17. The Griffith Criterion for Glass Fracture

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ABSTRACT

A review is made of the work pertaining to the Griffith criterion that has been published in the last decade. It is concluded that the Griffith criterion still retains the dominant position in explaining fracture, but it is not quantitatively complete. Numerous second-order corrections must be added to the theory, and a number of experiments are required to resolve conflicting evidence on the nature of the cracks.

Introduction

The purpose of this chapter is to correlate the experimental evidence related to the Griffith criterion. Excellent surveys of the early literature on this topic can be found in the review articles by Orowan [1] and by Jones.[2] In the decade that has passed since these reviews, enough new experiments have accumulated to warrant a reassessment of the situation.

The concepts behind the Griffith criterion and the resulting equations are well known and have been enumerated elsewhere in these proceedings. Poncelet [3] has objected to the Griffith hypothesis on the grounds that the initiation of a crack is an atomistic process and not treatable by the macroscopic equations of thermodynamics. However, Orowan [1] has derived Griffith's equation for critical stress (except for a minor numerical factor) using a strictly atomistic approach. Furthermore, Orowan [4] has demonstrated that when the stress reaches the value predicted by the Griffith formula, the material must break. This establishes the *necessary* condition which, in addition to Griffith's *sufficient* condition, places the Griffith criterion in a strong theoretical position.

The following discussion will be restricted to five topics: propagation velocity of cracks, time effects in static fatigue, size effect in small fibers and small indenters on plates, dispersion effect in strength, and experimental evidence for submicroscopic flaws.

Crack Propagation

When a crack propagates through a brittle isotropic material, part of the potential energy (excess elastic energy) is used to create a fresh surface, and part is required for the kinetic energy associated with the displacements around the tip of the crack.

Consider the case of plane strain. The kinetic energy (KE) resulting from a crack of length $2c$ propagating at each end with velocity $\dot{c}$, is

$$KE = \tfrac{1}{2}\rho\dot{c}^2 \int_S \left[\left(\frac{\partial x}{\partial c}\right)^2 + \left(\frac{\partial y}{\partial c}\right)^2\right] dx\, dy \tag{1}$$

where the integral is over the region that is elastically disturbed. The integrals can be reduced to a dimensionless form using the similarity transformations

$$x = \alpha c, \quad y = \beta c, \quad \text{and} \quad r = Rc = \sqrt{x^2 + y^2} \tag{2}$$

The derivatives in Eq. 1 depend on the strain (σ/E), so that the kinetic energy expression is of the form

$$KE = \tfrac{1}{2}\rho\dot{c}^2c^2 \frac{\sigma^2}{E^2} k \tag{3}$$

where k is a dimensionless number which can be determined provided that the stress field around the crack is known.

The essential assumption made is that the region of strain owing to a crack is limited to a zone centered on the tip of the crack. The mathematical problem is now focused on the parameter k, which, because of Eq. 2, can be expressed as an integral in the $r - \theta$ plane, bounded by the radius R:

$$k = \frac{2\pi}{(1-\nu)^2} \int_0^{2\pi} \int_0^R F(\nu, r, \theta)\, dr\, d\theta \tag{4}$$

where ν is Poisson's ratio. Thus k is determined if R is known since F is defined by the stress field around the crack (knowledge of the stress field implies some assumptions on the geometry of the crack).

It is logical that the zone of integration should be limited, since displacements cannot be communicated faster than the sound velocity. Mott [5] showed that, for plane strain, the displacements should be bound by the longitudinal wave front, so that $R = \dot{c}_m/v_l$, where $\dot{c}_m$ is the maximum crack velocity and v_l is the sound velocity.

The value of k can be found in terms of R by the requirement that the total energy of the system (elastic, surface, and kinetic) is minimized as the crack progresses. The elastic energy in a plate around a crack of length $2c$ that is available to drive the crack is $\pi\sigma^2c^2/E$, the surface

energy is $4\gamma c$, and the kinetic energy is given by Eq. 3. The energy condition during crack propagation is

$$\frac{d}{dc}\left(-\frac{\pi\sigma^2c^2}{E} + 4\gamma c + \tfrac{1}{2}k\rho\dot{c}^2c^2\frac{\sigma^2}{E^2}\right) = 0 \tag{5}$$

The energy condition that determines the critical crack size c_0 is

$$\frac{d}{dc}\left(-\frac{\pi\sigma^2c^2}{E} + 4\gamma c\right) = 0$$

or

$$c_0 = \frac{2\gamma E}{\pi\sigma_c^2} \tag{6}$$

Equations 5 and 6 yield

$$\dot{c} = \sqrt{\frac{2\pi}{k}}\sqrt{\frac{E}{\rho}}\left(1 - \frac{c_0}{c}\right)^{1/2}* \tag{7}$$

but, since $\dot{c} \to \dot{c}_m$ when $c \gg c_0$ and $v_l = \sqrt{E/\rho}$, we have

$$k = 2\pi\left(\frac{\dot{c}_m}{v_l}\right)^2 = 2\pi R^2 \tag{8}$$

There are two equations for k: Equation 4 arises from the general expression for the kinetic energy in a zone around the crack, and Eq. 8 arises from the minimization of elastic, surface, and kinetic energy. There is only one value of k that satisfies both equations; it is found by solving the following integral equation:

$$k = \frac{2\pi}{(1-\nu)^2} \int_0^{2\pi} \int_0^{\sqrt{k/2\pi}} F(\nu, r, \theta)\, dr\, d\theta$$

F corresponds to the quantity in the brackets of Eq. 1, when expressed in terms of the stress field around a crack and in polar co-ordinates.

Roberts and Wells [6] showed that, by taking the solution of the stress field around a Westergaard [7] crack for F and assuming $\nu = 0.25$, the above equation yields $\sqrt{2\pi/k} = 0.38$. The solution yields a slightly higher value of $\sqrt{2\pi/k}$ for a higher value of ν. The value $\sqrt{2\pi/k} = 0.38$ is in reasonable agreement with Schardin's and Struth's [8] measurements on vitreous silica; they found the maximum crack velocity to be about 0.40 of v_l. (See also Table 2 of Chapter 16 of this volume.)

It must be pointed out that better agreement between theory and experiment cannot be expected because of the assumptions in the theory.

* The equation of motion of the crack is found by differentiating Eq. 7 with respect to time.

In the first place, the theory assumes a constant stress field. However, as the crack moves through the glass, the stress may increase because the load is held by a smaller area, or it may decrease because some kinetic energy is imparted to the apparatus in contact with the specimen. Second, the theory assumes a condition of plane strain which results in $\dot{c}_m$ being proportional to the longitudinal wave velocity. In a three-dimensional case, however, shear waves and Rayleigh surface waves, in addition to the longitudinal waves, may limit the conversion of potential energy to kinetic energy. Third, the value of k derived above depends upon the assumption that $\nu = 0.25$. According to Eq. 4, $\sqrt{2\pi/k}$ varies approximately 5% about the value 0.38 as ν varies 40% about the value 0.25. Fourth, the value of k depends upon the geometry of the crack; in the case above, the stress field around a Westergaard crack is used; however, k may vary if the crack is changed to a different shape. Finally, Eq. 5 implies no dissipative processes, such as energy for deformation, during the fracture process. The conversion of energy to plastic flow in glass is small but cannot be completely ignored, since it is a matter of common observation that glass flows under small pyramid indenters in hardness tests. In view of the limitations of the theory, it is unreasonable to expect the relation $\dot{c}_m/v_l$ = constant to hold from glass to glass better than, say, to a few percent. Considerably more work must be done before the second-order effects described above can be explained; nevertheless, the experimental results clearly show that kinetic energy is the major reason why the crack velocity is a given fraction of the sound velocity.

Equation 7 can be written in terms of the stress at the crack tip σ,

$$\dot{c} = \sqrt{2\pi/k}\, v_l(1 - (\sigma_c/\sigma)^2)^{1/2} \tag{9}$$

which is plotted in Fig. 1. This result, labeled Mott, is compared with the published result of Poncelet,[9] who based his theory of crack propagation on radically different premises. Mott's result, which is based on the critical-flaw concept of Griffith, shows that the crack suddenly spreads catastrophically when the stress on a pre-existing flaw reaches σ_c but that the crack velocity is limited to about 0.38 of the longitudinal wave velocity. Poncelet's idea, the forerunner of several theories [10–12] based on the flaw-genesis hypothesis, is that there is no flaw until it is created by the stress (at about 0.7 of σ_c), whereupon it spreads catastrophically, the crack velocity being limited to about 0.5 of the transverse wave velocity.

The thermodynamic premises upon which the critical-flaw and the flaw-genesis hypotheses are based are divergent. It is difficult, however, to choose between them on strictly experimental evidence, since

they both predict virtually the same results. The maximum crack velocity is $0.5v_t$ by one theory and about $0.38v_l$ by the other, values that are reasonably close in view of the assumptions of the theory. The time to attain the maximum crack velocity when the stress is slightly above

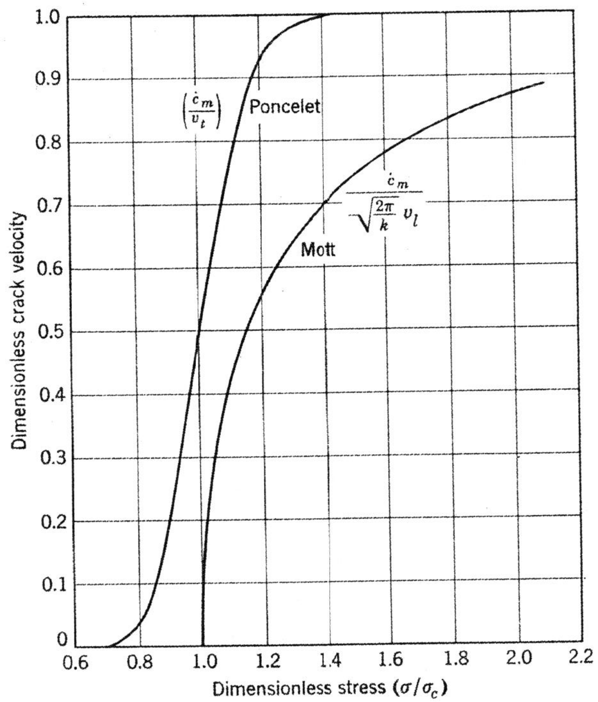

Fig. 1. Maximum crack velocity versus stress comparing the theory of Poncelet, based upon the flaw-genesis hypothesis, to that of Mott, based upon the pre-existing flaw hypothesis.

σ_c is very small in either theory, although, according to Mott's theory, the critical crack grows at a slightly smaller rate than the spread of a crack in Poncelet's theory. Integrating Eq. 7, the expression between crack depth and time is

$$t = \frac{1}{\dot{c}_m}\left[c\sqrt{1 - c_0/c} + c_0 \tanh \sqrt{1 - c_0/c}\right] \tag{10}$$

By Eq. 7, $\dot{c} \approx 0.99\dot{c}_m$ when $c \approx 100c_0$, and, consequently, the time to obtain 99% of $\dot{c}_m$ is of the order of magnitude of $1/\dot{c}_m$ for a 1 μ deep crack. This rapid rate of growth makes it very difficult to distinguish experimentally between the two hypotheses on the basis of growth dynamics.

Mott's theory is applicable for high values of the fracture velocity where the kinetic energy is appreciable. However, cracks sometimes move at very slow rates.[13] Roesler [14] pointed out that when the load is distributed on the same (or larger) area as the crack develops, a condition of quasi-equilibrium between surface and mechanical energy can be maintained over long periods. Furthermore, Orowan has shown that a crack will sometimes propagate under smaller stresses than required to initiate the propagation.[15] In this case, c_0 is the depth of the observed fracture and is controlled by γ and σ according to Eq. 6. A slowly spreading crack is consistent with a slow change in surface energy. This idea, first proposed by Orowan,[16] has been experimentally confirmed by Levengood,[17] who demonstrated that the diffusion of gases (in the glass) to the crack determines the rate of slow propagation.

Consequently, there appears to be no experimental work on fracture propagation that is inconsistent with the Griffith hypothesis of pre-existing flaws.

Static Fatigue

The expression "static fatigue" has been coined for that phenomenon particular to inorganic glass where the average breaking strength under *constant* load depends upon the time. Unlike metals, the effect of cyclic loading differs very little from that of static loading when compared to the same time interval.[18]

In terms of the pre-existing flaw hypothesis, this phenomenon is similar to that of slow fracture growth. In this case, the size of c_0, which determines the stress at which the fracture appears to occur spontaneously, depends upon time. The magnitude of c_0 depends upon stress and atmosphere, with the result that a lower stress causes fracture after a longer time interval. The stress-time equation, accordingly, results from the gradual penetration into the crack of substances that are adsorbed at the crack walls. There is a stress below which the glass appears to withstand fracture indefinitely. This is about one-third the breaking stress at rapid rate of loading. Orowan [16] showed that this factor of one third was about equal to the square root of the ratio of the surface energy γ in moist atmosphere to γ of a fresh surface in vacuum. Consequently, Orowan was led to conclude that the gradual change in γ was responsible for static fatigue.

Orowan predicted that there should be no static fatigue effect for specimens baked out and tested in vacuum. This was confirmed by the experiments of Gurney and Pearson,[19] who showed that the strength of rods tested below 10^{-5} mm Hg is about the same at 10^6 sec as at 1 sec. Further confirmation was made by the experiments of Kropschot and

Mikesell,[20] who found that the static fatigue effect had virtually disappeared at very low temperatures (about 76°K). Gurney and Pearson also varied the temperature in vacuum and were able to show that the fatigue effect could not be ascribed entirely to gaseous attack on the crack. They felt that some of the delayed fracture is due to the atmospheric constituents contained in the surface layers of the crack. That is to say, the material under intense stress at the tip of the crack is a reaction product and weaker than the pristine glass.

Elliott [21] has proposed that the actual depth of the crack may be deepened by the diffusion of corrosive agents to the root of the crack. For an assumption that the growth of corrosion products is the same as that for oxide films, the crack depth is related to time by

$$c_0 = A \log t + B$$

Substituting this into Eq. 6, we find

$$\sigma_c^{-2} = A_0 \log t + B_0 \tag{11}$$

This is one of the forms of the fatigue equation. This equation fits considerable experimental data.[21]

The loss of strength with time may be due to the growth of the crack size or the decrease of surface energy of a constant-sized crack or both, but in any event the Griffith hypothesis of pre-existing flaws seems adequate to explain the phenomenon of static fatigue.

Charles [22] has recently extended the above concepts to explain the increase of strength with rate of loading, which is additional evidence in support of the Griffith theory.

The Size Effect

A notorious feature of glass is its tendency to become stronger as the area under stress is decreased.* The size effect in strength is particularly evident for glass fibers with small diameters (5 μ), which attain strengths about fifty times that commonly found in bulk glass. A number of theories to explain the size effect have been fashionable, but recent experiments have discredited most of them. One such theory was the hypothesis of oriented structure.[23] However, a number of experiments have shown that the physical properties of strong fibers are isotropic. Otto and Preston [24] showed that the compaction and expansion of strong fibers is isotropic; Mould [25] found that the Poisson ratio of strong fibers was equal to that of bulk glass; Preston [26] has pointed out that no one has observed birefringence in fibers below about 50 μ; and Orowan [27] has

* This feature is not exclusive to glass. The size effect is also observed in metal whiskers and thin films.

pointed out that fracture morphology of glass fibers is unlike that of filaments with oriented structures (such as nylon). This evidence leads to the rejection of the oriented-structure hypothesis.

The most popular hypothesis of one decade ago was that of oriented flaws.[28] It was thought that the drawing process in fibers would arrange flaws longitudinally, parallel to the axis, and the longitudinal strength would therefore increase. However, Otto and Preston [24] found that the strength of strong fibers is just as high at 45° to the axis as at 90° to the axis. Furthermore, this hypothesis could not explain the high strength* (225,000 psi) found by Witucki [29] when 3-mm rods were acid-polished. The evidence is sufficient to reject the oriented-flaw hypothesis.

It has been suggested that the high strength of fibers is due to strains that arise from the cooling conditions during the fiber formation. Such strains have been observed by Stirling [30] for small rods (1 mm) cooled under tension, but they have not been observed in small strong fibers.[24] Furthermore, it can be shown that the thermal gradient in small fibers during the cooling process is so small, as is quenching time of the surface, that the assumption of "frozen-in" strains is difficult to justify analytically.[31]

Another hypothesis is that of surface strength, which supposes attractive forces in the surface layer to be different from the bulk attractive forces. This concept of surface strength was originally postulated by Preston [32] and has been revived by Tooley and his colleagues.[33] The physical meaning of this concept is obscure. Otto [34] has shown that the strength of fibers can be made independent of diameter by controlling the forming conditions. Furthermore, Bartenev [35] has shown that, if a thin layer of a fiber is etched off, the strength goes up, and Thomas [36] has shown that, with time (allowing the surface to thicken), the strength drops. These experiments all show that the variation in strength is due to a variation in surface defects and not due to a variation in the intrinsic strength. Flaw size naturally is limited by the fiber size, but this restriction cannot account for the large size effect that is sometimes observed.

A popular explanation of the size effect using flaw theory relies on the weakest-link argument. It is based upon the assumption that there exists an *a priori* probability of finding a flaw of a given severity, such flaws being distributed randomly in size and space. A number of theories have arisen, all based on somewhat different distribution functions, but they have been shown by Charles and Fisher [37] to be essentially equivalent. They all predict that larger samples will be weaker. This idea has been qualitatively confirmed, but quantitatively there are some discrepancies, which will now be described.

* By strength is meant the apparent stress that is required to fracture the material.

It is a necessary consequence of statistical theories of fracture that the dispersion must increase as the median increases. Charles' [22] recent work has confirmed this for moderate average breaking loads, as Table 1 shows.

TABLE 1. Statistics of Breaking Loads *

Variables	Sample Group		
	1	2	3
Number of Samples	200	200	200
Loading Rate (in./min)	0.5	0.05	0.005
Median Failure (psi)	13,870	12,320	10,770
Standard Deviation (psi)	1,140	1,040	903
Coefficient of Variation (%)	8.23	8.43	8.37

* This table is for glass rods where all conditions are identical except the loading rate.

Such excellent measurements of all the statistical parameters unfortunately do not exist over a large range of median breaking strengths. However, there is some information on dispersion of strength for very strong fibers.

Thomas [36] recently reported that very small fibers (2×10^{-4} in.) had a coefficient of variation of only 1% at 550,000 psi. Furthermore, Mould [38] showed that his fibers (2×10^{-4} in.) broke in two classes: One out of twenty broke at a low stress, while the rest were distributed symmetrically about 610,000 psi with a coefficient of variation of 2.4%. The only way to reconcile these results with the statistical flaw hypothesis is to assume that at the 500,000-psi stress level the fibers are being tested in the absence of flaws (in which case the theoretical strength is one order less than previously supposed), or that there is a very large number of very small identical flaws (which, by the Griffith formula, would be about 30 Å deep). The statistical flaw hypothesis predicts that fibers with strengths somewhat below 500,000 psi will have a very large dispersion, and that the strength distribution curves will be skewed toward lower strengths. This statistical prediction needs to be confirmed experimentally.

The experiments of Otto [34] and Thomas [36] show that the often-quoted result showing the average stress decreasing as the fiber size increases is an oversimplification of the true situation.[39] † This observation results from that method of pulling fibers whereby a smaller fiber is made by

† Bateson [39] was the first to present cogent reasons why the size effect in small fibers is incidental rather than fundamental. Anderson [31] later refined Bateson's suggestions.

pulling faster. If all conditions are held constant (including the pulling speed), fiber strength is nearly independent of diameter. Anderson [31] showed that the pulling speed affected the quenching time of the fiber nearly as much as the diameter and, consequently, fibers of slightly different diameters may or may not have drastically different thermal histories. In terms of the statistical flaw theory, this means that all the statistical parameters (including the flaw density) ordinarily change as fibers of different diameters are made and, therefore, great care must

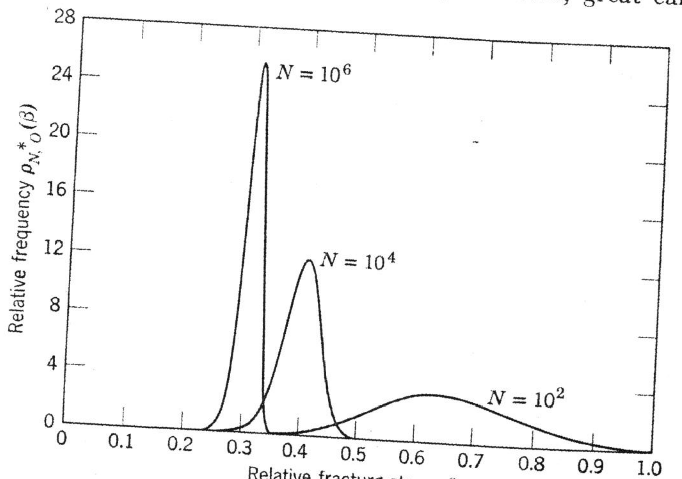

Fig. 2. Distribution of breaking strength with variations in stress according to the statistical distribution of Fisher and Hollomon.[40] As the flaw density decreases, the median strength and the dispersion increase.

be taken to apply the statistical theory to explain the loss of strength with diameter. Figure 2 shows how the median strength and dispersion change with the flaw number, according to the statistical theory of Fisher and Hollomon.[40]

A rapid variation of strength with diameter could be accounted for by the statistical flaw theory if it is assumed that the flaw density varies rapidly with the quenching time. Anderson [31] showed that the quenching time of very fine fibers was of the order of the relaxation time of the glass molecules at the bushing temperature. Consequently, all chemical phenomena at the surface, such as volatilization, adsorption, and devitrification, are restricted because of the limited time available for chemical processes at the surface. This limits flaw formation. Furthermore, the fiber cools uniformly because of a low temperature gradient that tends to arrest the same specific volume at the surface as in the interior. This

also restricts the formation of flaws. A schematic diagram showing strength versus diameter is shown in Fig. 3, which presupposes that the flaw density is proportional to the quenching time.

Thus, the explanation of the size effect in fibers using the statistical flaw theory is as follows: In Fig. 3, region A is one of maximum strength and very small dispersion, such as measured by Otto and Thomas; region B is a region of high strength with high dispersion; and region C

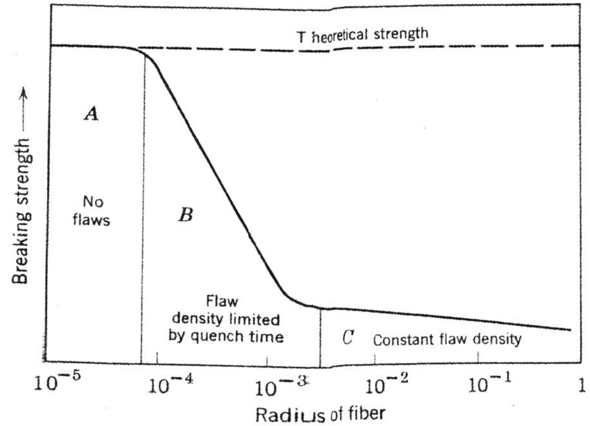

Fig. 3. Strength variation with radius of fiber according to the statistical flaw theory, in which it is assumed that the flaw density depends upon the quenching time of the fiber.

is one of modest strength where the dispersion is proportional to the median strength, such as shown in Table 1.

The statistical flaw theory can be directly tested on fibers if the flaw density is held constant from batch to batch. For example, if a number of batches of fibers of the same diameter (drawn sufficiently similar so that the flaw density can be assumed to be constant) are broken using progressively larger stressed lengths, the statistical theory can be tested.

Anderegg [41] reported such experiments where the stressed length of 13-μ fibers was varied over a factor of 300. The results showed that the statistical theory of Fisher and Hollomon [40] fits Anderegg's results excellently, as shown by Fig. 4, in which the flaw density is taken as 10^3 flaws/cm^2. This is a confirmation of the pre-existing flaw theory in its statistical form.

If further experiments, as yet not performed, show that the dispersion is very large in region B of Fig. 3, the statistical flaw theory would attain a very strong position, at least for fiber strength.

Another type of experiment that is often used to support the statistical flaw theory is the size effect found in the fracture of glass plate using spherical indenters. The measurement consists of the load required for a sphere of given size to produce a hertzian ring crack. The experiments by Tillett [42] show that this concept is greatly oversimplified. In her experiments, the stress changed by a factor of 2 as progressively smaller indenters were used, but she found that the deviation *did not* change. This result is quite contrary to the statistical theory. She further pointed out that if the breaking force is limited by the maximum stress on a flaw

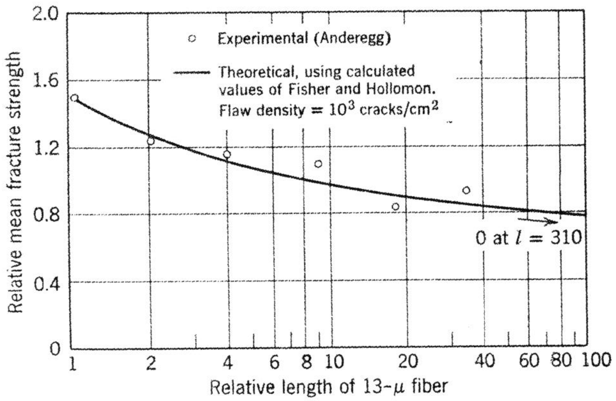

Fig. 4. Variation of average strength with length using the statistical distribution of Fisher and Hollomon.[40]

(Eq. 6) the breaking force is proportional to the square of the radius of the indenter, according to the hertzian equations ($F/r^2 = constant$). It turned out, however, that in the region where the stress varies with area, $F/r = constant$. This latter measurement, shown in Fig. 5, can only be reconciled with the hypothesis that failure occurs when the integrated strain energy, rather than the stress, exceeds some maximum. It was observed that at larger indenters the relation $F/r^2 = constant$ was observed, indicating that flaw statistics may be valid for large stressed areas. Tillett observed that in the region where the law $F/r = constant$ is valid the fracture proceeds very slowly, suggesting quasi-equilibrium between surface and mechanical energy, whereas in the region where $F/r^2 = constant$ is valid, the fracture occurs instantaneously. Tillett concluded from her experiments that "the variation of strength with indenter size is not due solely to the flaw distribution of the material, as has been previously suggested."

In view of Tillett's results, the statistical flaw theory can be considered only marginally successful in explaining the size effect. It is important that an experiment be performed to determine carefully the coefficient

of variation as the median strength is changed, both for strong fibers and spherical indenters on plate.

In spite of the lack of data on dispersion and Tillett's experimental evidence to the contrary, the statistical flaw theory appears to be the best explanation for the size effect. Experimental evidence relating flaw density to strength will be discussed in a later section.

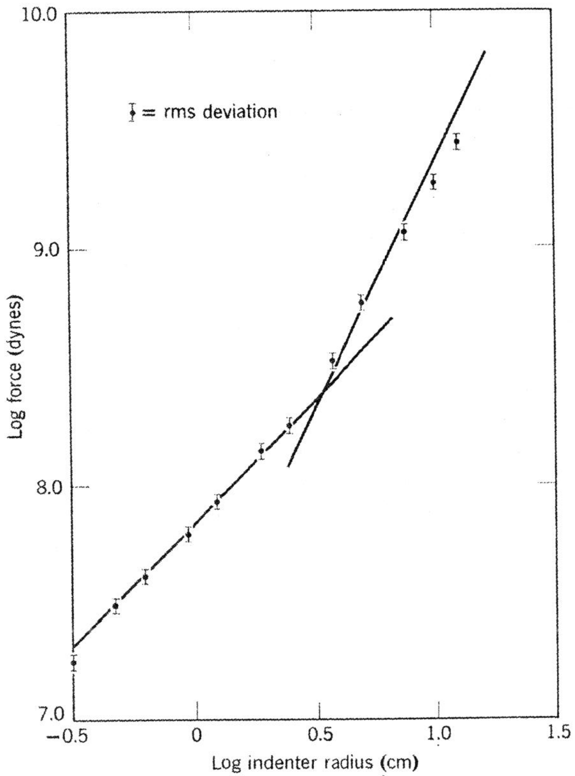

Fig. 5. Variation of breaking force with indenter radius after the experiments of Tillett.[42] Line on lower part of curve is inconsistent with concept that glass breaks when a maximum stress is exceeded.

Variability in Strength

The statistical flaw theories were originally intended to explain the variability of strength, which is usually reported to be high. A random distribution of flaw sizes leads to a scatter of the observed strength values, such as is shown in Fig. 2. However, a number of workers have pointed out that the variability in strength may be overemphasized owing to an incorrect assignment of the stressed area. What is measured is the breaking load, and from this value the stress is usually computed by dividing by a predetermined area.

Too often, the experimentalists have not made the necessary corrections for the area by determining the exact location of the origin of the fracture. Unpublished measurements by Brandt [43] illustrate this danger. He broke thin circular glass plates by hydrostatic water pressure. If he calculated the stress as though each fracture had originated in the geometric center, the coefficient of variation was about 27%, a typical figure usually found in the literature on this subject. However, when he determined the location of the origin of each break and computed the stress at that point, the coefficient of variation dropped to 11% with the median unchanged. Much of the literature on strength measurements is reported for rods broken in bending. In this case, the stress depends in a very sensitive way on location of the origin of the fracture. Unfortunately, in many cases, the reported values of stress have not been corrected, and the reported dispersion is, therefore, of no theoretical significance.

There is another correction for the stress that should be made because of the size of the mirror on the crack. When the fracture proceeds, the initial portion is smooth, forming the mirror, and the remainder of the fracture face is rough, forming the hackle.* Smekal [44] found that for circular rods the breaking force divided by the hackled area was nearly constant under a variety of conditions that were sufficient to change drastically the nominal stress. Terao [45] and Levengood [46] report similar results for other types of fractures. In general, the mirror is large when the stress is small.

Shand derived the stress concentration factor for a crack with the dimensions of the mirror.[47] His theory states that during the initial part of fracture the crack is in quasi-equilibrium but slowly expanding, the stress on the edge of the crack increasing as the radius of the mirror increases. Soon the crack becomes sufficiently large that the real stress at the edge of the mirror reaches the theoretical strength of the glass, whereupon the crack increases its velocity rapidly to the maximum velocity, thus causing cleavage. The stress concentration resulting from a flaw is $k = 2\pi\sqrt{c/\rho}$. Taking the mirror to be the flaw, Shand derives the value $k = K(D, r)/\sqrt{\rho}$ where K is a prescribed function of the rod diameter and the radius of the mirror. The radius of curvature of the crack ρ must be assumed. Shand showed that even though the mean

* There are actually several distinct regions of the fracture face, which all blend into each other, but the separation into mirror and hackle is well defined and practical.

breaking stress (no correction) varied by 20% as the rate of loading changed, the corrected stress (muliplied by the unknown value of $\sqrt{\rho}$) varied only 4%.

One objection to Shand's idea is that the concept of the fracture process being broken up into only two distinct phases is an oversimplification of the facts, as any microscopic examination of a fractured surface will show. Yet the concept has considerable merit because it is consistent with the concept of two distinct conditions for crack propagation (the catastrophic one determined by Eq. 5 and the quasi-equilibrium one determined by Eq. 6). It also coincides with the two different conditions found by Tillett for the formation of hertzian cracks. It is consistent with the phenomenon of static fatigue, where the initial part of the fracture process forming the mirror is the time-consuming process.

The concept of the fracture process composed of two distinct stages, although still a simplification, appears to be a refreshing idea that may reconcile the Griffith flaw theory with statistical methods. It needs further confirmation, but Shand's work is sufficient evidence to justify it as a working hypothesis.

If this idea is correct, the problem of strength is slightly changed in focus. Whereas the problem was once to determine at what nominal strength glass failed, it has now become a question of at what load the energy balance will be such that it will form a mirror from a submicroscopic crack. The mirror, when large enough, will cause catastrophic failure by exceeding the theoretical strength of the material. This idea raises a number of problems that will have to be resolved. For example, it is not clear whether failure (the growth of the mirror) would depend on a maximum stress or a maximum strain energy; the experiments of Tillett [42] and the papers by Roesler [18] support the latter view.

The Evidence for Submicroscopic Pre-existing Flaws

Much effort has been exerted in the attempt to verify the relationship between the crack depth c_0 and the nominal breaking stress σ_c (Eq. 6). In the region of crack depths of 10^{-2} to 10^{-3} mm (the corresponding stress is 7000 to 15,000 psi), Shand has shown that the relation holds up quite well.[47] However, if this relationship is extrapolated, as shown in Fig. 6,[47,49] a stress of 500,000 psi corresponds to a pre-existing flaw with a crack depth of one-quarter wavelength of light, which is beyond the power of direct detection by optical methods. This difficulty has caused some authors to doubt the reality of such small flaws.

The work of Shand [47] leaves no doubt that the effect of very minor surface injuries has a drastic effect on the strength of the glass. He took a group of $\frac{1}{4}$-in. rods, carefully prepared under uniform conditions, and divided them into several groups. The first untouched group broke at

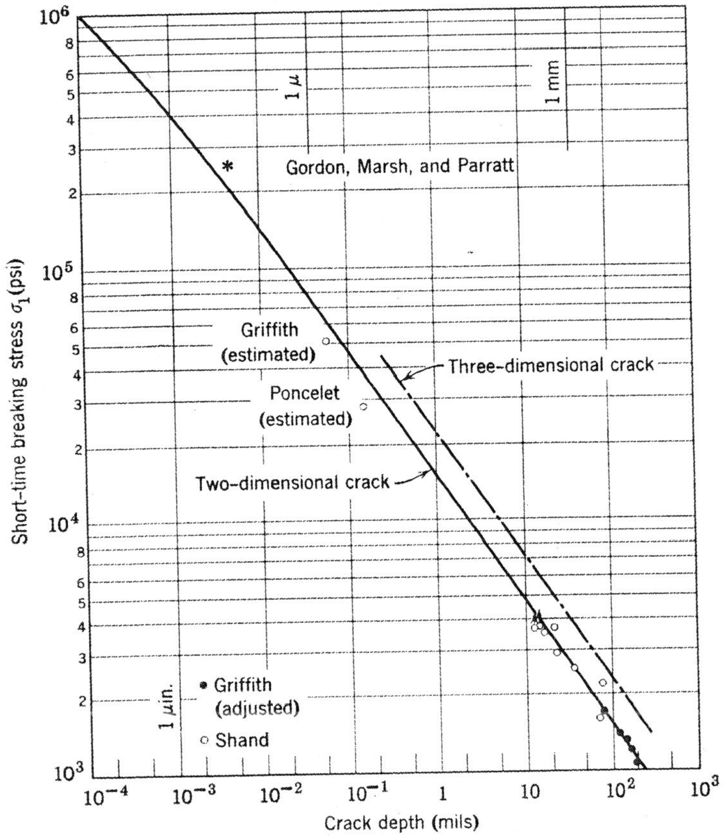

Fig. 6. Relation between crack depth and stress according to Eq. 6. Open circles represent measurements of Shand.[47] Asterisk represents findings of Gordon, Marsh, and Parratt.[49]

73,500 psi. The second group was damaged by dropping fine sand upon the glass from a height of 3 in., whereupon the strength dropped by 50%. The remaining groups were sandblasted under air pressure, whereupon the strength dropped to about 16,000 psi, the value customarily found in commercial rods. Shand showed that minor injuries on the surface were of greater destructive power than major injuries. He also took a second series of $\frac{1}{4}$-in. rods which he tested as-received at 6000

psi. When he sandblasted them, their strength dropped further, but, when he acid-polished them by removing about 2 mils of the surface, the strength rose to 250,000 psi, a value comparable with that of fine fibers. This evidence, although indirect, supports the concept of pre-existing flaws.

Direct evidence for submicroscopic flaws has been sought by using electron microscopy and decoration techniques. The former technique has not revealed any striking evidence of flaws in glass with strengths above 50,000 psi, although considerable detail of the topography (in fact, too much detail) is found for strengths below that value.[50] The latter technique has been more successful. Andrade and Tsien [51] found oriented cracks on the inside of tubes where abrasion was nonexistent. Gordon, Marsh, and Parratt [49] have developed further the sodium vapor decoration technique of Andrade and Tsien and have found convincing evidence of small flaws. Their photographs prove conclusively that long (50 μ), narrow (200 Å), and shallow (1000 Å) cracks exist in strong glass before the application of stress and that these cracks multiply with stress. They showed that the crack density is low whenever the glass breaks at high stress, thus supporting the statistical flaw hypothesis. From their Figs. 4 and 5, the linear flaw density is seen to be about 10^3 cracks/cm for a breaking strain of 0.3% and about 10^2 cracks/cm for a breaking strain of 3.0%. Since the cracks seem to have the same distribution of severities in both figures, the difference in strength most likely results from the difference in flaw density.

Cracks 200 Å wide and 1000 Å deep could easily account for strengths in the order of 200,000 psi. Consequently, the work of Gordon, Marsh, and Parratt is an excellent confirmation of the pre-existing flaw hypothesis, as well as incidental evidence for the statistical flaw hypothesis. The cracks were not related to the direction of drawing and were not characteristic of those cracks caused by abrasive damage. It appears from their work that large compressive stresses close the cracks so well that they will not decorate. Figure 7 shows the crack density for strong and weak glass, Fig. 8 shows the crack formation around abrasive scratches, and Fig. 9 shows the cracks that were decorated for both tension and compressive stresses in the surface layer.

In view of the experiments described above, it is easy to assume that cracks less than 1000 Å deep do exist. Furthermore, cracks much deeper than 1000 Å cannot be seen optically if the width is small. Elliott [52] showed that a flaw no more than several atomic diameters wide would account for loss in strength. It may be concluded that there are cracks which operate to reduce the strength that have not been revealed by decoration techniques.

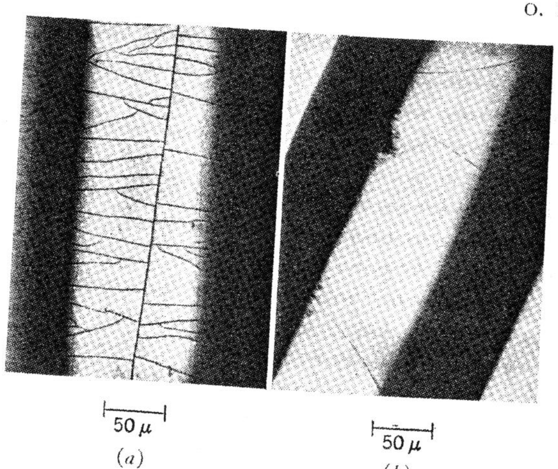

Fig. 7. Flaws found by Gordon, Marsh, and Parratt for strong and weak glass.[49] (*a*) Breaking strain about 0.3%. (*b*) Breaking strain about 3.0%.

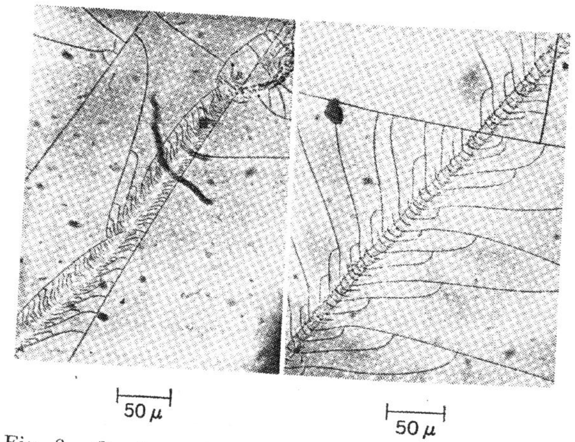

Fig. 8. Cracks created by abrasion.[49] Note that the system of cracks terminates at a previously existing crack. (*a*) Accidental scrape. (*b*) Deliberate scrape.

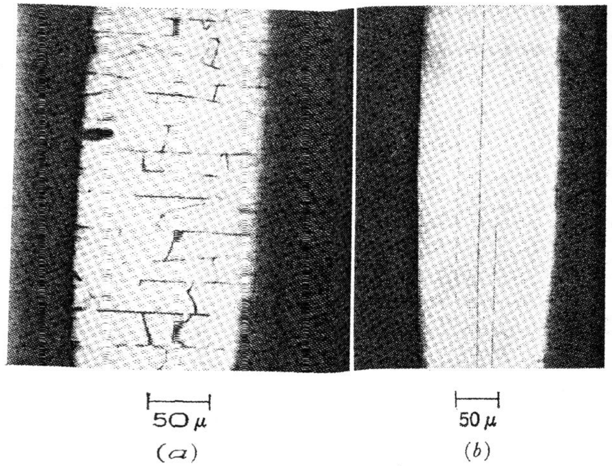

Fig. 9. Crack density of a rod in tension (left) and compression (right).[49]

An Alternative to the Griffith Theory

Cracks are a special type of defect. It has been observed that other types of defects could operate to reduce strength, such as inhomogeneities, inclusions, submicroscopic regions of strain, structural gradients at the surface, and devitrified products at the surface.* While these no doubt complicate the general picture, the Griffith criterion of pre-existing cracks is a self-consistent, experimentally verified theory that explains the main features of strength and fracture of brittle materials.

However, other theories have arisen to explain strength and fracture of brittle solids that do not use the concept of pre-existing flaws, of which the theory by Poncelet [53] is the outstanding example. According to Poncelet, flaws do not exist before the stress. This is called the flaw-genesis hypothesis and is based upon the statistical mechanics approach, which states that nonequilibrium processes are determined by the rate at which chemical bonds are broken, less the rate at which these bonds are formed. With no stress, the rates are exactly equal, but, with stress, the rates are biased, and the overwhelming molecular activity (10^{13} vibrations per second) is reflected in a probability function that is extremely stress and temperature dependent. The equation derived is really an expression for the probability of an event that shows the net rate of bonds breaking, and the assumption is made that this is equal to

* The possibility that devitrified products create the crack has been emphasized by Gordon and co-workers.[49]

the macroscopic rate of crack propagation. The virtue of this approach is that it could account for breakage of glass at stresses considerably below the theoretical strength without the *a priori* assumption of defects. However, while one hypothesis is removed, other assumptions are invoked. In place of assumptions about the depth and geometry of a crack, assumptions are made about the activation energy and the manner in which intrinsic energy is biased by the macroscopic load.

Poncelet has developed this hypothesis into a complete and self-consistent theory which accounts for time effects (static fatigue), crack propagation, variability of strength, and comminution.[54] The interesting feature of the flaw-genesis theory is that it leads to virtually the same final equations as the pre-existing flaw theory. This is demonstrated by the comparison of several stress-time equations for static fatigue, as follows:

$\log t = A_1 + B_1 \log \sigma$	(pre-existing flaws)	Charles [55]
$\log t = A_2 + B_2 \log \sigma$	(flaw genesis)	Poncelet [53]
$\log t = A_3 + B_3/\sigma^2$	(pre-existing flaws)	Elliott [21]
$\log t = A_4 + B_4/\sigma$	(flaw genesis)	Stuart and Anderson [56]

There are considerable experimental data that extend over a factor of 10^6 in elapsed time. Even so, no one equation fits the present data a great deal better than the others, since there is not enough difference between them to be of consequence.* Similar situations exist for the theory of crack propagation, as was shown in the second section.

From a fundamental point of view, the disagreement between the flaw-genesis and the pre-existing flaw hypotheses is a reflection of the difference between the point of view of statistical mechanics and classical thermodynamics.

There is some experimental evidence that stress creates flaws; this was shown by Gordon, Marsh, and Parratt.[49] McAfee [57] showed that tensile stress greatly enhances the diffusion of helium, but that compression and shear stresses have virtually no effect. Stress-enhanced diffusion begins to occur at a threshold in stress that is about half the nominal breaking stress. The effect is so marked that it can be explained only by reversible fissures which open and close with the stress. This observation lends some credence to the flaw-genesis hypothesis. On the other hand, these fissures cannot accelerate the passage of argon, so there is doubt that the fissures are ever large enough to concentrate stress.

* None of the above equations fits the data except for the intermediate ranges of a log time-log stress plot. The stress at very short time and very long time is independent of time.

Summary

The Griffith theory is today in a stronger position than it was one decade ago. A completely quantitative theory of strength and fracture is not yet available, but all the general features are explained very well. No experiments are in direct contradiction.

In the writer's opinion, the major developments of the past decade were: (*a*) the development of the equations of crack propagation by Mott; (*b*) the extension of Griffith's equation to include static fatigue by Orowan, Elliott, and Charles; (*c*) the experiments of Otto which proved Bateson's suggestion that the stress-diameter relationship for strong fibers is incidental rather than fundamental; and (*d*) the techniques developed for the observation of pre-existing flaws by Gordon, Marsh, and Parratt, and their discovery of devitrified layers at crack sites.

There are still some serious questions that have to be resolved. The experiments of Tillett concerning the energy criteria of fracture and the report of Mould and Thomas on the dispersion of fiber strengths need to be reconciled with the statistical flaw theory. In particular, many good experiments on the dispersion of strength are needed. For example, experiments need to be performed where the dispersion is reduced by exposing the glass to a large number of identical, relatively large, flaws. Shand's idea that fracture is at least a two-stage process, needs further checking. Further evidence of flaws by decoration technique is needed to determine their geometric characteristics and distribution. The speculation by Gordon and co-workers that cracks originate at devitrification sites warrants further experimentation.

Note added in proof: Dr. Sydney Bateson has communicated new results pertinent to the discussion in the section entitled "Variability in Strength." (These results have been presented orally and will be published by Dr. Bateson.) He measured fracture velocities of running cracks produced from edge flaws in plate. The standard deviation of the breaking strength was greater than 15%. However, the standard deviation of $\sigma_b\sqrt{R}$ (R is the radius of the mirror) is about 1%, tending to confirm Shand's hypothesis that fracture is a two-step process in which the mirror is the Griffith flaw. On the other hand, the mirror itself is growing so rapidly that it cannot be considered in quasi-equilibrium. Obviously, improvements on Shand's ideas are needed.

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