7. The Mechanism of the Brittle Fracture of Metals

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ABSTRACT

Detailed consideration is given in this paper to the factors controlling the growth of a crack, particularly the surface energy of the crack, the plastic deformation accompanying crack propagation, and the stress concentration responsible for crack initiation. Experiments reported suggest that fracture initiation is not invariably related to dislocation groups formed on yielding. Calculations on closely spaced groups of like dislocations show that crack formation under stress is possible. The existence of such groups in annealed metals requires further consideration.

Introduction

In this paper, certain ideas that have developed during the course of investigations at the National Physical Laboratory on the brittle fracture of iron are discussed. We have been particularly interested in the apparent discrepancies between the results of certain experiments and current theories of fracture, and we shall begin by considering these theories. In choosing illustrations and numerical examples, we shall have the case of iron mainly in mind, but many of the points we shall make will be of more general application.

Modifications of the Griffith Theory

Although the Griffith crack theory in its original form is not applicable to the fracture of metals,[1] it is nevertheless the basis of current ideas on the subject. Griffith considered a balance between two factors: the surface energy of the spreading crack and the release of the strain energy of the applied stress field. Subsequent modifications of this theory have retained the concept of the energy balance, other factors being added as their importance became apparent. Plastic deformation provides an alternative means of using up the strain energy released as the crack extends; [2] accordingly a term is added to the energy balance representing the energy dissipated in plastic deformation. With normally accepted surface energy values, the cracks required to give fracture at normal loads are much too large to be present initially in the unstressed material, and, to meet this difficulty, it is generally supposed that fracture begins at a point of local stress concentration such as might be provided by a pile-up of dislocations; [3–8] the local strain energy associated with this stress concentration must also be added to the energy balance.

The four energy terms controlling the growth of the crack which we shall consider are therefore:

(*a*) The surface energy of the crack, proportional to its length l,

$$+Al$$

(*b*) The reduction in energy of the applied stress field, proportional to the square of the applied stress and the square of the crack length,

$$-Bl^2\sigma^2$$

(*c*) The complex plastic-deformation factor that depends upon the crack length, the applied stress, and the yield point of the material,

$$+Cf(l, \sigma, Y)$$

(*d*) The energy provided by the local stress concentration at which fracture begins, which is generally assumed to be proportional to log l,[5,6]

$$-D \log l$$

The change in energy owing to the extension of the crack is then

$$\Delta E = \Delta[Al] - \Delta[Bl^2\sigma^2] + \Delta[Cf(l, \sigma, Y)] - \Delta[D \log l]$$

and the condition for extension of the crack is that ΔE should be negative; thus complete fracture will result if E decreases continuously as l increases. According to this equation, at certain stress values, E may pass through a minimum as l increases, and so stable cracks that do not propagate at certain stresses can be expected.

Apparently, grain boundaries are also a cause of minima in the E-l curve, for cracks that stop at grain boundaries have been observed. The propagation of cracks of this kind can be fairly simply understood in terms of the general equation; however, when they are observed, they are always fully formed and of lengths equal to the diameters of the grains in which they occur. The conditions for their initiation are not equally well understood, and this question seems of paramount importance in the explanation of brittle fracture.

The terms in the fracture equation will now be discussed individually, with some emphasis on the early stages of crack formation, and comments will be made about their reactions upon each other.

The Surface Energy Term

It has been suggested that in the case of metals the surface energy term will be negligible compared with the plastic-deformation term, and this will undoubtedly be so when gross plastic deformation occurs. Nevertheless, the surface energy term is of great importance, particularly in the very early stages of crack formation, for then the surface energy alone is large compared with the elastic energy available from the reduction of energy of the applied stress field (term *b*), and, if the crack is to extend at all, very little of the energy available must be dissipated in plastic deformation.

Experimental evidence on this point comes from the behavior at low temperatures of iron having embrittled grain boundaries.[9–12] The embrittlement is due to small quantities of dissolved oxygen, nitrogen, or phosphorus, and in the case of phosphorus additions it has been shown that the phosphorus segregates in the grain boundaries, leading to very low grain-boundary fracture stresses (in serious cases as low as 1 ton/in.^{2}). The degree of embrittlement is influenced greatly by appropriate heat-treatments, but these heat-treatments do not affect the yield stress markedly. It is therefore likely to be the amount of energy required to form the new surfaces, rather than the amount of energy absorbed in plastic deformation, which is principally affected.

Direct measurements on the effect of impurities on the surface energy of iron have not been made, but experiments are in progress on the effect of antimony on the surface tension of copper-antimony alloys in which a similar grain-boundary embrittlement can be produced.[13] The results appear to show quite definitely that the segregation of antimony can reduce the surface tension of copper to about one fifth of its original value,[14] which could explain the reduced grain-boundary strength of these alloys.

Brittle fractures in iron and steel take place predominantly by cleavage along {100} planes. No measurements of the surface energy of these planes are available, but it is generally assumed to be about 2000 ergs/cm^2. The fact that iron breaks along {100} planes suggests the surface energy of these planes is certainly low, but it is generally considered that it cannot be very widely different from the usually accepted value. On the other hand, impurities could segregate along {100} planes[15,16] as well as along grain boundaries and produce similar results, so that the possibility must not be neglected that in practice the effective surface energy of {100} planes could be substantially lower than is generally assumed.

If the energy absorbed during the passage of a brittle crack through a specimen having embrittled grain boundaries is measured, it is naturally many times less than the energy absorbed during the fracture of a corresponding less-brittle specimen, and it is many times larger than the simple surface energy required to separate the fracture faces. The surface energy is never in itself the total energy consumed in the grain-boundary fracture, therefore, but its value evidently strongly influences this total energy. To understand how this comes about, it is necessary to consider how the plastic-deformation energy term arises in the fracture energy equation.

The Plastic-Deformation Energy Term

The plastic-deformation energy term represents the energy consumed by the plastic deformation that occurs in the highly stressed region in front of the growing crack. The minimum size of this region is determined by the strain at which the force between adjacent atoms reaches its maximum, for the strain at the end of the crack has this magnitude. The strains at increasing distances from the end of the crack fall off in proportion to the reciprocal of the distance until they reach the strain generally prevailing in the material in which the crack is growing.

A second stress field is also present. This arises from the inability of the crack to transmit a tensile or shear stress and results in a concentration of the applied stress around the end of the crack. It increases rapidly with increases in applied stress and length of the crack and ultimately becomes the more important field of the two.

The occurrence of plastic deformation during the spread of the crack is governed by the presence of mobile dislocations within the stress field at the end of the crack. The important area is that within which the shear stresses are sufficiently large to move the dislocations, that is to say, in which the stress exceeds the yield point of the material. If the crack is small and the dislocations are widely spaced, the crack may grow for some time before its stress field encounters a dislocation (Fig. 1*a*). In this case no plastic deformation occurs, and the only energy absorbed by the growth of the crack is that attributable to the surface energy term. On the other hand, if the crack is large and the dislocations closely spaced (Fig. 1*b*), many dislocations may be included within the region stressed above the yield point, and the energy absorbed in plastic deformation will be correspondingly large. The surface energy term is therefore predominant during the initiation and early growth of the crack, and the plastic-deformation term predominates in its subsequent development.

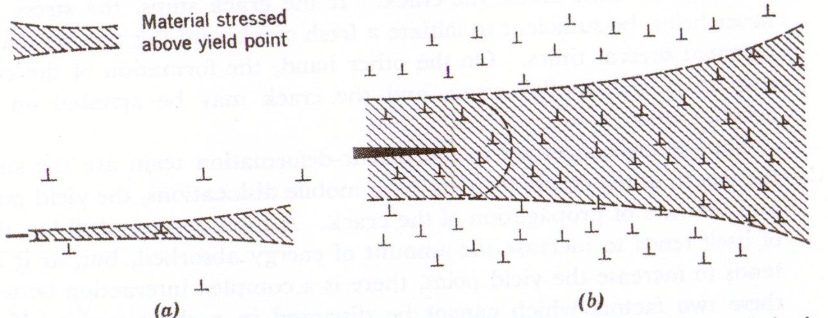


Fig. 1. The effect of size of crack and density of dislocations on the amount of plastic deformation occurring as a crack extends. (*a*) Small crack growing in region containing few dislocations. (*b*) Large crack growing in region containing many dislocations.

The surface tension term has nevertheless an important influence on the plastic-deformation term because of its effect on the stress at which the crack begins to develop. If the stress is near the yield point of the material, the region within which the stress is above the yield point is large, and many dislocations are included within it; but if the stress is low, for example, in embrittled iron-phosphorus alloys, the region is correspondingly narrow, and the energy of plastic deformation is less (Figs. 2*a* and *b*). The width of the band swept out by the stress field of the advancing crack governs the energy of plastic deformation. If the stress is maintained throughout the growth of the crack and rate of crack propagation is small, this width will be expected to increase continuously. Since the energy available for crack propagation increases

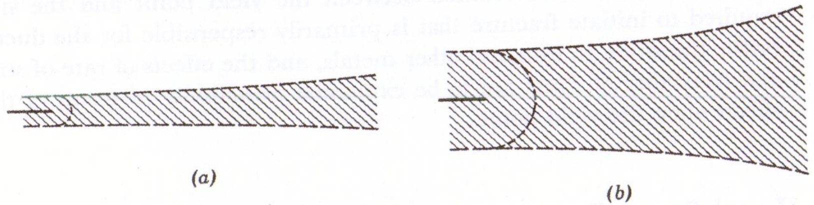


Fig. 2. The effect of the magnitude of the applied stress and the yield stress of the material on the amount of plastic deformation occurring as a crack extends. (*a*) Region within which plastic deformation occurs, low stress or high yield point. (*b*) Region within which plastic deformation occurs, high stress or low yield point.

as the crack lengthens, however, there is a tendency for the crack to accelerate, and this may result in the effective yield point being raised and the absorption of energy being lowered. The expansion of the stress field increases the energy absorbed in plastic deformation and thus tends to retard or even arrest the crack. If the crack stops, the stress may nevertheless be sufficient to initiate a fresh crack, and the process may be repeated several times. On the other hand, the formation of the crack may relax the applied stress, and the crack may be arrested on this account alone.

The factors concerned in the plastic-deformation term are the stress, the length of the crack, the density of mobile dislocations, the yield point, and the rate of propagation of the crack. A high density of dislocations of itself tends to increase the amount of energy absorbed, but, as it also tends to increase the yield point, there is a complex interaction between these two factors which cannot be discussed in a short space. If the yield point is raised, the energy absorption is decreased and brittleness thereby increased. If it is sufficiently lowered, it eventually becomes less than the stress required to initiate fracture as determined by the surface tension term. The area in which plastic deformation can occur is then extended indefinitely, and the energy absorbed by the plastic deformation is sufficient to ensure that no crack develops until the material has work hardened sufficiently so that it has a yield point above a critical value. Fracture in this instance is preceded by general plastic deformation, and the material shows considerable ductility. According to this view, it is this relation between the yield point and the stress required to initiate fracture that is primarily responsible for the ductile-brittle transition in iron and other metals, and the effects of rate of strain and triaxiality of stress are to be explained principally in terms of their effects upon the yield point.

Local Stress-Concentration Term

This term arises out of the need for some kind of stress concentration before cracking can occur at normal loads. According to most current theories, the primary requirement is some form of pile-up of dislocations occurring under the applied stress at a stage prior to the initiation of fracture.[3–8] These theories have been elaborated and used to explain many of the phenomena connected with brittle fracture, but they nevertheless present some difficulties. They lead to the conclusion that there should be a very close connection between fracture stresses and the corresponding yield stresses, and indeed, the experimental evidence supporting these ideas often takes the form of demonstrations that brittle cracks are initiated at the yield point of a material.

While it is clear that large groups of dislocations may form as a result of plastic deformation and set up high local stresses, and while it is also true that plastic deformation in many cases is closely followed by fracture, yet it seems contrary to general experience to suggest that yielding is a necessary condition of fracture, since fracture may occur at any stage of the deformation process, either after a large amount of deformation, or shortly after yielding, or below the yield stress without obvious deformation. Theories of fracture that depend on the occurrence of slip to provide the necessary stress concentration must explain how great amounts of slip may occur without fracture being initiated; and they differ in their method of dealing with this problem. Some effort has been devoted to showing that, when crystals break without major deformation under stresses lower than the expected yield point, some evidence of slip can nevertheless be found;[8] however, as has been suggested earlier, plastic deformation necessarily occurs during the extension of a crack, and its existence provides no evidence that the deformation was the cause, rather than the consequence, of the formation of the crack.

The essential problem in fracture has always been to know why the amount of deformation that accompanies fracture is sometimes large and sometimes extremely small. The simplest assumption is that deformation is small when the stress at which fracture occurs is low in relation to the yield stress, and large when the fracture stress is higher than the yield stress. That is to say, yielding and fracture are independent processes. Single crystals of iron behave as if this indeed were the case. At moderate temperatures, the yield stresses are low and the crystals are completely ductile; they begin to show traces of brittleness at −145°C when the yield stress is rising rapidly. At −196°C, they cleave without obvious prior slip when the orientation is such that the stress resolved normal to a $\{100\}$ plane is high, and they deform plastically with 100% reduction in area when the stress normal to the cleavage plane is low. This is the behavior that would be expected if yielding were determined by a critical resolved shear stress, and cleavage by an independent critical stress normal to the cleavage plane. The phenomena are complicated by the occurrence of twinning, which for the present purpose can be regarded as an alternative form of plastic deformation. The transition orientation changes sharply as the temperature is altered, so that at −145°C the crystals are ductile in practically every orientation, and at −253°C they are brittle in almost every orientation. The simple assumption that the critical shear stress varies with temperature more rapidly than the cleavage stress accounts for this behavior, and the rate of change of critical angle with temperature is roughly that which would be expected from the measured yield stresses and fracture stresses. The details of the experiments are given in Fig. 3.[17] Similarly, when the crystals are hardened in various ways, the ratio of critical resolved fracture stress to critical shear stress does not remain constant, and the transition orientation changes accordingly. Figure 4 contains data on the effects of irradiation damage, addition of carbon and nitrogen, and precipitation hardening which support these statements.[17]

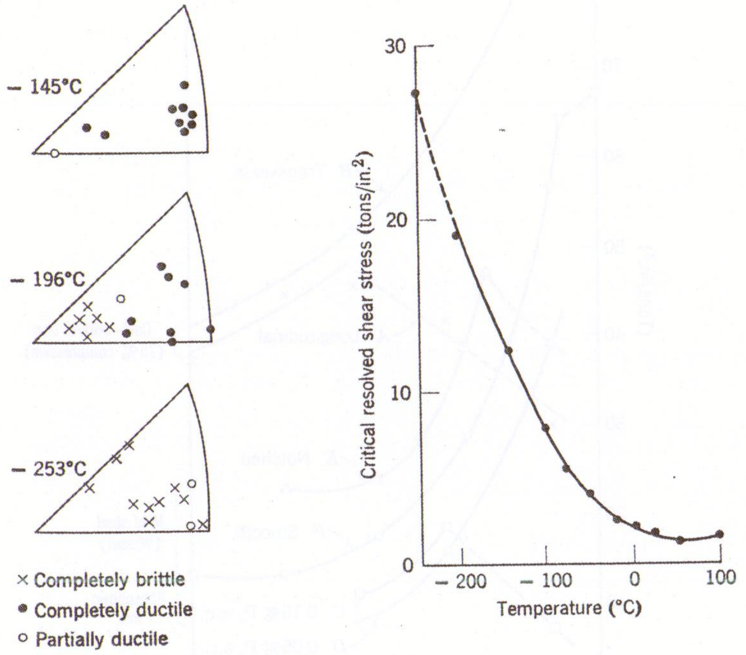
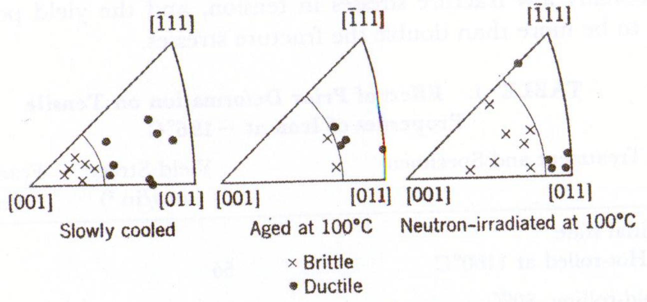


Fig. 3. The effect of orientation and temperature on the tensile properties of iron single crystals.

In connection with the table in Fig. 4, it may be remarked that in the work at the National Physical Laboratory the resolved tensile stress for cleavage has not been found to be readily influenced by the addition of impurities; certainly no element has yet been found that lowers the cleavage strength of the crystals to the extent that the strength of the grain boundaries can be altered, and it seems likely that brittleness in the crystal is generally due to an increase of its yield point.

A striking feature of the behavior of single crystals of iron at −196°C is the sharpness of the transition from extreme ductility to extreme brittleness. It is possible to have two crystals differing in orientation by no more than 2°, one of which shows a reduction of area well over 90%, while the reduction of area of the other is too small to be measured. If the cleavage of the brittle crystal had been occasioned by a small amount of initial slip, it is very difficult to understand why the larger amount of slip in the ductile crystal did not lead to fracture at an early stage. On the other hand, the view that the occurrence of slip leads to an increase of the fracture stress explains the behavior quite naturally. It is found that, if the stresses on the reduced area of the ductile crystal are measured during the course of a tensile test, they become much higher than the cleavage stress of the undistorted crystal.



Description	Resolved Shear Stress (avg) (tons/in.²)	Resolved Cleavage Stress (avg) (tons/in.²)	Ratio	Transition Angle (deg)
Iron: slowly cooled	18.8	26.6	1.41	21
Iron: aged at 100°C	19.8	25.7	1.30	34
Iron: neutron-irradiated at 100°C	23.1	28.5	1.23	38
0.05% nitrogen: quenched	24.9	31.5	1.26	39
0.05% nitrogen: quenched and aged at 100°C	35.0	40.4	1.15	41

Fig. 4. The effect of neutron irradiation, the presence of nitrogen in solution, and precipitation hardening on the behavior of iron single crystals tested in tension at −196°C.

The influence of plastic deformation on the fracture stress of polycrystalline iron is more complex than has usually been realized. It has been observed that tensile deformation at a temperature at which iron is ductile increases the fracture stress measured at a lower temperature at which iron is brittle. However, this increase of fracture stress is observed only when the metal is tested in the direction in which the metal was originally stretched: If tested at right angles to this direction, the fracture stress is found to be considerably reduced. If the initial deformation is compressive, the fracture stress is reduced in the direction of compression and increased in the direction at right angles. Since cracks are not produced across a direction of simple compression, this result eliminates the possibility that the changes of fracture stress were due to the formation of cracks. Table 1 illustrates these observations. Experiments were made to measure the yield point in compression of those samples that showed exceptionally low fracture stresses in tension, and the yield points were found to be more than double the fracture stresses.

TABLE 1. Effect of Prior Deformation on Tensile Properties of Iron at −196°C

Treatment and Specimen	Yield Stress (tons/in.^{2})	Fracture Stress (tons/in.^{2})
Initial state		
Hot-rolled at 1100°C	56	62
Cold-rolling, 80%		
Along direction of rolling	91	137
Transverse to direction of rolling	95 measured in compression	42
Cold compression, 75%		
Along direction of compression	—	40
Transverse to direction of compression	88	88

More light is thrown on the relation between the yield point and the fracture stress by studies of the effect of temperature on the behavior of tough and brittle samples.

Examples of this are given in Fig. 5, which shows the results of tensile tests over a range of temperatures on three pairs of specimens.[11,18,19] One of each pair is very brittle at low temperatures, whereas the other is less so. Curves *A* and *B* relate to a heavily cold-worked mild steel, tested along the direction of working (which was compressive) and transversely. The transverse specimens were ductile at all temperatures, and the yield point increased regularly as the temperature fell. The longitudinal specimens became brittle around room temperature, and their fracture stresses decreased with falling temperature. At −196°C, the fracture stresses were so much lower than the probable yield points that it is very unlikely that yield initiated the fracture.[18] Curves *C* and *D* relate to two samples of polycrystalline high-purity iron containing phosphorus, one of which had been embrittled by being quenched from 700°C, while the other had been slowly cooled.[11] The yield point of the embrittled sample above its transition temperature is not materially altered, but at a sufficiently low temperature it breaks without elongation at a stress which again decreases with falling temperature; at −196°C, this stress is far below the yield point of the corresponding unembrittled sample. Curves *E* and *F* are taken from Wessel's work,[19] and in this case embrittlement has been induced by the presence of a deep, sharp notch. The notched specimens become brittle at a higher temperature than the unnotched specimens, and the strength of the notched specimens in the brittle condition falls with falling temperature. In all these examples,

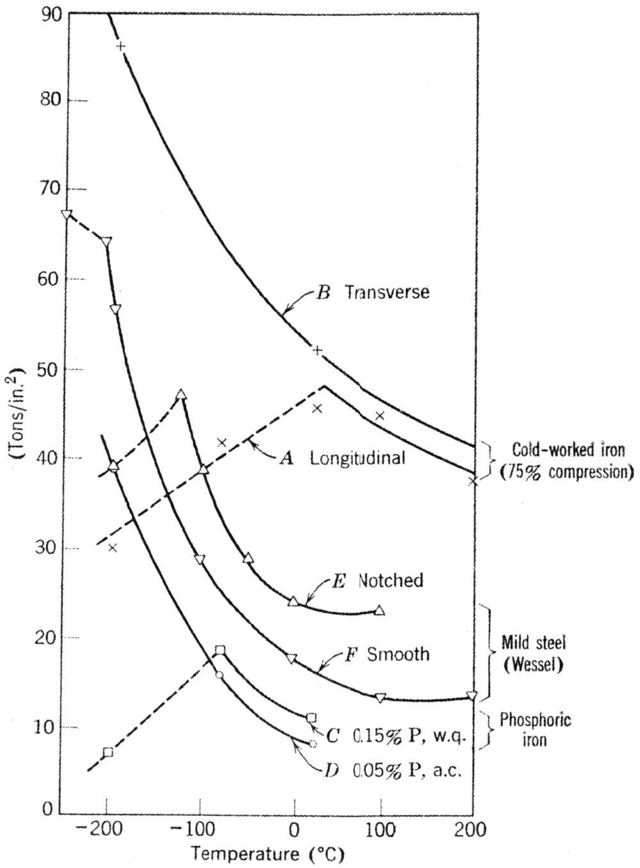


Fig. 5. The effect of variation of temperature on the yield stresses and fracture stresses of: (1) Heavily cold-worked iron tested (*A*): along direction of compression; (*B*): normal to direction of compression. (2) Iron-phosphorus alloy, (*C*): 0.015% P, quenched from 700°C; (*D*): 0.05% P, air-cooled. (3) Mild steel, (*E*): notched; (*F*): smooth.

the defect that led to the low fracture stress of the embrittled member of the pair was present before the specimens were stressed. In *E* and *F*, it was a machined notch that could have induced local yielding, but in the other specimens there is no question of the existence of a gross stress concentration arising from the shape of the test piece. Initiation of fracture must be attributed to some structural feature, which, in the case of the phosphoric iron, was undoubtedly an embrittled grain boundary, and, in the case of the cold-worked iron, may well have been a suitably disposed dislocation group. Since the unnotched mild steel tested by Wessel became embrittled at a sufficiently low temperature and the observed fracture stress was again lower than the expected yield stress as obtained by extrapolation of the yield-stress–temperature curve, it seems likely that in this case also some origin of-fracture initiation, independent of the yield phenomenon, was present.

For these reasons, it seems impossible to conclude that initiation of fracture is invariably due to dislocation groups formed on yielding, and it becomes necessary to inquire whether other dislocation groups sufficient to initiate fracture could be present. Dislocation groups of many kinds certainly exist even in annealed metals. Figures 6 and 7 show examples of closely packed arrays in titanium [20] and iron.[21] Electron micrographs of annealed iron show yet more closely packed arrays [22] (Fig. 8), some of which contain complex dislocations, probably with displacement vectors in a $<100>$ direction, which are formed by the combination of two normal $<111>$ slip dislocations. A question arises as to whether their strain energies could be sufficient to provide any substantial proportion of the surface energy necessary to produce a cleavage crack.

For the purposes of calculation, a group of 90 edge dislocations of the same sign, arranged on a rectangular grid with 9 vertical rows and 10 horizontal rows, has been taken, the vertical rows being the rows that contain the extra atom planes of the edge dislocations. The distribution of tensile or compressive stress across the vertical rows in such an array is illustrated in Fig. 9. The heavy curves show the general stresses in the bulk of the crystal, and the lighter curves the additional stresses in the immediate neighborhood of the dislocations. The strain energy is greatest in a region near and just outside the top and bottom rows of the group, and the stress distribution is such that it cannot be entirely relieved by a simple bending movement. The general stresses are proportional to the Burgers vector of the dislocations and inversely proportional to the distance between them. In Fig. 9, the highest general stress is $50 \times 10^{11} b/d$ dynes/cm^2, where b is the Burgers vector of the dislocation and d is the distance between the dislocations, the material being supposed to have the properties of iron.

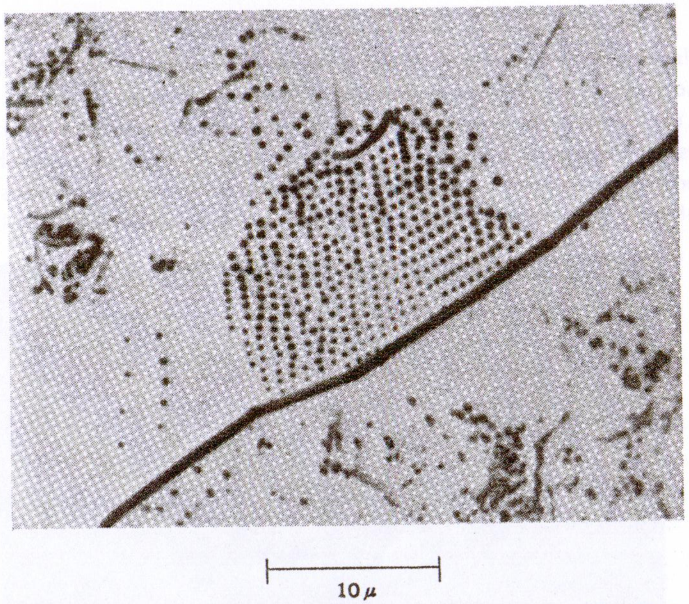


Fig. 6. Titanium–10% molybdenum alloy, rapidly cooled from 925°C. Etched in an aqueous solution of $\frac{1}{2}$% HF, 1% HNO$_3$. (After Schofield.[20])

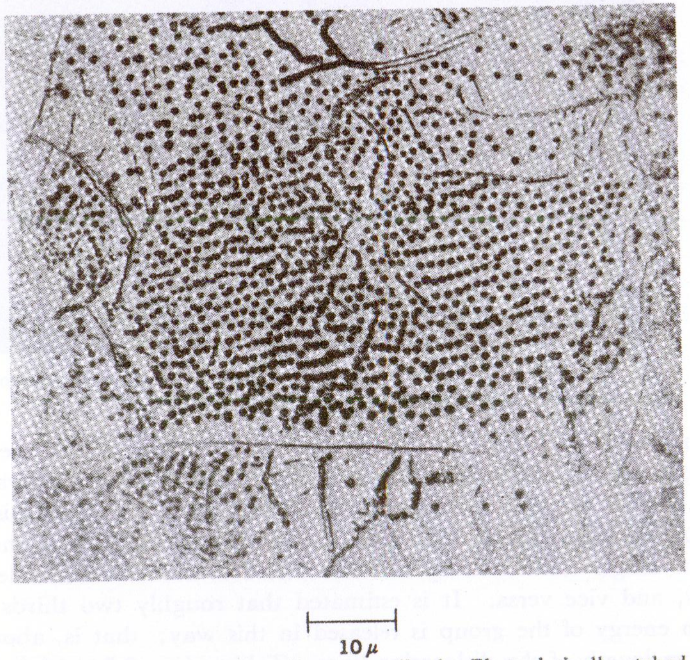


Fig. 7. Iron–0.086% phosphorus alloy, normalized. Electrolytically etched in solution of chromium trioxide in acetic acid. (After Earley.[21])

This is a very high stress, of the order of 300 tons/in.^{2}, and the existence of the group would depend upon the dislocations being very firmly locked. The total strain energy associated with these stresses within a region three times as wide as the group is approximately 0.35 erg/cm length of the dislocation lines when b/d is 1/100. This strain energy depends on the number of dislocations in the group and not upon the distance between them, since the reduction of stress brought about by an increase in d is compensated by the increase in area over which the stresses operate.

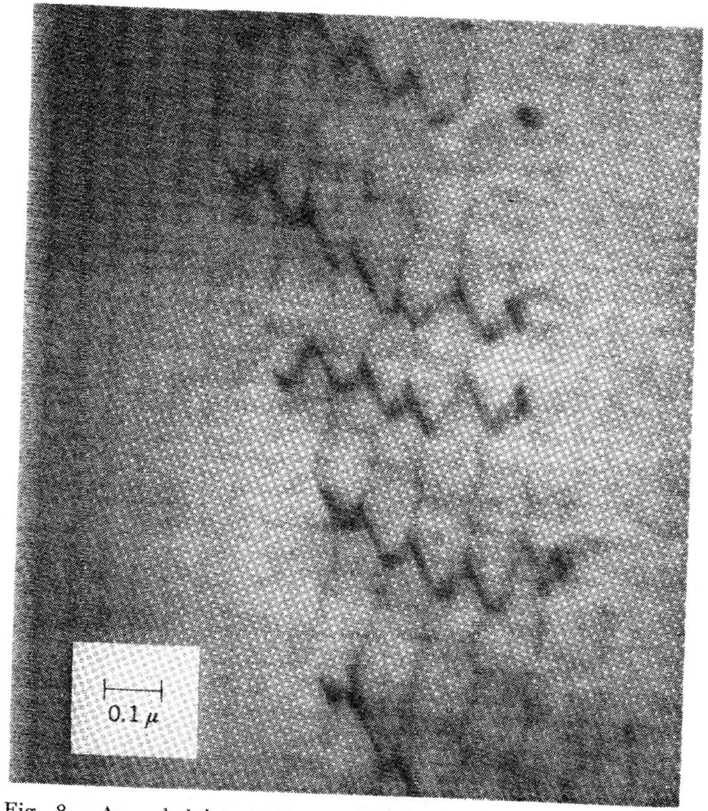


Fig. 8. Annealed iron. Transmission electron microscope photograph. (After McLean.[22])

If a crack passes down the center of the group, much of this strain energy is released, since the strains on the right-hand side of the crack are no longer influenced by the dislocations of the left-hand side of the crack, and vice versa. It is estimated that roughly two thirds of the strain energy of the group is released in this way; that is, about 0.25 erg/cm length of the dislocation lines. Taking b as 2.5×10^{-8} cm, the length of the crack that would release the majority of this energy is 3.75×10^{-5} cm, and its surface energy is approximately 0.15 erg/cm length of the dislocation lines (the surface energy is taken as 2000 ergs/cm^2). Consequently, the strain energy of this group is sufficient to provide the surface energy of a crack through it, and it would still be sufficient to do so if the dislocations were spread out to a distance of 200b or 500 Å, since the strain energy of the group would be unchanged while the length of the crack necessary to release the energy would be increased by a factor of 2.

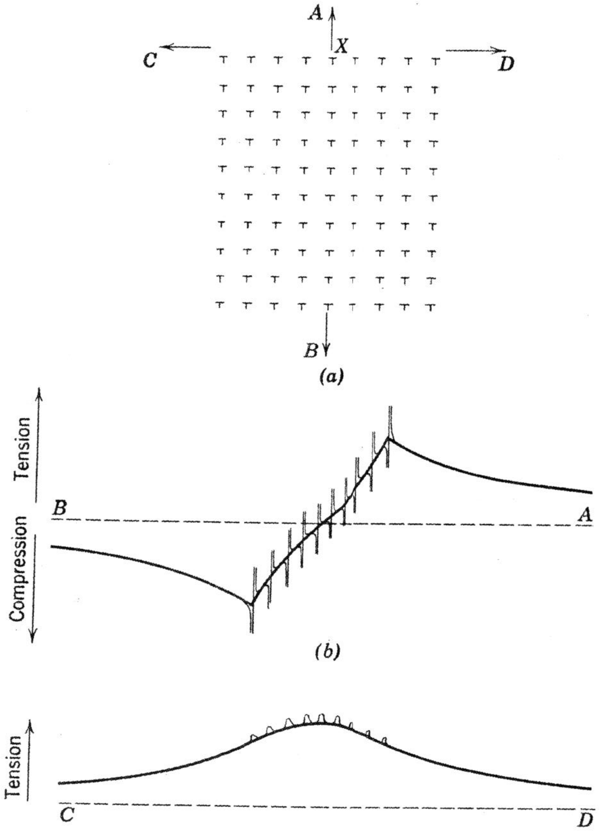


Fig. 9. (*a*) Array of dislocations. (*b*) Tensile stress normal to plane *AB*: distribution along direction *AB*. (*c*) Tensile stress normal to plane *AB*: distribution along direction *CD*.

If the number of dislocations in the group is sufficiently large, the releasable strain energy within it is proportional to the square of the number of dislocations, provided all the dislocations are of the same sign. A dislocation of opposite sign within the group will effectively cancel the strain field of a neighboring dislocation, so that in a group containing a mixture of positive and negative dislocations, the releasable strain energy is proportional to the excess of the numbers of one sign over the numbers of the opposite sign. However, it is not sufficient for the releasable strain energy of the dislocation group to exceed the surface energy of the crack passing through the group. The crack must still satisfy the condition that, at all stages from its initiation as a crack one or two atoms long to its growth to a size comparable with the width of the group, its growth shall be attended by release of energy. This means, in terms of the Griffith equation (neglecting the possibility of the occurrence of plastic deformation), that when the crack has a length $2c$ the stress in the neighborhood of the crack shall not be less than

$$\sigma = \left[\frac{2ES}{(1-\nu^2)c}\right]^{1/2}$$

where E is Young's modulus, S is surface energy, and ν is Poisson's ratio. In Fig. 10*a*, curve *A*, the stress required by this relation, is plotted against the length of the crack. The point at which the crack is most likely to be initiated is at the dislocation in the center of the first row on the tension side of the group (*X*, Fig. 9*a*); the calculated stress distribution in the neighborhood of this dislocation, along the line *XA* (Fig. 9*a*), is given in curve *B* (Fig. 10*a*). These curves suggest that under the conditions that have been assumed in the calculations (a group of 90 dislocations with Burgers vector 2.5 Å, spaced 250 Å apart), although a crack a few angstroms long might exist in the immediate neighborhood of the dislocation, it could not spread outward, and that a stress at least of the magnitude corresponding to the maximum vertical distance between the two curves (*XY*, Fig. 10*a*) is required to make it spread. This stress is about 400 tons/in².

A little modification of the assumptions is needed to arrive at conditions in which the spread of the crack is possible. Figure 10*b* shows the stresses that would arise if the group of 90 dislocations were compressed to 125 Å apart, and Fig. 10*c* shows the stresses when the distance between the dislocations remains unchanged at 250 Å and a single dislocation with an abnormally high Burgers vector of 5 Å is introduced at the critical position in the group. In both these cases, the stress necessary to enable the crack to spread is considerably reduced. Figure 10*d* illustrates the case when the dislocations are 125 Å apart, and a single dislocation with a Burgers vector of 5 Å is introduced. Here the possibility of a long crack forming without any application of stress appears to exist. The sensitivity of these calculations to the value of the surface tension is shown in Fig. 10*e*, where curve *A* relates to a surface tension of 2000 dynes/cm and curve *B* to a surface tension of 1800 dynes/cm. The breaking stress would not be less than 200 tons/in.² for 2000 dynes/cm and not less than 50 tons/in.² for 1800 dynes/cm.

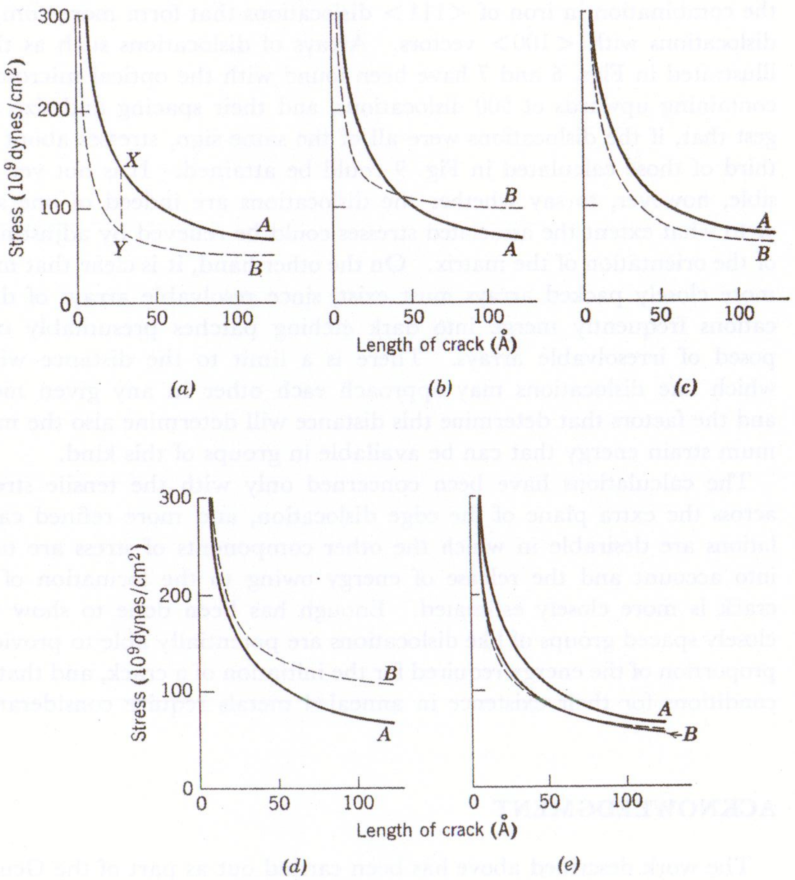


Fig. 10. Comparison of Griffith fracture condition with stress available along line *XA* (Fig. 9*a*) for various assumed circumstances.

It thus appears that large, closely packed groups of dislocations are required and that the chance of cracks being initiated at low stresses is much improved if dislocations of large Burgers vector, such as could be formed by the running together of two or more dislocation lines, are present in the group. The assumption of the existence of some dislocations of large Burgers vector avoids the necessity for assuming extremely closely packed groups of dislocations which are associated with very high stresses. Interest therefore attaches to the question of whether such groups of dislocations could be present in unworked metals, and whether evidence of their existence can be found.

So far evidence has been obtained with the electron microscope of the combination in iron of $<111>$ dislocations that form more complex dislocations with $<100>$ vectors. Arrays of dislocations such as those illustrated in Figs. 6 and 7 have been found with the optical microscope containing upwards of 500 dislocations, and their spacing and size suggest that, if the dislocations were all of the same sign, stresses about one third of those calculated in Fig. 9 could be attained. It is not yet possible, however, to say whether the dislocations are indeed of one sign, or to what extent the associated stresses could be relieved by adjustments of the orientation of the matrix. On the other hand, it is clear that much more closely packed arrays must exist, since resolvable arrays of dislocations frequently merge into dark etching patches presumably composed of irresolvable arrays. There is a limit to the distance within which like dislocations may approach each other in any given metal, and the factors that determine this distance will determine also the maximum strain energy that can be available in groups of this kind.

The calculations have been concerned only with the tensile stresses across the extra plane of the edge dislocation, and more refined calculations are desirable in which the other components of stress are taken into account and the release of energy owing to the formation of the crack is more closely estimated. Enough has been done to show that closely spaced groups of like dislocations are potentially able to provide a proportion of the energy required for the initiation of a crack, and that the conditions for their existence in annealed metals require consideration.

ACKNOWLEDGMENT

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DISCUSSION

W. S. Owen, *University of Liverpool.* In an earlier paper,[1] evidence was presented supporting the view that cleavage in mild steel is preceded by plastic deformation, and the results of an extensive study of the deformation associated with cleavage have been discussed recently.[2,3] These show that the formation of a cleavage microcrack in the materials studied is always preceded by slip, twinning, or both. In the temperature range in which twinning predominates, the fracture stress is often well below the expected yield stress of the material. However, in mild steel, there is a temperature range in which no twinning occurs before fracture and in which cleavage is initiated by slip. In view of Allen's suggestion that, in the cases so far described, the deformation observed could have occurred after cleavage, it seems necessary to return to this central question. The results presented here were obtained on specimens prestrained at 273°K. However, the maximum prestrain employed was 8.5%, and so they should not be compared with the experiments reported by Allen in which 75 or 80% prestrain was used. The prestrain applied in the present series of experiments was only a little greater than the minimum required to remove the discontinuous yield behavior.

All the fracture tests were carried out at 78°K, at which temperature the 1020 mild steel * used always broke with a complete cleavage fracture. Except for a few twins formed after cleavage in the vicinity of the fracture, no twinning was observed. In an earlier series of tests, the yield stress was varied by changing the applied strain rate.[1] At very slow rates, the yield occurred at a stress level below which microcracks in individual grains did not propagate. At a critical yield stress, microcrack formation was followed by propagation to complete failure. For this steel, the critical propagation stress at 78°K was 116,000 psi. As the yield stress was increased by using faster strain rates, the fracture stress increased accordingly.

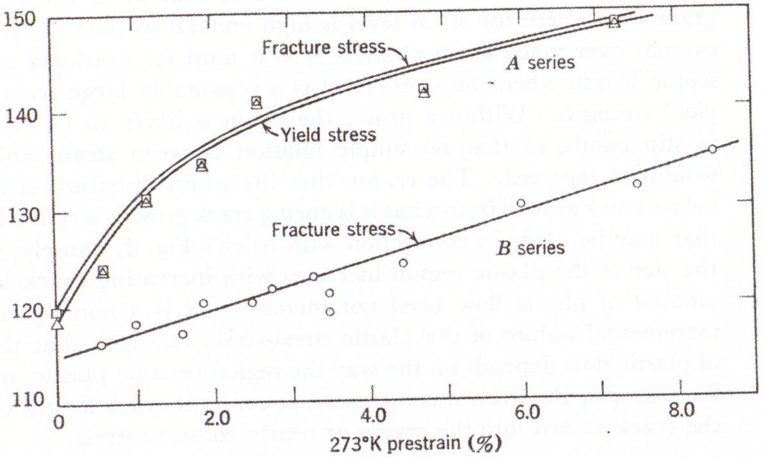

Fig. D.1. Variation of true fracture stress at 78°K with the extent of prestrain at 273°K, E steel, as received. Strain rate at both temperatures, 0.0054 in./min, Lüders strain at 273°K = 0.3%. Series *A*: aged 30 min at 373°K between 273° and 78°K test, yield discontinuous at both temperatures. Series *B*: no aging, no discontinuous yield at 78°K.

In the present tests, a different method of changing the yield stress was employed. All tests were carried out at a strain rate (0.0054 in./min) that is a little faster than that producing the critical yield stress for propagation. Prepolished strip specimens with 5-in. gage length were strained at 273°K to a permanent extension between 0.3% (the Lüders strain) and 8.5% and then aged for 30 min at 373°K before testing at 78°K. The yield at 78°K was discontinuous, and the upper yield stress increased with the amount of prestrain in the specimens. Fracture followed very quickly after the yield drop, and the fracture stress increased along with the yield stress (Fig. D.1, curve *A*). Clearly, in these experiments, slip was necessary *before* a cleavage crack could be nucleated.

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* Analysis and standard test data for this steel are given in Reference 1.

Allen has emphasized that a major problem in fracture is why the accompanying deformation is sometimes large and sometimes small. In annealed mild steel, the plastic strain is always large. Cleavage is initiated within a Lüders band, and, even if the engineering extension is very small, the strain within the band at, say, 78°K is seldom less than 3% and often as much as 10%.

A further series of experiments, designed to initiate cleavage with the minimum of strain at 78°K, gave results that are relevant to any consideration of the nature of the strain preceding cleavage. The same type of specimen as used in the previous experiments was prestrained at 273°K and cooled immediately to 78°K, so that no intermediate aging occurred. On testing at the lower temperature, there was no discontinuous yielding; rather, uniform plastic extension occurred before the specimen broke with a cleavage fracture (Fig. D.2). Although the fracture stress increased with the extent of prestrain (Fig. D.1, series *B*), the plastic strain at 78°K before cleavage was the same for all specimens (0.4% ± 0.1%). The plot of fracture stress against prestrain (Fig. D.1) extrapolated to 116,000 psi at zero prestrain, which is the same propagation stress found in the first experiments.

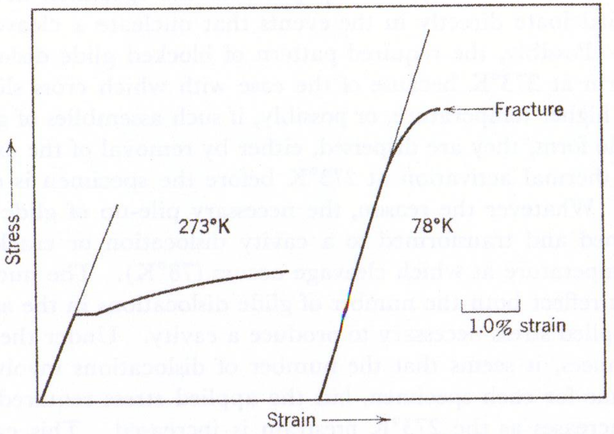

Fig. D.2. Form of stress-strain curve at 273°K and after quenching to 78°K.

Unlike the aged specimens, the fracture stress of the prestrained and quenched specimens was not affected by strain rate. For example, specimens prestrained 7.2% at 273°K and quenched to 78°K were tested at strain rates ranging from 0.0001 in./min to 0.02 in./min without any detectable change in the fracture stress and no significant variation in the precleavage plastic strain.

Upon unloading prestrained but unaged specimens after about 0.4% uniform plastic strain at a stress just a few pounds below the known fracture stress at 78°K and examining the prepolished surfaces microscopically, no microcracks were found. From this it is concluded that, although the energy absorption owing to cleavage step and river formation is expected to be increased by 273°K prestrain, the change in the stress necessary to propagate a microcrack through the strained crystals is insignificant compared to the increased difficulty in nucleating a microcrack. Evidently, the slip put into the specimen at 273°K does not participate directly in the events that nucleate a cleavage crack at 78°K. Possibly, the required pattern of blocked glide dislocations does not form at 273°K because of the ease with which cross slip can occur at the higher temperature, or possibly, if such assemblies of glide dislocations do form, they are dispersed, either by removal of the external stress or by thermal activation at 273°K before the specimen is quenched to 78°K. Whatever the reason, the necessary pile-up of glide dislocations is formed and transformed to a cavity dislocation or crack nucleus at the temperature at which cleavage occurs (78°K). The nucleating conditions reflect both the number of glide dislocations in the assembly and the applied stress necessary to produce a cavity. Under the present circumstances, it seems that the number of dislocations involved is about the same for each specimen, but the applied stress required for nucleation increases as the 273°K prestrain is increased. This can easily be accounted for by the increased resistance to the passage of dislocations (increased frictional force) resulting from the residual dislocation networks put into the specimen by prestraining at 273°K.

These experiments also show that, as pointed out earlier by Hahn, Averbach, Owen, and Cohen,[3] interstitial or impurity locking of dislocations is not a prerequisite of fracture. Nor is a time delay before slip essential, although it is obviously needed if the fracture stress is to be attained solely by rapid loading.

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F. A. McClintock, *Massachusetts Institute of Technology.* A reference to studies showing that the strains in front of a ductile crack fall off as the reciprocal of the distance from the end of the growing crack would be appreciated. While this has been found to be the case for the strain distribution before growth in elastic-plastic longitudinal shear and in fully plastic longitudinal torsion, it is not the case in front of growing cracks under these modes of loading,[1,2] nor even for the strain before growth in fully plastic, doubly notched specimens under tension.[3] The abovementioned results were obtained from classical plasticity theory, and hence, they deal with average strains over regions of the order of the grain size, where the stress level is high enough so that the plastic zone extends over many grain diameters, as it must for cracks of any macroscopic length when the stress level is a reasonably large fraction of the yield strength. Within a grain, the strain is likely to be concentrated in slip bands, so that no simple relation between strain and distance would be expected. The reason that the strain distribution is different before crack growth from what it is during crack growth is related to a point that may be made in connection with Allen's Fig. 2, namely, that while the size of the plastic region increases with increasing crack length, the amount of plastic flow need not increase. It is a consequence of the incremental nature of the plastic stress-strain relations that the amount of plastic flow depends on the way the region became plastic, on whether for example, the stress was increased at constant crack length or whether the crack moved into the region at nearly constant stress.

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N. P. Allen (Author's Reply). Owen's new experiments add to the cases in which cracking is clearly associated with yield, but the cases in which cracking is clearly *not* associated with yield remain, and the theory of fracture must be sufficiently wide to cover both. The experiments appear to show clearly that the local shock associated with a sharp yield is not an essential feature of cleavage fracture. Owen feels that the dislocation groups surviving after deformation at 273°K cannot nucleate cracks, whereas those formed during deformation at 78°K can. It should be possible to discover whether there are any essential differences between these two types of groups that might support this idea.

The statement regarding the distribution of strain at the end of a small crack relates to the elastic strains in a material assumed to be elastically isotropic and homogeneous and does not take into account the modifications that arise when plastic strain occurs. Figures 1 and 2 are indeed drawn to illustrate the importance of the accident of the presence or absence of dislocations in the strained region, which in turn governs the incidence of the local strains mentioned by McClintock. It is agreed that the size of the plastic region is not the sole factor in determining the amount of plastic flow, but it is nevertheless an important factor, and it limits the amount of plastic deformation that can occur when the crack is very small.