the yielded zones of unnotched specimens.

When the glide dislocations change into cavity dislocations, not only does the stress σ_i that resists their motion disappear, but the whole applied stress then acts on them, instead of only the resolved shear stress σ. Thus, p in Eq. 4 becomes the tensile stress normal to the crack, and, for fracture at the yield point, this can be written as σ_y multiplied by a factor for the ratio of normal stress to shear stress. Substituting this, together with Eq. 9, into Eq. 4, we find that fracture should just be able to occur at the yield point when

$$\sigma_y k_y d^{1/2} = \beta\mu\gamma \tag{11a}$$

that is, when

$$(\sigma_i d^{1/2} + k_y)k_y = \beta\mu\gamma \tag{11b}$$

where $\beta \simeq 2$ (torsion), 1 (tension), and $\frac{1}{3}$ (in the plastically constrained zone at the root of a notch). The material becomes ductile when the left side of this expression becomes smaller than the right side.

* Professor Petch and I both arrived at this view at about the same time.[40] In the above, I have summarized the theory in the form in which I developed it.[12] Professor Petch's treatment [41] is a little different in detail, but the main points are the same.

Equation 11 was obtained originally from a detailed analysis of the process in Fig. 1c, but we see from the above argument that this type of equation should be valid quite generally for the growth of cracks created by the conversion of glide dislocations into cavity dislocations. By contrast with Eq. 10, it predicts effects arising from crystal size, friction stress, and the type of stress system; these predictions seem to be borne out in practice.[12] Even when $\sigma_i = 0$, and the grain-size effect then disappears, Eq. 11 still differs detectably from Eq. 10, since the influence of the type

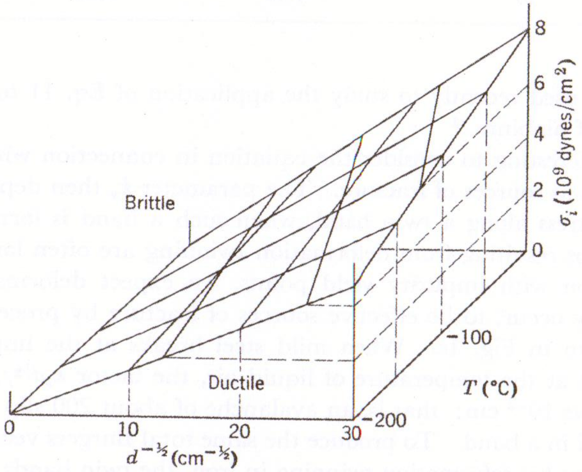

Fig. 2. A ductile-brittle transition surface for unnotched tensile specimens of a mild steel, constructed from the measurements made by Hull and Mogford.[42]

of stress system, represented by β, remains. When the variation of k_y with temperature is known, the effects of grain size and friction stress on the brittle temperature can be represented by means of a transition surface, such as that shown in Fig. 2. This surface was constructed from the experimental measurements made by Hull and Mogford,[42] who used neutron irradiation to obtain controlled variations of the friction stress in a mild steel (En 2 steel). These experiments also enabled the constancy of $\sigma_i d^{1/2}$ at the brittle point at constant temperature to be tested. Table 1 shows the results obtained on unnotched tensile specimens tested at −196°C after irradiation to various neutron doses. The grain size at the ductile-brittle transition was found in each case by examining specimens of different grain sizes. We see that, apart from the unirradiated sample, $\sigma_i d^{1/2}$ remains constant to within a few percent, even though σ_i and $d^{-1/2}$ themselves change by more than 50%. Neutron irradiation has

TABLE 1. Values of Friction Stress σ_i and Transition Grain Diameter $2d$ at the Brittle Point in a Steel Tested at −196°C after Neutron Irradiation [42]

Irradiation (10^{18} neutrons/cm^2)	σ_i (10^9 dynes/cm^2)	$d^{-1/2}$ (cm$^{-1/2}$)	$\sigma_i d^{1/2}$
0	2.66	17.5	1.52
1	2.98	20.0	1.44
15	3.61	26.0	1.39
57	4.15	30.0	1.38
90	4.40	32.0	1.38

also been used recently to study the application of Eq. 11 to the brittle fracture of niobium.[43]

It is interesting to consider the equation in connection with deformation twins as sources of fracture. The parameter k_y then depends on the drop in stress along a twin band, when such a band is formed. Since yield drops resulting from deformation twinning are often large even by comparison with impurity yield points, we expect deformation twins, when they occur, to be effective sources of fracture by processes such as that shown in Fig. 1*c*. When mild steel breaks at the impurity yield point and at the temperature of liquid air, the factor $k_y d^{1/2}/\mu$ is usually a few times 10^{-6} cm; that is, an avalanche of about 200 slip dislocations is released in a band. To produce the same total Burgers vector of cavity dislocations by deformation twinning in iron, the twin bands would have to be not more than about 10^{-5} cm thick. Actual measurement of the thickness of the twin (knowing the angle of shear) is probably the simplest way of determining $k_y d^{1/2}/\mu$ and na for twinning. At very low temperatures, where impurity locking of dislocations is extremely strong, the first process of plastic deformation to occur may be twinning, nucleated at points of stress concentration, such as the sharp edges of foreign inclusions; we would then expect cracks to start from twins when Eq. 11 is satisfied. But at somewhat higher temperatures, where fracture occurs at the impurity yield point σ_y, the interrelation of slip, twinning, and fracture becomes very complicated. The same applied stress acts on the cavity dislocations, whether these are produced from slip or twin bands, so that what matters is whether slip can nucleate twinning at the yield stress and whether $(\sigma_y - \sigma_i)d/\mu$, which determines the total Burgers vector of these dislocations, is larger for twinning than slip, that is, whether σ_i is larger for slip dislocations than twinning dislocations. There is little to guide us on these questions, although in some fractures of mild steel it appears that deformation twins are not formed until after fracture has begun,[24] a fact which suggests that in this particular material it may be more difficult to nucleate twins than cracks from slip bands.

On the ductile side of the brittle point, microcracks are formed, as Low [34] and Owen, Averbach, and Cohen [44] have shown, but the yield stress is too small to grow them. There then occurs a stage of "delayed cleavage fracture" in which the sample deforms plastically until its yield stress has been raised to the level at which cracks that can propagate are formed. The metal is ductile, but the fracture occurs by the same process as before. The ductility improves so rapidly, however, with increasing temperature and decreasing grain size that this stage does not extend far beyond the brittle point, and failure by ductile fibrous fracture is soon preferred.

Cleavage fractures, on the other hand, can still occur at much higher temperatures than this. As a fibrous fracture spreads through the specimen at increasing speed, the combination of high strain rate and a large ratio of normal to shear stress (in a thick plate) at its tip can enable Eq. 11 to become satisfied and so change the fracture into a brittle cleavage. In mild steel, the range of temperature in which brittle cleavage appears can be raised about 150°C by this effect.[12] Since extremely large plastic strains culminating in ductile fracture can be produced at the root of a sharp notch without expending all the energy necessary to work plastically a large volume of the sample, the metal is not completely protected against semibrittle fractures starting from plastically worked roots of notches until its temperature has been raised to the range in which, even when the maximum allowance is made for strain rate and stress system effects, it is no longer possible to satisfy Eq. 11. Crack propagation tests such as that developed by Robertson [45] enable this upper limiting temperature to be found.

Tensile Fracture of Ductile Metals

We know that in the familiar tensile "cup-and-cone" fracture of a really ductile material, such as copper, the fracture starts at the center of the necked portion of the test piece, at first grows roughly perpendicularly to the tensile axis forming the "cup," and turns into the "cone" by fracturing along a surface at about 45° to the tensile axis as it nears the outer surface. We also know that the cup consists in detail of many irregular surfaces inclined to the tensile axis, which give the fracture its fibrous appearance, and that this part of the fracture develops gradually, being accompanied by much plastic deformation. It does not close up when the stress is removed and is obviously very different from an elastic crack.

A commonly held view of this process of fracture is that each element of material in front of the crack must first be worked plastically to change the stress or strain there to meet some fracture criterion, and that it then breaks. It is sometimes supposed that the material has a finite capacity for plastic deformation, so that its ductility has to be "exhausted" before it can break, which suggests a critical strain as the criterion for fracture. Sometimes it is supposed that a certain amount of work hardening has to occur at the front of a growing crack before the stress there can rise to the level needed to break the material. This is of course very similar to the idea of "delayed cleavage fracture" which we used to discuss the special behavior of iron just above its brittle point.

However, the fibrous fracture of a metal such as copper is quite different from a "delayed brittle fracture," and the whole idea that the plastic strain is needed to fulfill some criterion for a cohesive failure seems wrong. As Orowan, Nye, and Cairns [46] have emphasized, although a bar of ductile metal may break in tension at a certain reduction of area, the same material can be plastically worked to a much greater extent without breaking if there is a compressive stress present (for example, by wire drawing or in Bridgman's high-pressure apparatus); furthermore, if a tensile specimen is then cut from this highly deformed and work-hardened material, it will not break until it has undergone a reduction of area almost as large as that of the unworked sample. Table 2 shows this effect in annealed and cold-worked samples of two types of copper.[47]

TABLE 2. Tensile Properties of Copper [47]

Type	Condition	Yield Strength (psi)	Ultimate Tensile Strength (psi)	Reduction of Area (%)
Tough pitch	Annealed	5,500	31,500	71.4
Tough pitch	Hard	46,000	46,900	58.5
Oxygen-free	Annealed	5,000	30,900	92.1
Oxygen-free	Hard	47,000	47,500	86.4

Such results suggest that the plastic deformation is required, not as a preliminary to fracture, but as part of the process of fracture itself. Against this, it may be objected that the preliminary cold-working does not develop the same distribution of stress in the specimen as exists in a tensile neck and that, if the latter distribution is essential to fracture, the results are not in fact inconsistent with a "delayed brittle" type of fracture. However, Bridgman [48] has pulled tensile specimens under hydrostatic pressure to large strains, producing deep unfractured necks, and then pulled these necked specimens again under atmospheric pressure.

More plastic deformation always occurred at such a neck before fracture even though, in many cases, the specimen had already been pulled under pressure to a stress and strain well beyond the usual values for fracture at atmospheric pressure.

It seems then that, irrespective of the mechanical state of the material, more plastic strain is always needed to spread the fracture; hence, the function of this strain seems to be to cause the material in front of the fracture not to break but to recede away from the fracture by plastic distortion. In other words, we ought to think of the fracture as a plastic cavity or an "internal neck," rather than as a "crack," and to regard it as growing outward, without any real fracture at all in the strict sense of the term, to meet the external neck which is growing inward.

Clearly, a cavity grown in this way ought to be longer in the direction of the tensile axis than it is wide. This is occasionally observed, for instance, in aluminum alloys pulled at high temperatures; [49] but usually a fairly thin lens-shaped cavity is formed. In such cases, we expect that many small cavities have become nucleated simultaneously, each of which has then grown outward in all directions, becoming slightly elongated in the direction of the tensile axis, and eventually coalesced with neighboring cavities alongside it to become part of a large thin cavity. Examples of this have been seen and have been described in detail by Mrs. Tipper.[50] Figure 3, photographed in the region of a tensile fracture in copper, shows the process of internal necking and the coalescence of small cavities.[51] The main fracture and many small cavities in this specimen are shown in Fig. 4.

How are the cavities nucleated? One view is that they are small cracks formed from piled-up glide dislocations by processes of the type described earlier.[52] This may well be possible in iron.[53] In metals such as copper and aluminum, however, which normally do not show yield drops, we expect k_y to be very small and no large pile-ups to be formed. Recent electron-microscopical studies of dislocations in cold-worked f.c.c. metals have also shown that pile-ups do not form appreciably even during the work-hardening stage of deformation.[54]

The experimental evidence in fact suggests that the plastic cavities are nucleated at foreign particles, and that, if such particles were not present, the specimen would pull apart entirely by the inward growth of the external neck, giving nearly 100% reduction in area. In Table 2, for instance, the oxygen-free copper necked down to 92.1% reduction in area, whereas the tough pitch copper necked only to 71.4%. Similarly, pure aluminum necks down to over 90% reduction in area, whereas commercial grades often break at about 30%. Mrs. Tipper, who first suggested that ductile fractures start at inclusions, has photographed

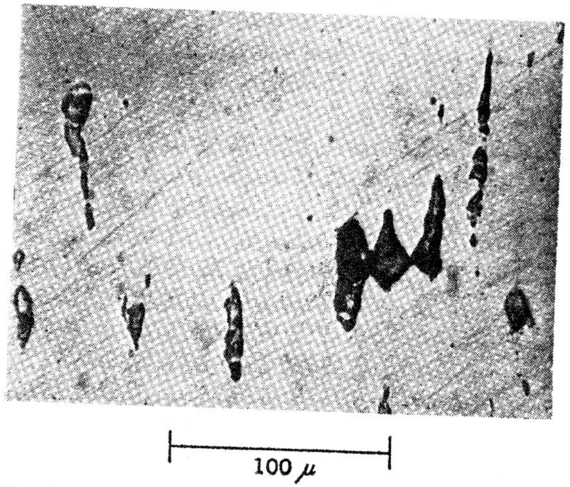

Fig. 3. Cavities coalescing in the neck of a tensile specimen of copper. (After Puttick.[51])

Fig. 4. Section through the neck of the tensile specimen from which Fig. 3 was taken. (After Puttick.[51])

several examples of such nuclei.[50] In Fig. 5 we show similar photographs obtained by Puttick; [51] the cavities on these oxide particles in copper were formed sometimes by the separation of the metal from the inclusion and sometimes by fracture of the inclusion itself.

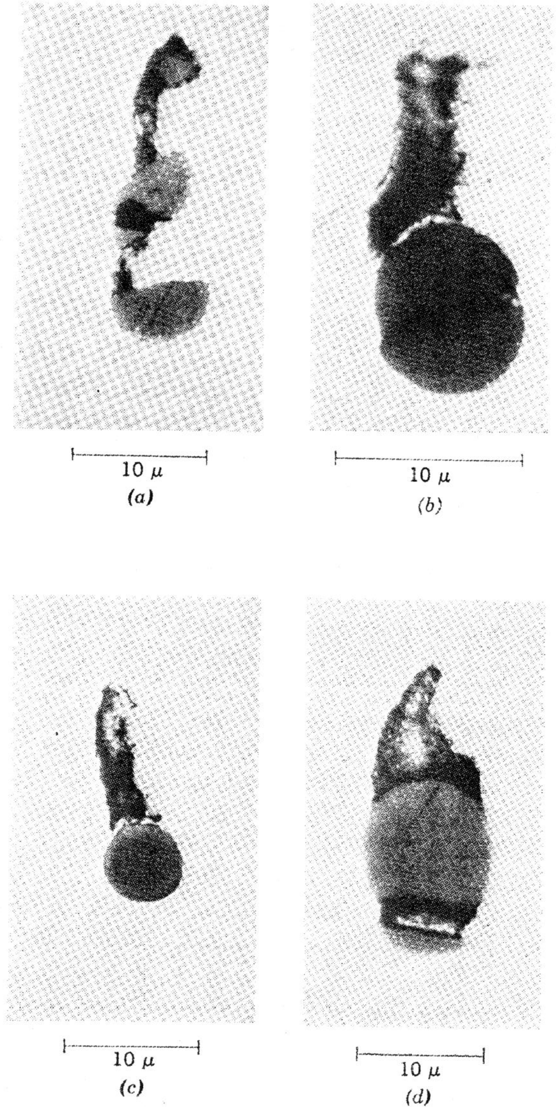

Fig. 5. Examples of cavities forming at oxide inclusions. (Tensile axis is vertical.) (After Puttick.[51])

One conclusion from this theory of ductile fibrous fracture is that the reduction of area should not vary significantly with temperature, provided the inclusions responsible for the fracture are not altered by temperature. For then the same factor that determines the growth of the external neck, that is, the reduction in the coefficient of work hardening at large strains, also determines the growth of the internal necks, so that external and internal growths occur together irrespective of the effect of temperature on the yield and work-hardening behavior of the metal. The reduction of area then depends on the distribution and properties of the inclusions. The insensitivity of the reduction of area to temperature is well confirmed in practice. According to the data collected by Teed,[55] the reduction of area in pure aluminum remained within the limits 87 to 92% over the range between room temperature and −180°C. Armco iron remained virtually constant at 70% reduction in area from room temperature down to −120°C, at which temperature the transition to cleavage fracture began. Copper, annealed or cold-drawn, alpha-brass, and nickel, all showed constant reductions in area between room temperature and that of liquid air.

As regards the final stage of tensile cup-and-cone fracture in which the material parts along a conical surface, Zener [56] has given good reasons for believing that the fracture here follows a path of intense, localized, and rapid plastic shear in which the material is softened by the heat of plastic working. A recent experiment by Puttick supports this view:[51] By straining the specimen in a rigid frame so that the rapid adiabatic shear upon which the effect depends was suppressed, it was possible to eliminate the cone part of the fracture and extend the cup to the surface of the specimen.

It hardly seems possible that, in this stage of fracture, the conical surfaces of the sheared zone simply slide right off each other, since at different points around the cone the directions of sliding are mutually incompatible, and heavy deformation in the wall of the cup would be necessary, as shown in Fig. 6*a*. Some deformation of the wall often does take place,[48] but not nearly enough to allow complete sliding off.

It is perhaps more reasonable to suppose that the conical surfaces do not merely slide but also move away from each other under the influence of the tensile stress that acts normally to them, so that plastic cavities spread rapidly between them. The process of fracture thus envisaged is similar, but in reverse, to the process of adhesion that occurs when two clean metallic surfaces are made to slide past each other under a normal pressure (Fig. 6*b*). Parker and Hatch [57] and McFarlane and Tabor [58] have shown that the junctions formed between asperities on such surfaces grow rapidly in area when sliding first takes place. The two surfaces

move together while the junctions are growing, and their initial angle of approach is 45°.[59]

The important factors in the processes of fracture discussed in this section are work hardening, plastic instability, and nuclei for plastic cavities. There is no stress criterion for fracture other than through the fact that the coefficient of work hardening becomes small at high stresses. Even so, it is possible to have fully ductile fractures that resemble brittle fractures in obeying a Griffith criterion and that propagate catastrophically once they have grown beyond the critical size. Orowan [60] emphasized

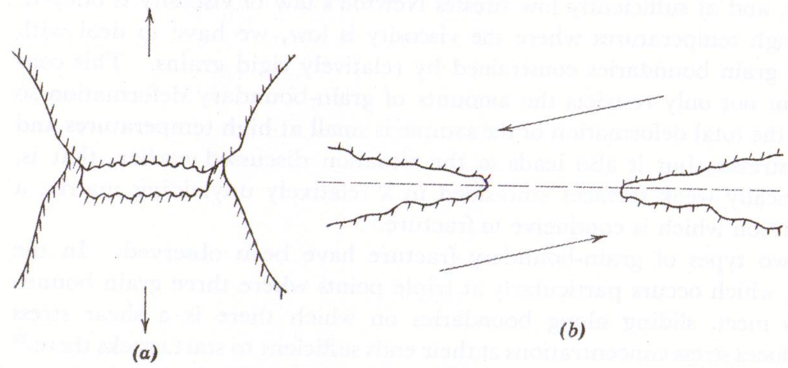

Fig. 6. Formation of the cone in a ductile tensile failure. (*a*) Deformation of the wall of the cone to allow complete sliding to occur. (*b*) Growth of plastic junctions during sliding of two metal surfaces under pressure.

that there is no reason why Griffith's formula should not be applied widely to all fractures in which the work of fracture takes place locally in the zone of fracture, provided that the surface energy term γ in Griffith's formula is interpreted as this work of fracture. This localization of the plastic deformation is possible for a long crack in a plate (plane stress conditions). When the plate is very thin, the plastic work of fracture is small because, even though the reduction of area to form the crack may approach 100%, the volume of the necked or sheared region in front of the crack, in which the plastic work is done, is extremely small (of order of the square of the thickness of the plate, per unit distance along the path of the fracture). Once the crack has reached a length at which the energy that can be drained by it from the applied stress field is greater than this plastic work, a catastrophic and rapid spreading of the fracture is possible. This effect, which Irwin has demonstrated in aluminum foil,[61,62] is extremely important in thin-skin constructions such as aircraft hulls.[63] Fully ductile fractures at speeds of several hundred feet per second have been produced in aluminum alloy sheets.[63]

Grain-Boundary Fractures under Creep Conditions

The atomic processes of flow in an incoherent boundary between grains are more like those in a viscous liquid than like plastic glide in a crystal. The activation energy to overcome barriers between various atomic configurations in the boundary is supplied almost entirely from thermal fluctuations. By contrast, the applied stress plays only the minor role of guiding the otherwise random thermal jumps of the atoms so that slightly more jump with the stress than against it. There is no elastic limit, and at sufficiently low stresses Newton's law of viscosity is obeyed. At high temperatures where the viscosity is low, we have to deal with fluid grain boundaries constrained by relatively rigid grains. This constraint not only restricts the amounts of grain-boundary deformation so that the total deformation of the sample is small at high temperatures and low stresses, but it also leads to the situation discussed earlier, that is, plastically weak surfaces embedded in a relatively unyielding matrix, a condition which is conducive to fracture.

Two types of grain-boundary fracture have been observed. In the first, which occurs particularly at triple points where three grain boundaries meet, sliding along boundaries on which there is a shear stress produces stress concentrations at their ends sufficient to start cracks there.[64] Such processes are geometrically very similar to the plastic-shear processes discussed earlier, and, if the shear gradient along a grain boundary is regarded as a pile-up of infinitesimal dislocations (using Eq. 5 to define their Burgers vector), then Fig. 1*c* can be used to illustrate grain-boundary fracture at a triple point. McLean [65] has developed the theory of such fractures in terms of a Griffith criterion.

When small stresses are applied for long times, the second type of grain-boundary fracture appears. Small holes form on grain boundaries, especially those perpendicular to the tensile axis, then grow and coalesce. The appropriate analogy in this case would seem to be with the multiply nucleated internal necking discussed in the preceding section. Since the original suggestion of Greenwood [66] that the growth of these holes was due to the movement of lattice vacancies, most people have been drawn to this point of view by the appearance of the holes themselves, which are rounded rather than elongated, and by the slowness with which they grow.

Of the many vacancy theories that have been put forward, the one that has appealed most to the writer is the theory of Balluffi and Seigle.[66] Their point is that, if a small hole happens to have formed in some way (for instance, as a result of the pile-up of shear deformation at a foreign particle), the problem of its growth under low stresses at high temperatures can be handled as if it were a problem of sintering. There can be little doubt that the final stages of closed-pore sintering occur by the migration of vacancies from holes to sinks.[67] The driving force for this Nabarro-Herring vacancy creep is provided by the equivalent pressure,

$$p = \frac{2\gamma}{r} \tag{12}$$

owing to the surface energy γ per unit area of a hole of radius r. Holes near incoherent boundaries close up rapidly because such boundaries are good sources and sinks of vacancies.[68] When such holes contain gas under pressure,[68] or when there is a suitable tensile stress present so that the outward forces on the holes exceed $2\gamma/r$, we expect those holes near incoherent boundaries to grow by the same Nabarro-Herring creep process, although acting in the reverse direction. In the case of uniaxial tension, the driving force is derived from the fact that, when atoms are taken from the surface of a hole, then deposited and crystallized on a face of the crystal that is perpendicular to the tensile axis, the specimen elongates in the direction of that stress, and work is done. It follows that holes on grain boundaries exactly perpendicular to the tensile stress should grow when that stress is greater than $2\gamma/r$. For other orientations in which the plane of the boundary lies at an angle θ to this best position, the applied stress needed increases inversely as $\cos^2 \theta$. For holes nucleated on typical foreign inclusions in metals, we take $r \simeq 10^{-4}$ cm. Thus, with $\gamma \simeq 10^3$ ergs/cm^2, the minimum stress at which holes can grow is of order 2×10^7 dynes/cm^2 (or 300 psi).

As a result of these considerations, some experiments were started in the writer's group at the Atomic Energy Research Establishment, Harwell, on the creep of copper containing oxide inclusions, at temperatures in the range of 400°C.[69] Tensile stresses of up to 4730 psi were applied to produce creep strains of a few percent per day, and superposed on these tensile stresses was a hydrostatic pressure of argon of up to 6000 psi. Under suitable combinations of stress, holes formed on the grain boundaries and eventually caused fracture. Using the time of fracture as an index of the rate of growth, an activation energy of 1.1 ev was determined for the process, which suggests grain-boundary diffusion rather than lattice diffusion as the mechanism of transport.

Experiments in which the hydrostatic stress equaled the tensile stress produced no holes, even though the tensile stress and the temperature were such that holes grew in a few minutes in the absence of the hydrostatic stress. Thus, a tension across the boundaries was necessary to make the holes grow; a shear stress without this tension could not make them grow, even though it produced creep. On the other hand, it appeared that a shear stress was needed to nucleate the holes; for, although the fracture time varied approximately as $(\sigma - p)^{-1}$ when the hydrostatic pressure p was varied at constant σ, as would be expected from the Newtonian nature of vacancy creep, it varied much more rapidly than this when the tension σ was varied at constant pressure. Metallographic examination showed that more holes were formed at the higher tensile stresses. Thus, we may have to consider shear stress for the nucleation of holes and tensile (or hydrostatic) stress for their growth.

Fatigue Fracture

Although we are still some way from understanding fatigue cracks, recent experiments have narrowed the range of possibilities considerably. The following are some of the main experimental facts:

1. The crack starts at the surface early in the life of the specimen and grows slowly.[70,71]

2. The fatigue damage from which the crack is formed is localized near the surface of the specimen.[70]

3. The crack starts in broad slip bands, traces of which persist during electropolishing treatments.[70]

4. Grooves and ridges are formed on the surface of the specimen along slip bands. The ridges sometimes develop into thin tongues of metal which, from their appearance, are usually called "extrusions."[72] The grooves, which develop into shallow cracks or "intrusions," are formed in comparable abundance to the extrusions.[73] Sometimes extrusions and intrusions are found on the same slip band.[72,74] The intrusions appear to be responsible for the persistent character of some slip bands.[74]

5. Extrusions, intrusions, and fatigue cracks can be formed at extremely low temperatures where thermally activated movements of vacancies or chemical processes can hardly occur. Fatigue fracture has been produced at 4.2°K.[75] The growth of an intrusion into a fatigue crack is strongly affected by the atmosphere to which the specimen is exposed,[70,74] but this does not seem to affect the formation of the intrusion.[71,74]

It therefore seems that the fatigue crack starts at the surface from intrusions, that it starts there because only there can intrusions and extrusions form, and that the formation of these extrusions and intrusions is a purely geometrical process that does not depend on chemical or thermal action but upon cyclic stressing.[76]

Two such processes have been suggested. In the first, shown in Fig. 7, it is supposed that intersecting slip bands shear one another, forward and backward, during the various phases of the stress cycle, with the result that the Frank-Read sources operating such a band are shifted to one side of the band during the forward phase of the cycle and to the other side during the backward phase.[73] Since forward slip occurs on one side of the band and backward slip on the other side, the material within the band is displaced in the slip direction.

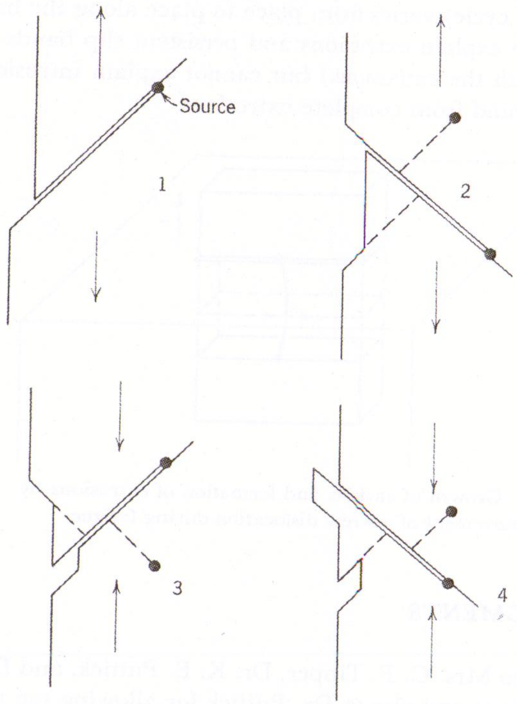

Fig. 7. Formation of extrusions and intrusions by intersection of slip bands during cyclic straining.

In the other theory, suggested by Mott,[77] the backward slip again takes place on the side of a slip band opposite to the forward slip, but this is accomplished by screw dislocations which, during the first half-cycle, glide along one face of the band and then cross slip to the other face, return along this other face, and finally cross slip back again during the second half-cycle. Figure 8 indicates the process. The inner end of such a dislocation is connected to a crack which lies in the slip band underneath the extrusion and which may have formed from the uniting of edge dislocations on neighboring parallel planes, as in the theory suggested by Fujita.[78]

There are several differences between the two theories that could usefully be put to test. The first process makes use of two slip systems with different directions and planes of slip, whereas the second can be operated using a single slip direction provided there is cross slip. The first theory can explain extrusions, intrusions, and persistent slip bands but cannot explain the occurrence of intrusions and extrusions on the same band unless the phase of the slip (and, hence, the stress at which slip begins during the cycle) varies from place to place along the band. The second theory can explain extrusions and persistent slip bands (from the cavities underneath the extrusions) but cannot explain intrusions except as cavities left behind from complete extrusions.

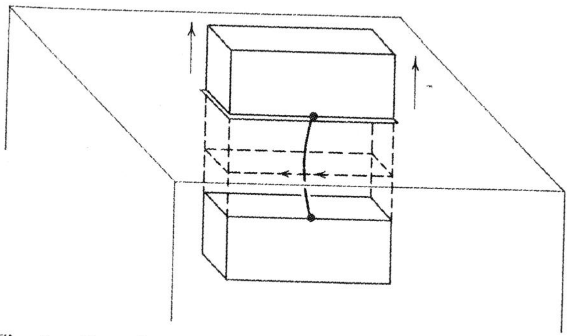

Fig. 8. Growth of cavities and formation of extrusions by cyclic movement of a screw dislocation during fatigue.

ACKNOWLEDGMENTS

I am grateful to Mrs. C. F. Tipper, Dr. K. E. Puttick, and Dr. D. Hull for useful discussions and also to Dr. Puttick for allowing me to show his photographs in Figs. 3, 4, and 5.

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DISCUSSION

T. KANAZAWA, *University of Tokyo, Japan.* Cottrell concludes that the growth of the crack is the critical stage for fracture. Hahn and others show that stable microcracks, about one grain diameter in length, are observed in steels at certain temperatures. Since the growth process will, therefore, be influenced by heterogeneities such as grain boundaries and impurities, the conditions for unstable microcrack propagation should be considered in heterogeneous, rather than in homogeneous material.

Cottrell also suggests that the metal is not completely safe from semi-brittle fracture starting from plastically worked roots of notches until the temperature is raised to the range where Eq. 11 can no longer be satisfied. According to Cottrell, a Robertson-type crack propagation test can be used to establish this upper limiting temperature. Equation 11 is similar to the Griffith relation and describes the conditions for starting semibrittle crack propagation. On the other hand, crack propagation tests determine whether a running crack will be stopped, and this depends on kinetic energy effects and the dynamic stress distribution. The conditions for starting the crack and the conditions in the Robertson test are, therefore, not the same.

E. S. MACHLIN, *Columbia University.* Experimental data obtained by Intrater [1] show that the number of voids produced along bicrystal boundaries in copper is a single-valued function of the amount of grain-boundary sliding, independent of the test temperature. Also, Kramer [2] has found in interrupted tests on polycrystalline nickel that the fraction of grain-boundary area lacking cohesive contact (cracked) is a monotonically increasing function of the prior creep strain. There are many measurements [3,4] that reveal that the amount of grain-boundary sliding that occurs during a creep test in a polycrystalline specimen is proportional to the total creep strain. These data are consistent in that the data on polycrystalline specimens imply that the total grain-boundary area occupied by voids and cracks is a function of the amount of grain-boundary sliding, as observed directly in the bicrystal experiments. These experiments imply further that vacancy condensation is not important to the production of grain-boundary fracture under the experimental conditions of these tests.[1,2]

There exist numerous investigations [4–7] that show an inverse relation between time of fracture (the index used by Cottrell) and minimum creep rate. It is likely, therefore, that the activation energy associated with the time of fracture is in reality that for the controlling creep process. It is possible that the activation energy for creep at low temperatures (400°C in copper) may be less than that for self-diffusion. A value of 0.7 ev has been observed in silver at 500°C.[8] Grain-boundary sliding activation energies for copper vary from 1 to 2 ev.[9] Thus the activation energy observed by Hull and Rimmer in their experiments can be interpreted as that for the controlling creep process in copper at 400°C. The fact that no voids were produced when the hydrostatic stress equaled the tensile stress can be interpreted as an indication that a net tensile stress is required to separate the surfaces of fracture to prevent their recohesion. Chen and Machlin [10] found the same effect in their tests on copper bicrystals, namely, that voids were produced only when there was a net shear stress parallel to the bicrystal boundary that produced sliding.

No voids were produced when only a tensile stress normal to the bi-crystal boundary was present, and such tension applied subsequent to a shear application resulted in the opening up of the voids produced in the prior shear. Under similar conditions, Intrater [1] observed the unique relation between the area of voids along the grain boundary (and number of voids as well) and the amount of grain-boundary sliding. It appears, therefore, that Hull's and Rimmer's experiments are not critical with respect to a demonstration of the existence of vacancy condensation at voids during creep.

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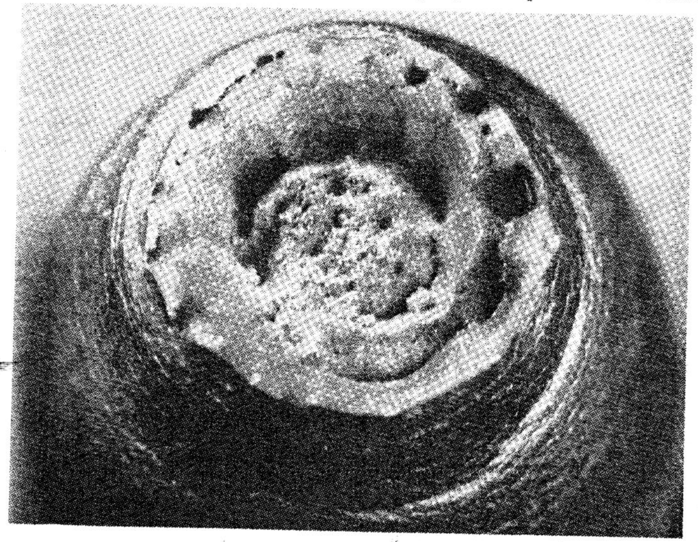
H. C. ROGERS, *General Electric Research Laboratory*. It has been shown [1] that the failure of polycrystalline ductile metals occurs in two or three stages, as illustrated in Fig. D.1 for fine-grained OFHC copper. Stage I is the formation, growth, and coalescence of individual voids in the central region of the neck to form the central crack. The fracture surface does not mate except in those cases where void linkage occurs by the Stage II-type failure.

In Stage II, the crack produced in Stage I propagates by causing a shear strain localization at its tip and at a large angle to the transverse plane. In this thin sheet of heavily deformed material, an array of small voids is nucleated; this void-weakened region fails under the action of the applied tensile stress. The specimen geometry forces the crack to zigzag back and forth across the plane of the minimum section as it propagates radially. With this type of failure, the fracture surfaces in Stage II mate on a gross scale; on a fine scale, however, the coalescence of the sheet of small voids leaves a distorted half-void on each surface. The extent of the zigzag steps is a function of the stress level, the rate of strain hardening, the increase in the rate of stress after the failure of each step, and the elastic energy stored in the system. If the zigzag crack propagates to the surface, the result is a "cup-cone" failure. This type of final failure is to be expected in strong materials that do not work harden rapidly and has been observed for brass, iron, and dural (in all

conditions) at room temperature. Unpublished work shows that the finer the grain size, the smaller the region of large void coalescence, and the sooner Stage II begins. The appearance of the fractures of steel specimens pulled in tension under hydrostatic pressure as described by Bridgman [2] indicates that both the size of the coalescing voids in Stage I and the percent of the fracture occurring by this process decrease with



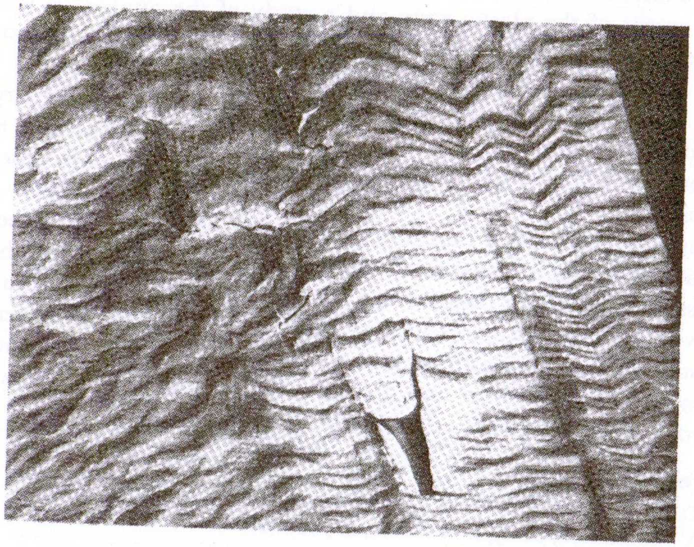
1 mm

Fig. D.1. One half of the fracture of a fine-grained OFHC copper tensile specimen showing three regions that failed by different mechanisms.

increasing pressure, the crack propagating primarily by the Stage II process.

Puttick's suppression of the cone of a cup-cone fracture by the use of a rigid tensile machine is interpreted as a reduction in the amount of elastic energy available for extension of the crack step out of the plane of maximum tensile stress, which forces the crack to proceed to the surface by a series of much smaller steps as in the normal growth of the central crack. A possible effect of adiabatic heating, if primary, should be manifested in the region of the zigzagging central crack as well as in the region of cone, since the failure mechanism is identical.

Stage III in the tensile failure of OFHC copper and other "pure" face-centered cubic metals at room temperature is by a mechanism that results in a "double-cup" fracture.[1] A negative electron micrograph of the inner surface of one of the cups is shown in Fig. D.2. Rather than being covered with the remains of small voids as are the surfaces of the cups in a cup-cone failure, voids are incidental to this fracture. (One is shown in the foreground.) The inner surface of large voids is like that of the fracture surface of the double cup, indicating that the mechanism of nongrain-boundary void growth is the same as that by which the final stage of fracture occurs.



2 μ

Fig. D.2. Negative replica of the fracture surface of a "double-cup" failure in OFHC copper.

Although the sufficiency of foreign particles in a ductile matrix to act as void-nucleating agents is unquestionable, it is doubtful that there is very good evidence for the necessity of such agents. In the first place, even if one were to suppose that the initial void formation in a material such as 99.999%-purity copper takes place by void nucleation at foreign particles, it is difficult to believe that the relatively uniform arrays of fine voids that are nucleated in the regions of heavy shear are the result of particle nucleation. In such materials, no evidence of particles associated with either the large voids or the smaller voids [1] has been seen. Second, by changing the heat-treating atmosphere from vacuum to hydrogen, one can suppress the normal nucleation of voids within the grains, voids occurring primarily at grain boundaries. Even though the formation of voids at grain boundaries will alter the stress pattern within the grain to some extent, if voids are nucleated within the grains by

foreign particles, the high local stresses in the neighborhood of such particles would still exist, and voids attributable to them should still appear.

References

1. H. C. Rogers, submitted to *Trans. AIME*.
2. P. W. Bridgman, *Studies in Large Plastic Flow and Fracture*, McGraw-Hill Book Co., New York (1952).

F. A. McClintock, *Massachusetts Institute of Technology*. On a macroscopic scale, fracture may be defined as the separation into two parts of the smallest element of volume to which the macroscopic concepts of stress and strain can be applied. This stage will be called fracture completion of the material to distinguish it from the process of fracture on the microscopic scale or the fracture completion of the entire specimen. In the sample shown by Cottrell (Fig. 4 of Chapter 2), the fracture process has begun at the half-radius, judging from the cavities visible there. Not as much additional strain will be required to complete the processes leading to fracture completion of the material at that point as would have been required if the cavities were not already visible. In other words, fracture completion of the material by the growth and agglomeration of cavities, defined for the smallest region to which the macroscopic concepts of stress and strain can be applied, does depend on the history of stress and strain in that region.

Cottrell cites Bridgman's work as a demonstration of the fact that stress and strain do not determine fracture. The quite different histories of two of Bridgman's specimens are sketched on this basis in Fig. D.3. It is reasonable to assume that the plastic deformation occurring while the material was under mean compression allowed nuclei to be reoriented, or broken up and dispersed, so that when the mean principal stress did become tensile, the growth and agglomeration of cavities was delayed. Thus in Bridgman's experiments, there is no contradiction to the hypothesis that the fracture completion of the material depends on the history of stress and strain. In working out the mechanics of macroscopic crack growth with such a criterion, there will also appear the minimum size of the region to which the macroscopic concepts of stress and strain can be applied. This size is set, at least in part, by the spacing of cavity nuclei.

While the above dependence of fracture on stress and strain history was illustrated by the growth and agglomeration of cavities, a similar argument can be made for the case in which cleavage is repeatedly nucleated in regions just in front of the growing crack. In harder alloys, there is debate about the continuous growth of sharp cracks without repeated initiation in advance of the main crack. If such growth does exist, the plastic deformation that can initiate a crack should also affect its propagation, so that here also the fracture completion of the material would depend on the history of stress and strain in the region being considered.

Because the history of stress is so complicated in the necked tensile specimen and is different for points at different radii, it would be desirable to study fracture in simpler cases. In particular, in a circum-

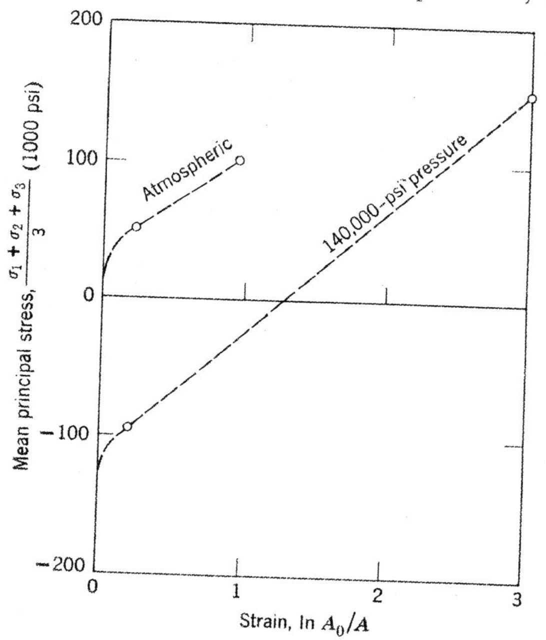


Fig. D.3. Effect of history of stress and strain on fracture.

ferentially notched bar in torsion, the mean principal stress is zero at all times throughout the specimen, and Mackenzie [1] has shown that fracture from notches of various radii can be correlated in terms of the shear strain in a region whose size is of the order of a few grain diameters.

It seems to be a common misconception that the extent of the plastic region in front of a crack in thin foil is of the order of the foil thickness. That this is not so can be seen even from an elastic analysis, which indicates that the stress σ at a point r in front of a crack of length $2c$ in a sheet subject to a nominal stress at infinity of σ_∞ is approximately given by

$$\sigma \simeq \sigma_\infty \sqrt{c/r}$$

Equating the stress at the radius of the limit of the plastic zone R to the flow stress Y gives

$$R \simeq c(\sigma_\infty/Y)^2$$

For example, if a stress of one-third the flow stress is applied to a 6-in. crack in foil 0.00025 in. thick, the radius of the limit of the plastic zone is 0.3 in. (or 1000 times the foil thickness, which is thus irrelevant). Relaxation of the stresses within this region will cause the plastic zone to be even greater. An exact elastic-plastic analysis for the amount of energy absorbed remains to be done, but microscopic observations of the lateral extent of the plastic zone and a study of the analogous case for shear indicate that the amount of energy absorbed in the entire plastic region is much larger than that locally absorbed in the neck. Thus energy balances considering only the elastic energy and the energy in the neck overlook a more important term. These considerations led to the more complete analysis of the mechanics of ductile fracture described elsewhere.[2]

It must be emphasized that it is necessary to develop, from dislocation mechanisms on up, a fracture criterion expressible in terms of the history of stress and strain in a region large enough to permit the criterion to be used with the macroscopic concepts of stress and strain encountered in the usual theories of elasticity and plasticity. These theories must be used to find how the forces and deformations applied to specimens affect the processes in the region at the tip of the crack.

References

1. A. C. Mackenzie, M.S. Thesis, M.I.T. (1958).
2. F. A. McClintock, *J. Appl. Mechanics*, **25**, 582 (1958).

J. J. Gilman, *General Electric Research Laboratory, Schenectady, New York.* There is a mode of fracture nucleation that often operates in addition to those discussed by Cottrell. One place where this new mode is important is the case of fracture at kink planes in zinc.

The way in which cracks form at kink planes is illustrated in Fig. D.4. At (*b*) in this figure, the situation coincides quite closely with an actual crack in a zinc crystal shown at (*d*). Here the kink boundary below the crack has split into two parts, and the part with the larger angle has moved toward the left, while the part with the smaller angle has moved toward the right. Since the applied stress was a compression in the horizontal direction, it could not have supplied the necessary shear stress to move these two boundaries in opposite directions. Therefore, I propose that tensile forces like those indicated by the vertical arrows in Fig. D.4*b* must have acted. The way in which such tensile forces can arise is suggested by Fig. D.4*c*.

If plastic shearing occurs on one side of a boundary and on a single glide system, and if the boundary does not move to bisect the new misorientation angle, then a situation arises as shown in Fig. D.4*c*, in which

h_2 is not equal to h_1. This will cause large tensile stresses parallel to the boundary which may lead to cracking. The tensile strain that is produced is $h_1/h_2 - 1 = \epsilon$, but since $h_2 = h_1 \cos \Delta\theta$, we have $\epsilon + 1 = 1/\cos \Delta\theta$. Then, if a strain of 2% is required to cause fracture, the change of misorientation angle that is needed is $\Delta\theta = 7°$.

It is suggested that this is a common mechanism by which cracks are nucleated in crystals. Wherever there is a subboundary or another type of boundary (grain, twins, or precipitate) that cannot move to accommodate a shear strain discontinuity, large tensile stresses will result parallel to the boundary. This may be the way, for example, in which tensile stresses are produced perpendicular to the cleavage planes of iron (100) by plastic flow on planes that make almost any angle with respect to the cleavage planes. The only requirement is that a shear strain discontinuity be produced of the type shown in Fig. D.4*c*. It may be noted that dislocation pile-ups are not necessary in this mechanism, although they may act to intensify the stresses locally.

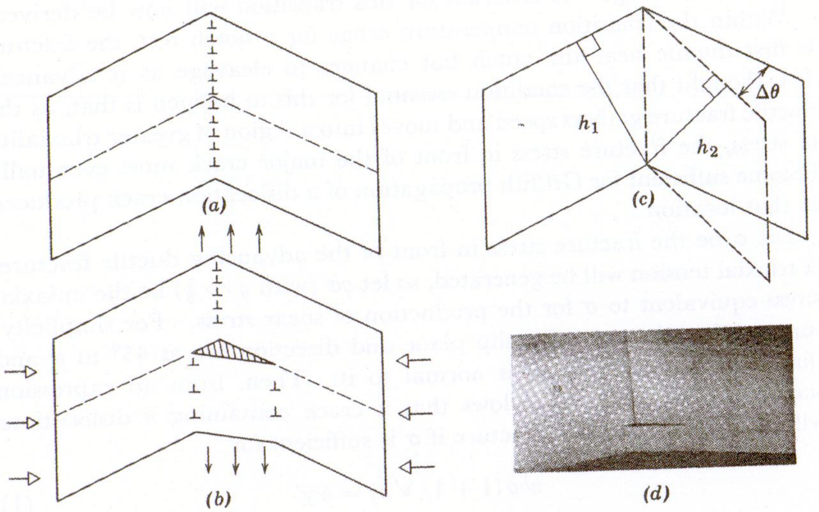


Fig. D.4. Formation of cracks at kink planes in zinc.

A. H. Cottrell (Author's Reply). When a crack propagates in a heterogeneous material, the heterogeneity of the material must of course be taken into account, as Kanazawa points out. In such cases, the specific energy of the fracture surface is initially small and then increases at a later stage in the growth of the crack, especially when a grain boundary is penetrated. Contrary to the view expressed by Kanazawa, the author believes that the crack-starting temperature and the Robertson temperature are related, because the propagation of a long crack at a temperature just below the Robertson temperature often occurs by the starting of new cleavage cracks in front of the main crack. Under these *particular* conditions, the processes of starting and propagating appear to be similar.

The observations described by Machlin are consistent with the view that grain-boundary sliding is important in the nucleation of voids. However, the growth of voids is believed to be caused by a vacancy mechanism, because it occurs at a rate that is consistent with the Nabarro-Herring mechanism for vacancy flow from nearby grain boundaries, and because the variation in time to fracture with hydrostatic pressure observed in Hull and Rimmer's experiment agrees with the Newtonian character of vacancy creep. The "zigzag" growth of ductile fracture reported by Rogers may be related to the process of reverse adhesion discussed by the author in connection with the cone fracture. It is important to establish whether such types of ductile fracture can be nucleated in ways other than at foreign particles or pre-existing cavities. If they can, we shall have to find an explanation for the formation of cavities in fully ductile metals in terms such as the piling-up of dislocations or the aggregation of lattice vacancies. In connection with McClintock's remarks, the author believes that a complete understanding of fractures in ductile materials can be gained only by combining the atomistic and mathematical plasticity theories, and he is very glad to see that mathematical plasticity theory is now being applied to the problem.