# 3. The Ductile-Cleavage Transition in Alpha-Iron

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#### ABSTRACT

A criterion for the transition in fracture mode from ductile to cleavage is calculated for fracture at a notch. It is concluded that the temperature dependence of the transition arises more from the Peierls-Nabarro stress than from the strength of the locking of a dislocation source. The transition temperature is found to be a function of grain size, the friction on a free dislocation, the strength of the dislocation locking, and the degree of triaxiality of the applied stress. The quantitative agreement with experimental measurements is quite good.

It is also shown experimentally that dislocation locking by nitrogen is appreciably stronger than by carbon. This is contrary to the customary assumption, and it is suggested that there may be chemical as well as elastic interaction with the dislocations.

# The Transition Temperature

Some time ago, Cottrell  $^{1,2}$  and the author  $^{3,4}$  independently put forward ideas that have a number of features in common and seem to go quite a long way in explaining some aspects of the ductile-brittle transition.

The essence of these ideas is that the fracture process is divided into two parts: (a) the production of a crack by dislocation coalescence, and (b) the propagation of this crack. The first depends upon a critical resultant shear stress on unlocked dislocations, but the second depends upon the applied tensile stress; the distinction between these two types of stress has important consequences. Cleavage is thought to arise if the stress at fracture can propagate the crack produced by the dislocations as a Griffith crack. In ductile fracture, however, it is thought that the dislocation

eracks cannot propagate in this way, but that eventually the bridges between a number of such cracks give way and they link together into a major crack. This then grows by plastic deformation in front of it, which produces further dislocation cracks, and these, in turn, join up with the major crack.

Cottrell considers the possibility of Griffith crack propagation at the lower yield point and so obtains a condition for the energy transition. On the other hand, the author considers the transition in fracture mode from ductile to cleavage. A criterion for this transition will now be derived.

Within the transition temperature range for a notch test, the fracture is first ductile near the notch but changes to cleavage as it advances. It is thought that the condition essential for this to happen is that, as the ductile fracture gathers speed and moves into a region of greater triaxiality of stress, the fracture stress in front of the major crack must eventually become sufficient for Griffith propagation of a dislocation crack produced in that location.

Let  $\sigma$  be the fracture stress in front of the advancing ductile fracture. A triaxial tension will be generated, so let  $q\sigma$  (with  $q\sim \frac{1}{3}$ ) be the uniaxial stress equivalent to  $\sigma$  for the production of shear stress. For simplicity, suppose that the operative slip plane and direction are at 45° to  $\sigma$  and that the dislocation crack is normal to it. Then, from an expression stated first by Stroh,5 it follows that a crack containing n dislocations will spread as a cleavage fracture if  $\sigma$  is sufficient for

$$nb\sigma(1+1/\sqrt{2}) = 4\gamma' \tag{1}$$

where b is the Burgers vector and  $oldsymbol{\gamma}'$  is the effective surface energy associated with the growth of the crack.

If the tensile stress required simply to overcome the friction opposing the motion of an unlocked dislocation is  $\sigma_0$  and cracks form at both ends of a slip plane (length l) with the source at its center, then the back stress resulting from the cracks will halt dislocation movement into the crack when

$$n = \pi (1 - \nu)(q\sigma - \sigma_0)l/4\mu b \tag{2}$$

where  $\nu = \text{Poisson's ratio}$  and  $\mu = \text{rigidity modulus}$ . If, as Cottrell suggests, the crack forms by coalescence of dislocations on intersecting slip planes rather than on the same slip plane, this would only make a small numerical difference to the expression for n.

Consider now the value of  $\sigma$ . With unnotched  $\alpha$ -iron specimens, the ductile fracture stress  $\sigma_f$  is given by

$$\sigma_f = \sigma_0 + k * l^{-1/2}$$

where  $k^*$  is a constant and  $\sigma_0$  again appears to be the friction on an unlocked dislocation.<sup>6</sup> The ductile fracture stress is at least approximately a shear stress criterion, <sup>7.8</sup> so in the present case we put

$$q\sigma = \sigma_0 + k^* l^{-1/2} \tag{3}$$

Thus, from Eqs. 1, 2, and 3, it would appear that ductile fracture will change to cleavage when  $\sigma$  comes up to the value

$$\sigma \simeq 4\mu\gamma'/k^*l^{\frac{1}{2}} \tag{4}$$

In this transition criterion, increase in grain size favors cleavage because the number of dislocations in the crack is greater; therefore the crack is easier to propagate (Eq. 1). This is, however, partly compensated for by the lower fracture stress available for propagation (Eq. 3). The friction stress  $\sigma_0$  and the triaxiality factor q do not affect the resultant shear stress required on the dislocations for crack formation, but they do increase the tensile stress at fracture and so favor cleavage. On the other hand, increase in  $\gamma'$  makes propagation of the dislocation crack more difficult, and this favors ductile fracture.

## The Transition Temperature

Temperature dependence comes into the transition criterion (Eq. 4) mainly through the  $\sigma_0$  part of  $\sigma$  and through  $\gamma'$ . An increase in temperature decreases  $\sigma_0$ ; it also weakens the dislocation locking, which will increase the number of dislocation sources that can be operated by the growing crack and so will increase  $\gamma'$ . Both these effects of temperature increase will favor ductile fracture.

Experimental evidence from the lower yield point suggests that the temperature dependence of  $\sigma_0$  is more important than that of  $\gamma'$  in the transition temperature of notched  $\alpha$ -iron specimens.

The lower yield point  $\sigma_{lyp}$  obeys a relationship similar to Eq. 3,

$$\sigma_{lyp} = \sigma_0 + kl^{-\frac{1}{2}} \tag{5}$$

In the interpretation of this equation, k appears as a measure of the concentrated stress required where a slip band is held up by the grain boundary in order to unlock a dislocation and so transmit yielding from one grain to the next during the propagation of a Lüders band. Thus, the temperature variation of k and  $\gamma'$  both depend on the temperature variation of the locking strength.

Figure 1 shows some  $\sigma_{lyp}$  values for a 0.11% C steel. It is apparent that the temperature dependence of  $\sigma_{lyp}$  in the range of interest for notched-impact transition temperatures is principally due to  $\sigma_0$ ; the slope k is almost constant, showing that the dislocation locking and there-

fore  $\gamma'$  are almost constant. This suggests that the temperature dependence of the transition temperature of notched specimens arises mainly from  $\sigma_0$ .

Previous work has shown that  $\sigma_0$  has two parts, a temperature-independent friction  $\sigma_0^*$  and a temperature-dependent  $\sigma_0^{\dagger}$ . The former arises

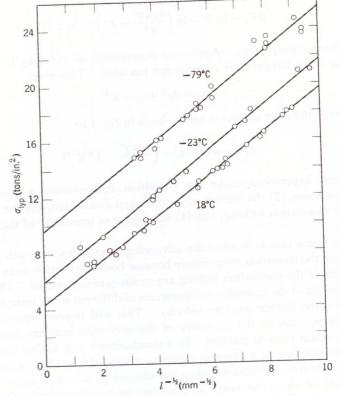


Fig. 1. Dependence of lower yield point on grain size.

from the resistance of random solute atoms, fine precipitates, and lattice defects to dislocation movement and is about 2 tons/in.<sup>2</sup> at the lower yield point of an annealed mild steel.<sup>9</sup> It has been suggested that  $\sigma_0$ † represents an appreciable Peierls-Nabarro stress, and it varies from about 2 tons/in.<sup>2</sup> at room temperature to 24 tons/in.<sup>2</sup> at  $-196^{\circ}$ C for ordinary strain rates.<sup>10</sup>

The temperature dependence of  $\sigma_0 \dagger$  can be expressed rather well by an expression of the form

$$\sigma_0 \dagger = \text{const exp}(-\alpha T)$$

N. J. PETCH where  $\alpha$  is a constant. Provided  $\sigma_0^*$  is not too big, the whole  $\sigma_0$  approximately obeys a similar relationship, so that with B and  $\beta$  as constants,

$$\ln \sigma_0 = \ln B - \beta T$$

Thus from Eq. 4, taking  $\gamma'$  as constant, the transition temperature  $T_e$  is given by

$$\beta T_c = \ln B - \ln \left( \frac{4q\mu\gamma'}{k^*} - k^* \right) - \ln l^{-\frac{1}{2}}$$
 (6)

Less accurately, the temperature dependence of  $\sigma_0\dagger$  may be taken as linear if the temperature range is not too wide. This gives

$$\sigma_0 = \sigma_0^* + C - \epsilon T$$

where G and  $\epsilon$  are constants, and so leads in Eq. 4 to

$$\epsilon T_c = \sigma_0^* + C - \left(\frac{4q\mu\gamma'}{k^*} - k^*\right)l^{-1/2} \tag{7}$$

These expressions make the transition temperature a function of (1) grain size, (2) the friction on an unlocked dislocation, (3) the strength of the dislocation locking, and (4) the degree of triaxiality of the applied

The strain rate in front of the advancing ductile fracture will have an effect on the transition temperature because both the friction term and the strength of the dislocation locking are strain-rate dependent. Thus, any comparison of the transition temperatures of different steels must be made at the same ductile fracture velocity. This will depend upon the test conditions, such as the geometry of the specimen and the amount of plastic strain prior to fracture. In a standardized test, taking the transition temperature at a constant percentage cleavage in the fracture area, the ductile fracture velocity when the change to cleavage occurs should normally be about constant. If, however, the test conditions are altered, or if the amount of plastic strain prior to ductile fracture changes drastically, then the ductile fracture velocity will alter, and the transition temperatures will not be strictly comparable.

## The Effect of Grain Size

Although Eqs. 6 and 7 look complicated, they predict simply a linear dependence of  $T_c$  on  $\ln l^{-\frac{1}{2}}$  or, less accurately, upon  $l^{-\frac{1}{2}}$ .

Figure 2 shows that the transition temperature (75% cleavage) for a 0.11% C steel varies between +60° and -30°C for the grain-size range 250 to 6500 grains/mm<sup>2</sup>, and that the variation is linear with  $l = l^{-\frac{1}{2}}$ .

The predicted slope is also simple:

$$\frac{dT_c}{d\ln l^{-1/2}} = -\frac{1}{\beta} \tag{8}$$

Strain rate has a large effect on the friction  $\sigma_0$ , 11 and the value of  $\beta$ required is at the strain rate experienced in front of the advancing ductile fracture in the notch, which is probably at least a rate of 10<sup>3</sup>/sec. From

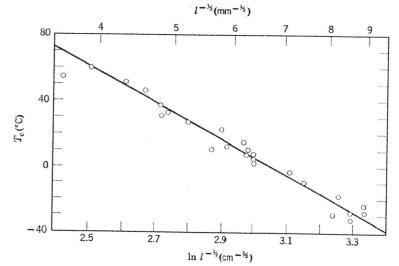


Fig. 2. Dependence of transition temperature on grain size.

lower yield point measurements by Baron 12 at 102/sec, the following (in egs units) have been estimated for the temperature range of interest.

$$\beta = 0.62 \times 10^{-2}$$
 ln  $B = 23.4$   $\epsilon = 2.0 \times 10^{7}$   $C = 79 \times 10^{8}$ 

This value of  $\beta$  agrees quite well with the value  $0.88 \times 10^{-2}$  obtained by applying Eq. 8 to the transition temperatures in Fig. 2.

## The Temperature-Independent $\sigma_0^*$

An increase in the friction  $\sigma_0^*$  produces a direct increase in the lower yield point through  $\sigma_0$  (Eq. 5). The effect on the ductile fracture stress is less certain because the change made to alter  $\sigma_0$  \* may alter the strain at fracture, and this will alter the fracture stress. Measurement indicates, however, that an increase in  $\sigma_0^*$  at the lower yield point increases the ductile fracture stress by very closely the same amount.11 This increases the tensile stress available at fracture for Griffith propagation of a dislocation crack and so favors cleavage.

Equation 8 gives another simple expression,

$$\frac{dT_e}{d\sigma_0^*} = \frac{1}{\epsilon}$$

Thus, with  $\epsilon = 2.0 \times 10^7$  cgs units, a 1-ton/in.<sup>2</sup> increase in  $\sigma_0^*$  should raise the transition temperature by 7.5°C. This is probably an overestimate because the value of  $\epsilon$  used is for a strain rate that is probably too low.

Since  $\sigma_0^*$  is due to interaction of unlocked dislocations with precipitates, with impurity atoms in solution, and with lattice defects, it is a fruitful source of variations in  $T_e$ . Thus, the effects of quench aging, alteration in nitrogen and phosphorus contents (for example, between Bessemer and open-hearth steels), irradiation damage, and parts of the effect of strain and of strain aging are due to this cause.

Table 1 shows some measurements on the effect of  $\sigma_0^*$  on  $T_c$  for a 0.11% C steel. The change in  $\sigma_0^*$  was obtained from the lower yield point. On the average, a 1-ton/in.<sup>2</sup> increase in  $\sigma_0^*$  raises  $T_c$  by 4.7°C.

TABLE 1. The Effect of  $\sigma_0^*$  on  $T_c$ 

Treatment	$T_c$ Increase (°C)	$\sigma_0^*$ Increase (tons/in.2)	$\Delta T_c/\Delta \sigma_0^*$
0 1 5 65000	15	3.0	5.0
Quench from 650°C	22	4.7	4.7
Quench from 690°C Quench from 700°C; age 20 hr, 50°C	61	15.0	4.1
Quench from 690°C; overage 150°C	7	1.5	4.7
Nitride 580°C; quench	18	3.6	5.0

In some recent measurements of Churchman, Mogford, and Cottrell, 13 who showed that irradiation damage increases  $\sigma_0^*$ , an increase of 25,000 psi corresponded to a rise in the transition temperature of about 60°C. On the basis of the present measurements, a rise of 52°C, owing to this  $\sigma_0$ \* change, would be predicted.

The effect of strain and strain aging has been examined.11 Prestrain will increase  $\sigma_0^*$  at the beginning of plastic deformation in a subsequent notch test, and it will also introduce unlocked dislocation sources. The effect on fracture in a notch is uncertain, but with 2% prestrain a rise of  $\sim 5^{\circ}$ C in  $T_c$  was observed for each 1-ton/in.<sup>2</sup> increase in  $\sigma_0^*$ , if the  $\sigma_0^*$ increase is taken simply as the work hardening during the prestrain. The agreement of this figure with that observed in Table 1 suggests that at this strain the main effect on the transition temperature is due to an increase in the tensile stress at fracture by the amount of the hardening in the

prestrain. At 10% prestrain, however,  $\Delta T_c$ /(pretreatment increase in  $\sigma_0$ \*) was only 2.5°C/ton/in.2

Subsequent aging locks the dislocation sources and also increases  $\sigma_0^*$ by precipitation. By taking this increase in  $\sigma_0$ \* as the change in flow stress produced by aging,  $\Delta T_c/\Delta \sigma_0^*$  is found to be 4.8°C/ton/in.2, showing that the principal effect of aging on  $T_e$  arises from the  $\sigma_0^*$  change. It follows that the susceptibility of a steel to embrittlement by strain aging depends on the amount of precipitation that occurs, and this in turn depends mainly on the nitrogen content and upon the presence of alloying elements that interact with nitrogen and so affect its precipitation.

## The Effect of Dislocation Locking Strength

The easier the operation of a dislocation source, the greater is the amount of plastic work that accompanies the opening of a crack; this results in a larger effective surface energy  $\gamma'$  and a lower transition temperature. Weakening the dislocation locking lowers k in Eq. 5, and in Cottrell's treatment of the energy transition this reduction in the tensile stress available at the yield point for Griffith propagation will also lower the transition temperature.

The important effect of manganese on the transition temperature of mild steel appears to be partly due to refinement of grain size and partly to weakening of the dislocation locking. This weakening is shown by a reduction in the lower yield point slope k.<sup>14</sup>

Taking the strength of the locking as proportional to k, a rough estimate of the width of the plastic zone around a growing crack suggests

$$\gamma' = \text{const } k^{-\alpha}$$
 (9)

where  $\alpha = 1$  to 2.11

Equation 6 applied to Fig. 1 gives  $\gamma' = 50,000 \text{ ergs/cm}^2$  for a lowmanganese steel. With use of Eq. 9,  $\gamma'$  will be increased to 60,000 to 72,000 ergs/cm<sup>2</sup> at room temperature for 1.9% Mn steel. From Eq. 6, this causes a decrease of Te by 66° to 110°C. On the other hand, the presence of the manganese increases  $\sigma_0^*$  at the lower yield point by 4.5 tons/in.2, which will raise To by 21°C, so that a net decrease of 45° to  $90^{\circ}$ C is predicted. The observed decrease at constant grain size is  $43^{\circ}$ C. <sup>14</sup>

The effect of the manganese on the dislocation locking is thought to be due to the 0.2-ev interaction observed by Dijkstra and Sladek  $^{15}$  between a manganese atom and a nitrogen atom. 14 Locking in a mild steel should under normal conditions be principally due to nitrogen. It is suggested that either the manganese causes displacement of some of the nitrogen

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atoms within the condensed atmosphere, so weakening the locking, or else that the nitrogen atoms prefer to remain by manganese ones in the body of the crystal, provided carbon is available for the dislocations. To explain the lower k by this second possibility would require carbon locking to be weaker than nitrogen locking.

In recent work, the dislocation locking in  $\alpha$ -iron has been examined further by Codd and Petch. It has been found that killing a steel with

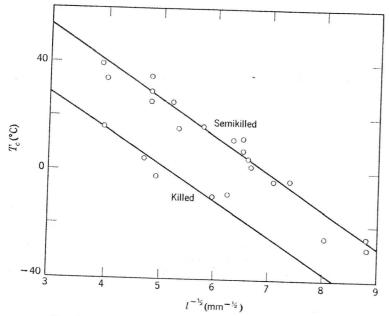


Fig. 3. Influence of killing on the transition temperature.

aluminum and silicon, which removes nitrogen as well as oxygen, also weakens the locking strength. Deoxidation of a mild steel with 0.3% Si and 0.08% Al was observed to lower the yield point k at  $18^\circ$  and  $-196^\circ$ C to values close to those observed with 0.9% Mn steel. The transition temperature was also lowered by about the same amount as observed with 0.9% Mn of Fig. 3.

In subsequent experiments, nitrogen and carbon were removed from a semikilled steel by wet hydrogen until the yield point was suppressed, and then carbon only was returned using an atmosphere of purified hydrogen and hexane vapor. This pure carbon locking gave k values at 18° and -196°C that were practically the same as those observed with the 1.9% Mn steel. Nitriding returned k to the semikilled values (Fig. 4).

It is apparent that carbon locking is weaker than nitrogen locking, and this leads to a lower transition temperature.

In the original Cottrell-Bilby idea, dislocation locking in  $\alpha$ -iron was due to a single line of carbon or nitrogen atoms. It is now recognized that the atmosphere is more extensive than this and that precipitates may occur in the outer regions. Since the solubility of carbon in  $\alpha$ -iron is considerably less than that of nitrogen at the temperatures where atmospheres form, one possible explanation of the weaker carbon locking

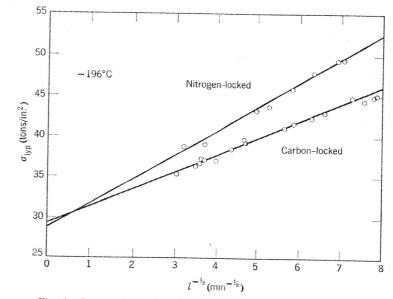


Fig. 4. Lower yield points for carbon- and nitrogen-locked steels.

is simply that the outer regions of the atmosphere are less concentrated. However, quenching the carbon-locked steel from  $650^{\circ}$ C was found to have no effect upon k, although the available concentration of interstitial atoms in solution was then greater than in the annealed semikilled steel that showed the high k value. Thus, it is concluded that the weaker strength of the carbon locking is not a concentration effect but must arise from an interaction energy between a dislocation and a carbon atom that is lower than the interaction energy between a dislocation and a nitrogen atom. This is, of course, contrary to the customary assumption that the interaction energies are closely similar because of the closely similar elastic dilations.

Recent theoretical work has emphasized the directional nature of metallic bonding, 18 and the observation of what appears to be an ap-

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preciable Peierls-Nabarro stress in  $\alpha$ -iron  $^{9.14}$  also suggests some directionality in the bonding. Possibly, therefore, in addition to the elastic interaction in the dislocation locking, there is a chemical interaction arising from a degree of residual valency at the dislocation to which the impurity atoms become attached, such as occurs in covalent silicon and germanium.17

### Conclusion

There is still much that is unknown about fracture in  $\alpha$ -iron. In particular, it is still not clear whether the first step is the coalescence of dislocations on one slip plane, or on intersecting slip planes, or even possibly some quite different event. Nevertheless, the idea that the first step is the formation of a crack, proportional in length to the grain diameter, at a critical resultant shear stress on the unlocked dislocations and that the second step is concerned with the Griffith propagation of this crack by the applied tensile stress appears to go a long way toward explaining the characteristics of the ductile-brittle transition. In particular, it brings out the importance of grain size, of the friction on an unlocked dislocation, and of the strength of the dislocation locking.

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### DISCUSSION

F. DE KAZINCZY, W. A. BACKOFEN, and B. KAPADIA, Massachusetts Institute of Technology. Different views on the grain-size dependence of the lower yield point (the expression  $\sigma_y = \sigma_i + k_y d^{-1/2}$ ) have been advanced by Cottrell and Petch. Cottrell suggests that  $k_y$  decreases with

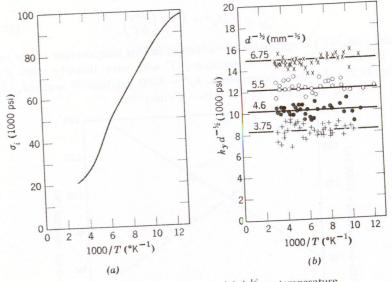


Fig. D.1. Dependence of  $\sigma_i$  and  $k_y d^{-1/2}$  on temperature.

increasing temperature, while Petch gives data in his Fig. 1 showing an insensitivity to temperature. Measurements made at Massachusetts Institute of Technology agree with those by Petch. The lower yield stress of a 0.15 C-1.18 Mn steel, represented by the terms  $\sigma_i$  and  $k_y d^{-1/2}$ , is shown for a range of temperature and different average grain diameters din Fig. D.1. The lines correspond to  $k_y = 2200 \text{ psi/mm}^{-1/2}$  over the entire range. As Petch points out, this suggests that dislocation locking is constant; since atmosphere pinning would be expected to show temperature dependence, it would also suggest some other reason for the impediment to source operation. Dependence on grain size of stress and temperature at the ductile-brittle transition (highest temperature obtained during fracture at the yield point) is summarized in Fig. D.2.

With these findings, a direct statement can be made of the grain-size dependence of the transition temperature. For  $\sigma_y$   $(T, d^{-\frac{1}{2}})$ :

 $d\sigma_y = \left(\frac{\partial \sigma_y}{\partial T}\right) dT + \left(\frac{\partial \sigma_y}{\partial d^{-\frac{1}{2}}}\right) dd^{-\frac{1}{2}}$ 

or

$$\frac{d\sigma_{y}}{dd^{-\frac{1}{2}}} = \left(\frac{\partial\sigma_{y}}{\partial T}\right) \frac{dT}{dd^{-\frac{1}{2}}} + k_{y}$$

At the ductility transition,  $\frac{d\sigma_y}{dd^{-1/2}} = k_y^*$  (slope of  $\sigma_y$  vs.  $d^{-1/2}$  in Fig. D.2). Therefore,

$$\frac{dT}{dd^{-1/2}} = (k_y * - k_y) / \left(\frac{\partial \sigma_y}{\partial T}\right)$$
 (D.1)

Since  $(\partial \sigma_y/\partial T)$  is approximately constant in this temperature range and  $k_y$  is not a function of temperature, T will vary linearly with  $d^{-1/2}$ (Fig. D.2). In these experiments,  $k_y^* = 8700 \text{ psi/mm}^{-1/2}$  and  $\partial \sigma_y / \partial T$  $\simeq -750 \text{ psi/°K}$ , so that  $dT/dd^{-1/2} \simeq -9^{\circ}/\text{mm}^{-1/2}$ .

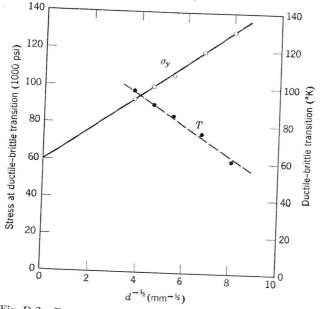


Fig. D.2. Dependence of stress and temperature at the ductilebrittle transition on grain size.

Equation D.1 is insensitive to whether or not  $\sigma_y$  vs.  $d^{-\frac{1}{2}}$  in Fig. D.2 has a nonzero intercept on the stress axis. However, for the special case of zero intercept (treated by Cottrell), it may be rewritten by substituting  $k_y^* = \sigma_y/d^{-\frac{1}{2}}$  and  $k_y = (\sigma_y - \sigma_i)/d^{-\frac{1}{2}}$ , with the result that

$$\frac{dT}{d\ln d^{-\frac{1}{2}}} = \frac{dT}{d\ln \sigma_i} \tag{D.2}$$

which is also Petch's Eq. 8 used in describing the grain-size dependence of the fracture mode transition.

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N. J. Petch (Author's Reply). There is no real disagreement between the views expressed by Petch and those by Cottrell. Although the slope k of the yield stress-grain diameter relation ( $k_u$  according to the Cottrell notation) does not change very much in the temperature region immediately below room temperature, a sharp increase in the slope khas been found at still lower temperatures. The author is surprised that this increase was not observed by de Kazinczy and co-workers.