

II. Cleavage, Ductility, and Tenacity in Crystals

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ABSTRACT

This chapter is a review of the state of our present knowledge about cleavage as a mechanical process. In addition to being a review, it includes new experimental results and ideas about cleavage and cleavage resistance.

Rules that are useful in the practical art of cleavage are given, and the mechanics of cleavage is developed. An equation of motion for cleavage cracks is derived, as well as equations relating surface energy to the forces required for cleavage. Results of a photoelastic study of the stress distributions around cleavage cracks are described.

A simple theory that can be used to estimate the anisotropic surface energies of crystals is given and is used successfully to predict surface energies and cleavage planes.

It is pointed out that terminal crack velocities are limited to values less than sound velocity by inertia and that experiments agree with theory.

Processes that absorb and release energy during crack propagation are discussed. Cleavage steps, electrical discharges, anelastic effects, and plastic deformation all absorb energy from moving cracks. Plastic deformation is usually the most important of these processes, and a relation is derived for estimating the magnitude of its effect. Energy may be released to a moving crack by chemical effects and by intersections with centers of internal stress, such as dislocations and point defects.

A parameter called the tenacity of a crystal is defined. It is a measure of the resistance of a crystal to cleavage or ductility. In addition to the surface energy of a crystal, the tenacity depends on such things as the dislocation mobility in a crystal.

It is pointed out that the brittleness of ionic crystals may be an intrinsic characteristic because electrostatic faults are produced in them when plastic flow occurs at the tips of cracks. These faults tend to suppress stress relaxation at crack tips.

Introduction

Cleavage is a fascinating natural phenomenon that is also the fundamental process in the final failure of many engineering materials. It is *resistance to cleavage* that determines the ductility of a crystalline material. Furthermore, cleavage provides a simple and relatively controlled means for creating fresh surfaces. Therefore, it can be used to measure surface energies. For this purpose, it is more straightforward and, hence, more reliable than most other methods. Finally, cleavage is an excellent method for shaping crystals because none of the material is wasted as cutting chips.

In spite of the importance of cleavage, it has received relatively little study as a mechanical process. Some of the questions that arise are the following:

1. How much force is required to cause a crystal to cleave?
2. What determines how fast a cleavage crack moves?
3. Why are cleavage surfaces sometimes rough and sometimes smooth in crystals of the same chemical composition?
4. What determines the crystallographic planes on which cleavage occurs most easily?
5. How can cleavage resistance (hence ductility) be increased or decreased?
6. Why do some crystals cleave easily while others do not?

This chapter discusses our present knowledge of the answers to these questions. Some of the answers seem to be quite good, but others are not completely satisfying as yet. The topics that are considered are the practical art of cleavage, cleavage mechanics, cleavage planes, speed of cleavage cracks, energy absorption and release during cleavage, ductility, and tenacity.

The Practical Art of Cleavage

Many a crystal has been mutilated in an attempt to cleave it. However, through experience gained in the course of cleaving a few thousand crystals, the author and his colleague, W. G. Johnston, have gradually acquired a set of guiding principles. The use of these principles does not remove all of the art from the practice of cleavage, but they do give considerable help.

Proper hammers and chisels are essential for the best results. The hammer should have a ball head, so that the blow struck on the chisel is always perpendicular to the struck surface. The chisel must be harder

than the crystal being cleaved, and the harder the better, provided a small amount of toughness is retained. Thus, tool steels make better chisels than plain carbon steels. A good chisel material for many purposes is hardened high-speed steel, tempered to a hardness of about 62 Rc. In addition to hardness, the acute angle of the tip is an important feature of a chisel. If the angle is too small, the tip will tend to buckle and chip; if the angle is too large, the chisel will tend to crush the crystal without wedging it open. A good compromise angle seems to be about 30°. For crystals that cleave readily, such as LiF and KCl, a chisel with a straight knife-edge is satisfactory. More resistant crystals like calcite and MgO can be cleaved better with a chisel whose knife-edge has convex curvature. The curved edge makes it possible to start the cleavage crack at the center of a surface rather than at an edge, thereby reducing chipping.

Specimens of specific size can be cleaved readily if guide lines are first scribed onto a surface of the crystal. This can be done conveniently with a machinist's vernier layout tool that can be set to 0.001 in. The cleavage chisel will readily settle into the centers of the scribed lines so that specimens can be cleaved within tolerances of about ± 0.001 in.

The most important factor in good cleavage practice is the strategy of placing the cuts. Good strategy allows crystal slabs as thin as 0.010 in. to be cleaved perfectly and minimizes the damage that occurs in larger crystals. There are essentially two starting shapes (blocks and rods) that require somewhat different strategies. They will be considered in turn.

Blocks. It is usually desired to cut either thin plates or slender rods from the initial block. This is best done by always making cuts that bisect an existing crystal. It is usually foolhardy to try to cut a thin plate from a thick block or a thin rod from a thicker but slender rod. In fact, the more slender the rod, the more essential it is to use only cuts that bisect the existing rod. The "bisection rule" means that it is always best to plan to cut a block into 2^n parts where n is an integer. If a crystal is not too slender, it is often possible to cut one third from it, but attempts to cut small fractions from an existing crystal will usually fail.

Rods. A common problem is that of shortening a slender rod. Instead of trying to do this with one cut, it is often better to cleave thin pieces off one end of the rod until the desired length is reached.

The reason why bisection is so important in cleavage lies in the stress distribution. As we shall see later, the maximum tensile stress does not lie in the plane of a cleavage crack but off to either side of it. It is only the anisotropy of the strength of the crystal that makes a cleavage crack proceed along a plane surface. In the case of bisection, the angle between the plane of the crack and the plane of maximum tensile stress is

smaller than it would be if a thin piece were being cleaved from a thicker one. Therefore, unless the strength anisotropy of the crystal is very large, good cleavage is most likely to occur if a crystal is bisected. If the anisotropy is zero (as it is for glass and plastics), a material cannot be cleaved along a plane surface by the ordinary method.

Crystals are often very difficult to cleave if their surfaces are not orthogonal to the chisel and cleavage crack. Calcite is a good example. Attempts to cleave natural rhombs of calcite by simply placing them on a flat surface and hitting them with the chisel are almost sure to fail. On the other hand, if surfaces are cut or ground onto a calcite crystal parallel and perpendicular to the proposed cleavage plane, this material can be cleaved as readily as rock salt. The purpose of using perpendicular faces is to make the stress distribution symmetric in the crystal. This keeps the maximum tensile stress nearest to the plane of the crack. In crystals such as calcite, a single cleavage plane is not highly preferred; hence, cracks tend to wander from one plane to another unless a favorable stress distribution is provided so that their small-strength anisotropy can keep a crack moving in its own plane.

The support of long crystals and stubby blocks during cleavage is important. The ends farthest from the cleavage chisel should not be restrained from rotating away from the chisel. This is easily accomplished by placing fulcrums under the side opposite the place where the chisel is applied to the crystal.

The blow that is struck on the chisel should be sharp but not too heavy. Otherwise, compressive waves will be generated, which will travel down the crystal ahead of the crack and, after reflection from the end of the crystal, will travel back to the crack tip and interfere with the stresses there. The result may be the generation of cracks perpendicular to the primary crack.

The Mechanics of Cleavage

The energy balance treatment of fracture that was originated by Griffith¹ is the basis of this discussion of cleavage mechanics. Parts of the discussion have been given previously by Obreimov² and Benbow and Roesler,³ but new features are presented here concerning the motion of cleavage cracks. The energy balance method is a suitable approximation because Orowan⁴ has shown that it yields essentially the same result as an analysis of the stresses at the tip of a crack.

Figure 1 is a schematic drawing of a crystal that is undergoing cleavage. We assume the crystal to be homogeneous and isotropic and to have a distinct cleavage plane. Each "arm" that is separated from its mate by

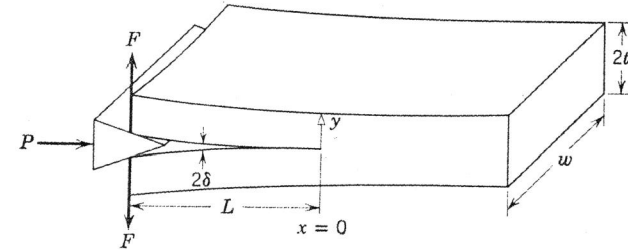


Fig. 1. Schematic cleavage of a crystal.

the crack has a length L and is treated as a cantilever beam built in at one end. The force F causes a bending moment $M(x)$ that has a distribution along the length of the beam:

$$M(x) = F(L - x), \quad 0 < x < L \quad (1)$$

According to simple beam theory, this bending moment puts an amount of strain energy U into the beam:

$$U = \frac{1}{2EI} \int_0^L M^2(x) dx = \frac{F^2 L^3}{6EI} \quad (2)$$

where I = moment of inertia of cross section = $wt^3/12$. The deflection of the beam at the place of application of the force is obtained by applying Castigliano's theorem:

$$\delta_0 = \frac{\partial U}{\partial F} \Big|_{x=L} = \frac{FL^3}{3EI} \quad (3)$$

If this is substituted into Eq. 2, the strain energy is obtained in terms of the deflection:

$$U = \frac{3EI\delta_0^3}{2L^3} \quad (4)$$

The surface energy S of one side of the crack is equal to the specific surface energy γ times the area that has been exposed:

$$S = \gamma Lw \quad (5)$$

As Mott first pointed out,⁵ if the crack is moving with velocity v_c , the beam also has some kinetic energy T , owing to its motion in the y -direction. Since the velocity in the y -direction is $d\delta/dt$, the kinetic energy is

$$\begin{aligned} T &= \frac{1}{2} m \left(\frac{d\delta}{dt} \right)^2 \\ &= \frac{1}{2} \rho wt \int_0^L \left(\frac{d\delta}{dt} \right)^2 dx \end{aligned}$$

From simple beam theory:*

$$\delta = \frac{F}{EI} \left(\frac{Lx^2}{2} - \frac{x^3}{6} \right)$$

Now,

$$\frac{d\delta}{dt} = \frac{\partial\delta}{\partial x} \left(\frac{dx}{dt} \right) = v_c \frac{\partial\delta}{\partial x} = \frac{v_c F}{EI} \left(Lx - \frac{x^2}{2} \right)$$

then,

$$T = \frac{\rho\omega t}{2} \left(\frac{v_c F}{EI} \right)^2 \int_0^L \left(Lx - \frac{x^2}{2} \right)^2 dx$$

Remembering that the velocity of longitudinal sound waves is $v_s^2 = E/\rho$, we have

$$T = \frac{\omega t}{15} \left(\frac{v_c}{v_s} \right)^2 \frac{F^2 L^5}{EI^2}$$

In terms of the strain energy,

$$T = \frac{24}{5} U \left(\frac{L}{t} \right)^2 \left(\frac{v_c}{v_s} \right)^2 \quad (6)$$

Thus for high velocities and long cracks, the kinetic energy is an appreciable fraction of the total strain energy.

Next, we obtain the equation of motion of the crack for the case of a constant applied force at $x = L$. In this case the total energy H is

$$H = S + T + U - F\delta_0(L) \quad (7)$$

where $F\delta_0$ is the work done by the applied force as the crack moves along. This work is $F^2 L^3 / 3EI = 2U$, so we have

$$\begin{aligned} H &= S - U + T \\ &= \gamma\omega L - \frac{F^2 L^3}{6EI} + \frac{24}{5} \left(\frac{L}{t} \right)^2 \left(\frac{v_c}{v_s} \right)^2 \frac{F^2 L^3}{6EI} \\ &= AL - BL^3 + Cv_c^2 L^5 \end{aligned}$$

where

$$\begin{aligned} A &= \gamma\omega \\ B &= \frac{F^2}{6EI} \\ C &= \frac{24}{5} \left(\frac{B}{t^2 v_s^2} \right) \end{aligned}$$

* There is additional deflection owing to shear, but since it is $\approx \frac{1}{2}\delta(t/L)^2$, it is usually negligible.

The potential energy is $P = S - U = AL - BL^3$, so the Lagrangian is $\mathcal{L} = T - P = Cv_c^2 L^5 - AL + BL^3$. Substituting into Lagrange's equation of motion:

$$\frac{d}{dt} \left(\frac{\partial\mathcal{L}}{\partial v_c} \right) - \frac{\partial\mathcal{L}}{\partial L} = 0$$

we have

$$2Ca_c L^5 + 5Cv_c^2 L^4 + A - 3BL^2 = 0$$

where a_c is the crack acceleration.

The Griffith condition for the beginning of crack motion states that the surface energy of the crack must be provided by the strain energy and the work done by the applied force, $W = F\delta_0$. That is,

$$\frac{dS}{dL} = -\frac{dU}{dL} + \frac{dW}{dL}$$

or

$$S = U$$

This defines the critical crack length L^* at which motion begins:

$$L^{*2} = \frac{A}{B}$$

In terms of this critical length, the equation of motion may be written:

$$a_c + \frac{5v_c^2}{2L} - \frac{5t^2 v_s^2}{16L^3} \left[1 - \frac{1}{3} \left(\frac{L^*}{L} \right)^2 \right] = 0 \quad (8)$$

When $L = L^*$ and the crack is at rest, $a_c = 0$, as would be expected. For L large compared with the critical value L^* , the crack velocity reaches a steady value (that is, $a_c = 0$) if

$$v_c = \frac{1}{2\sqrt{2}} \left(\frac{t}{L} \right) v_s \quad (9)$$

Thus, the terminal velocity is limited by the crack length and the specimen thickness as well as by the sound velocity. This is unlike the case of an infinite body where only the sound velocity determines the terminal crack velocity. It means that cracks can never move extremely fast in long slender bars.

Another feature of cleavage specimens is that the stresses at the tip of the crack are independent of the crack length when the crack is just beginning to move. It results from the fact that the bending moment at the crack tip is $M = FL$, and when the crack begins to move, $S = U$. This latter relation gives

$$F = \sqrt{\frac{6\gamma\omega EI}{L}} \quad (10)$$

so that

$$M = \sqrt{6\gamma\omega EI} \quad (11)$$

The bending moment is independent of L . Since the local bending moment determines the local stresses, the stresses are also independent of L . This makes cleavage-type specimens especially good for studies of crack propagation.

It may be noted that no consideration of the concentrated stresses near the tip of a cleavage crack is given in the above discussion. This is justified for the case of cleavage cracks because their speed is determined by the inertias of the "arms" of a cleavage specimen and not inertial effects near the crack tip. Therefore, since the crack velocity never becomes a large fraction of the speed of sound, relativistic effects are not important.

The forces that resist cleavage of a crystal are provided by its surface energy. This is defined in this case as the energy that is needed to pull apart two adjacent lattice planes of the crystal from their equilibrium separation distance d_0 to infinity. Thus, if the law of attraction of the surfaces is $F(y)$, then the surface energy is

$$\gamma = \frac{1}{2} \int_{d_0}^{\infty} F(y) dy \quad (12)$$

The surface energy can be measured by one of two methods. Either the force that is needed to cause a crack to advance or the width of the crack 2δ can be measured at a known distance from a crack tip at the instant when the crack begins to advance. From the Griffith condition, $S = U$, and Eqs. 2, 3, and 5, equations for γ can be obtained in the two cases:

$$\gamma = \frac{F^2 L^2}{6EIw} \quad (\text{in terms of the force}) \quad (13)$$

$$\gamma = \frac{EI\delta^2 L^2}{6w} \left(\frac{Lx^2}{2} - \frac{x^3}{6} \right)^{-2} \quad (\text{in terms of deflection}) \quad (14)$$

Determination of the surface energy through the applied force has the advantage that Eq. 13 holds even when cleavage is not completely elastic. The applied force is transmitted to the crack tip by the elasticity of the two "arms" of the crystal, so its magnitude is independent of any inelastic deformation that occurs. The use of the deflections to determine γ is subject to large errors if the deflections are not completely elastic. However, the deflection method has the advantage of accuracy (interference fringes formed from the two crack surfaces can be used to measure the deflections), and it is convenient to use for fast-moving cracks.

Stresses at Cleavage Cracks

No analytic description of the stresses at the tip of a cleavage crack exists at present. However, a photoelastic study has been made by Guernsey and Gilman,⁶ and some of their results are shown in Fig. 2a, in which the maximum tensile stresses are shown. Very near the crack

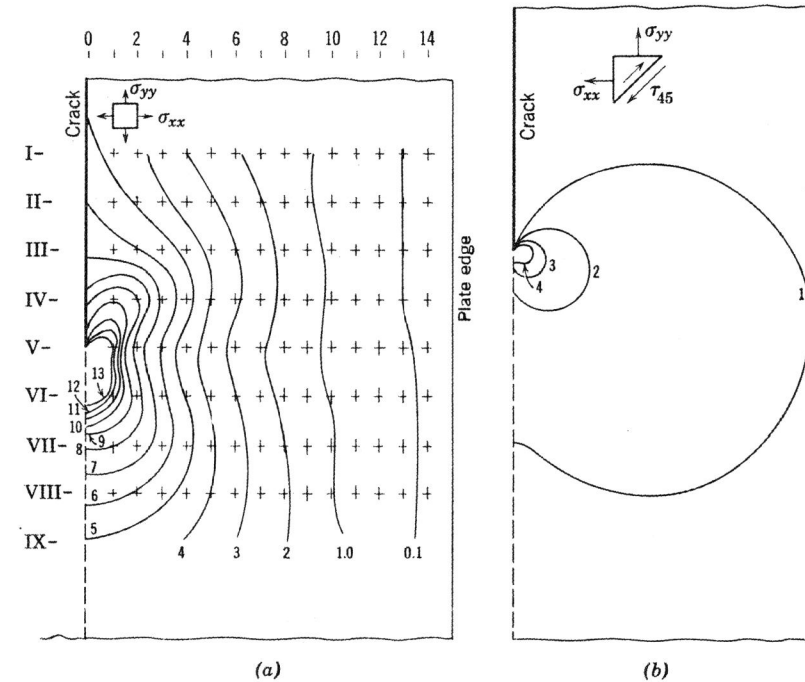


Fig. 2. Comparison of contours of maximum tensile stress. (a) Cleavage crack. (After Guernsey and Gilman.⁶) (b) Crack in infinite plate. (After Irwin.⁷)

tip, the stresses are proportional to $1/\sqrt{r}$ (r is the distance from the crack tip), just as in the case of a crack in an infinite body. At larger distances the stresses diminish more rapidly than this because they must become equal to zero at the bounding surfaces of the cleavage specimen.

The tensile stress distribution near the tip of a crack in an infinite plate is shown in Fig. 2b (Irwin⁷). Comparison with the distribution for a cleavage crack (Fig. 2a) shows that the chief qualitative difference between the two distributions is the position of the steepest gradient of the maximum tensile stress (closest spacing of the contour lines). For a crack in an infinite body, this gradient is coincident with the plane of the crack; but for a cleavage-type crack, it lies in a direction nearly per-

pendicular to the plane of the crack. The reason for the difference is, of course, the fact that a bending moment is present in the latter case but not in the former. Since cracks tend to run in the direction of the steepest gradient of the maximum tensile stress, a cleavage-type crack in an isotropic body will not run straight but will veer out of its own plane and run out the side of the specimen. The anisotropy of the cleavage strength of crystals makes it possible to obtain straight plane cleavages.

The Cleavage Plane

Several criteria have been proposed for predicting or rationalizing the observed cleavage planes of crystals. First, Haüy suggested that the unit cells of crystals are bounded by cleavage planes; this fails for fluorite. Then Tutton stated that the cleavage planes of a crystal are its most closely packed planes,⁸ but this criterion fails for such crystals as zinc blende (ZnS), fluorite (CaF₂), and calcite (CaCO₃). Next Huggins⁹ discussed the criterion that the cleavage plane is the plane that cuts a minimum number of chemical bonds per unit of area (consistent with the condition that the cleavage surfaces must be neutral). The discussion was continued by Wooster⁸ and in some detail by Shappell.¹⁰ This bond-density criterion suffers because of the rather vague meaning of "chemical bond," especially in the case of ionic crystals. Stepanov¹¹ argued that the anisotropy of elastic constants determines the cleavage plane of a crystal. Others have argued that the plane of minimum surface energy should be the cleavage plane.

This last criterion seems to be the best one, although there is some evidence that it may not apply for body-centered cubic crystals like iron and tungsten. Some experiments¹² and calculations¹³ indicate that the (110) planes have lower energies than (100) planes in these crystals, yet they prefer to cleave on (100) planes. On the other hand, tantalum¹⁴ and vanadium¹⁵ prefer to cleave on (110) planes, which is the expected behavior. All of these results for b.c.c. metals are subject to some doubt because the metals have not been completely pure. In fact, in the case of vanadium, no cleavage occurs in the absence of hydrogen, and oxygen is required for cleavage in tantalum.

It is suggested here that a mechanical criterion for the cleavage plane might be best. This would state that the observed cleavage plane should be the one that requires the least force to propagate a cleavage crack along it. From Eq. 9, we see that the material constants that determine how much force is required for cleavage are the elastic modulus E and the surface energy γ .

The elastic moduli for many crystals are known, but very few measure-

ments of surface energies have been made, and virtually no measurements have been made of the anisotropies of surface energies. Therefore calculated estimates must be used. Detailed calculations have been made only for the alkali halides,¹⁶ so we use a simple estimate. It is assumed that the attractive stress $\sigma(y)$ between two surfaces can be approximated by a sine function:

$$\sigma(y) = \sigma_0 \sin \frac{\pi y}{a}, \quad 0 \leq y \leq a$$

where a is the "range" or relaxation distance of the attractive forces. For small displacements, the attractive stress should obey Hooke's law, and

$$\sin \frac{\pi y}{a} \approx \frac{\pi y}{a}$$

Therefore,

$$\sigma = E \left(\frac{y}{y_0} \right) = \sigma_0 \left(\frac{\pi y}{a} \right) \quad (y \ll a)$$

where y_0 is the equilibrium lattice constant perpendicular to the plane. Thus, σ_0 is determined, and the attractive stress becomes

$$\sigma(y) = \frac{Ea}{\pi y_0} \sin \frac{\pi y}{a} \quad (15)$$

Substituting this into Eq. 12 gives

$$\begin{aligned} \gamma &= \frac{Ea}{2\pi y_0} \int_0^a \sin \left(\frac{\pi y}{a} \right) dy \\ &= \frac{E}{y_0} \left(\frac{a}{\pi} \right)^2 \end{aligned} \quad (16)$$

Equation 16 indicates that the cleavage planes should be those with minimum elastic stiffness normal to themselves, maximum separation distance, and minimum relaxation distance for the attractive forces between them. This latter distance is taken to be the diameter of the atoms in the cleavage surface, measured parallel to the surface. This is not true for nonneutral cleavage surfaces in ionic crystals, because the Coulombic attractions between such charged planes have very long ranges. Therefore, their surface energies are very high, and such planes are not considered to be possible cleavage planes.

Calculations based on Eq. 16 are given in Table 1. It may be seen that the equation predicts not only the correct magnitude of the surface energy, wherever measured values are available, but also the correct anisotropy in most cases. The correct cleavage plane is also predicted with the exception of the potassium halides, tungsten, and beryllium.

TABLE 1. Calculated Surface Energies of Crystals

Crystal	Structure	Young's Moduli (10^{11} dynes/cm ²)			Lattice Constant (Å)	a_0	Spacings of Possible Cleavage Planes		
		E_{100}	E_{110}	E_{111}			γ_{100}	γ_{110}	γ_{111}
MgO	rock salt	24.5	30.8	33.7	2.10	1.05	2.10	1.48	*
LiF	rock salt	7.35	10.8	12.8	2.00	1.00	2.00	1.41	*
NaCl	rock salt	4.37	3.45	3.22	2.81	1.40	2.81	1.99	*
NaBr	rock salt	3.48	2.72	2.53	2.97	1.48	2.97	2.10	*
KCl	rock salt	4.80	2.30	2.00	3.14	1.57	3.14	2.22	*
KBr	rock salt	3.70	2.15	1.88	3.29	1.64	3.29	2.32	*
KI	rock salt	2.55	1.31	1.56	3.53	1.76	3.53	2.49	*
PbS	rock salt	8.34	4.1	3.5	2.98	1.49	2.98	2.10	*
AgCl	rock salt	3.21	2.27	2.06	2.87	1.43	2.87	2.03	*
AgBr	rock salt	3.29	2.02	1.79	2.77	1.38	2.77	1.96	*
CaF ₂	fluorite	14.1	10.1	9.2	5.45	1.36	*	*	1.56
ZnS	zinc blende	4.88	7.4	8.9	5.43	0.96	*	1.92	*
InS	zinc blende	4.14	6.2	7.5	6.45	1.14	*	2.28	*
Glass		45.				0.8	> 1.6		
Ge	diamond	10.2	13.7	15.5	5.66	1.23	1.41	2.00	3.26
Si	diamond	13.1	17.5	19.9	5.42	1.17	1.35	1.91	3.12
C	diamond	105.0	116.0	120.0	3.57	0.77	0.89	1.26	2.06
W	b.c.c.	38.9	38.9	38.9	2.87	1.24	1.58	2.23	0.91
α -Fe	b.c.c.	13.2	22.2	28.4	3.16	1.37	1.43	2.02	0.83
Na	b.c.c.	0.21	0.55	1.41	4.29	1.85	2.14	3.03	1.24
Cu	f.c.c.	6.67	13.1	19.4	3.62	1.28	1.81	2.56	1.04
		E_{0001}	$E_{10\bar{1}0}$				γ_{0001}	$\gamma_{10\bar{1}0}$	
Be	h.c.p.	29.7	26.6		2.29, 3.58	1.11	1.80	1.98	
Zn	h.c.p.	12.5	10.9		2.66, 4.93	1.33	2.46	2.30	
Cd	h.c.p.	2.8	8.1		2.98, 5.62	1.49	2.81	2.58	
C	graphite	1.8	113.0		—	0.71	3.4	2.46	
Te	selenium	2.06	4.28		4.5, 5.91	2.44	1.73	3.86	

* Nonneutral planes

For these exceptions, there are several possible interpretations: (a) Equation 16 is a poor approximation; (b) surface energy does not control the cleavage; (c) the observed cleavage planes result from some impurity effect; (d) the measurements of the elastic constants are in error. The last possibility seems quite reasonable for the case of the potassium halides because they are very soft crystals.

Compared with Observed Cleavage Planes

Surface Energies $\gamma_{hkl} = \frac{E_{hkl}(a_0)^2}{\gamma_{hkl}\pi}$			Cleavage Plane		Surface Energy (ergs/cm ²)		
γ_{100}	γ_{110}	γ_{111}	Observed	Predicted	Experimental	Theoretical	Reference
1310	2330	*	100	Yes	—	1362	17
374	780	*	100	Yes	—	700, 169	18, 19
310	345	*	100	Yes	330	77-188	17, 20-24
248	288	*	100	Yes	—	76-177	17, 24
318	271	*	100	No	—	56-163	17, 24
306	253	*	100	No	—	54-151	17, 24
233	165	*	100	No	—	58-136	17, 24
625	440	*	100	No	—	—	
230	230	—	—	—	—	—	
230	200	—	—	—	—	—	
*	*	1100	111	Yes	—	—	
*	360	*	110	Yes	—	—	
*	359	*	110	Yes	—	—	
< 1820					1214	—	25
1110	1050	726	111	Yes	—	—	
1350	1270	887	111	Yes	—	1232	26
7050	5500	3500	111	Yes	—	5650	13, 27, 28
4680	3320	8130	100	No	—	2680, 5510	13, 29
1440	1710	5340	100	Yes	—	1450, 1600	29, 30
33	63	389			—	94-190	29, 31-33
590	820	2980			1100, 1370	500-1210	29-35
γ_{0001}	$\gamma_{10\bar{1}0}$						
2060	1680		0001	No	—	—	
185	850		0001	Yes	—	600, 772	29, 35
226	730				—	546, 800	29, 35
27	2340		0001	Yes	—	—	
720	670		10 $\bar{1}$ 0	Yes	—	—	

The Speed of Cleavage

Although measurements of crack velocities have been made for many engineering materials, LiF is the only *crystal* for which measured values of cleavage crack velocities are available.³⁶ The terminal velocity for cleavage cracks in LiF was determined and is compared with theory in Fig. 3 (along with data for other materials).

The fastest cleavage cracks are short ones, so the theory of crack propagation in infinite bodies is more appropriate to terminal velocities than Eq. 9. The theory of Roberts and Wells³⁷ predicts that the limiting velocity for this case should be $0.38\sqrt{E/\rho}$, and Stroh³⁸ has pointed out that cracks resemble surface waves and so should not be able to move faster than about $0.94\sqrt{G/\rho}$ (the Rayleigh surface-wave velocity). These

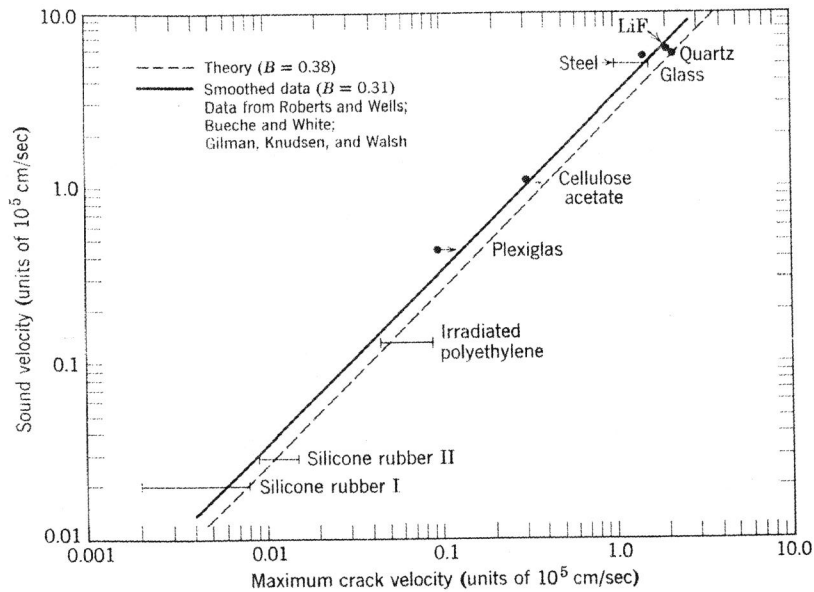


Fig. 3. Terminal velocities of cracks. (After Gilman, Knudsen, and Walsh.³⁶)

predictions are almost numerically equal. The former one is compared with experiments in Fig. 3, and it may be seen that the theory agrees with experiments fairly well for polycrystals, high-polymeric bodies, and in one case, for a monocrystal (LiF). Therefore, it is confirmed that inertial effects limit crack speeds, and the theory is essentially correct.

Energy Absorption During Cleavage

In ideal or "elastic" cleavage, the work that is done by the applied forces F is all converted into true surface energy. However, in most real crystals, the work is absorbed by a variety of processes, in addition to the creation of new surfaces. These include plastic deformation, anelastic effects, electrical effects, and the formation of cleavage steps. In order to take these factors into account, we define an effective surface energy γ_c , called the "cleavage surface energy." It is the sum of the true

surface energy γ and the energies of other absorbing processes. We consider these additional processes in turn.

Cleavage Steps

Surface roughness can arise during cleavage in several ways. If a cleavage crack is not started strictly parallel to the crystallographic cleavage plane, it will become segmented in order to maximize the fraction of its surface that does lie parallel to the preferred plane. Also, if one crystallographic plane is not substantially preferred over the others, a cleavage crack will tend to wander from one plane to another. This happens in calcite and in iron, for example. If a crystal contains sub-grain boundaries of the twist type, a crack cannot pass through the crystal without becoming segmented as it passes through the twist boundary. Finally, when cracks pass through the screw components of dislocation lines, the cracks acquire cleavage steps (Fig. 4). The screw dislocations may be grouped together in glide bands or dispersed through the crystal. Experimental demonstrations of step formation at dislocations have been discussed elsewhere³⁹ by the present author and by J. R. Low in Chapter 4 of this volume. If the "risers" (the parts roughly perpendicular to the flat surface) of cleavage steps are formed by cleavage on planes that are equivalent to the primary cleavage plane, then the additional energy γ_s that is absorbed because of the steps is simply $\gamma(k-1)$, where k is the ratio of the true surface area to the nominal surface area. Since it is unlikely that $k \gtrsim 2$, the energy absorbed with steps will not be greater than about double the energy absorbed without steps. If cleavage on some secondary cleavage plane is required to form the risers, γ_s could exceed γ in principle, but since k will tend to be less than 2, γ_s will still not exceed $\sim\gamma$. Thus for crystals in which no plastic deformation accompanies cleavage, $\gamma_c = \gamma + \gamma_s$ will not be large compared with γ . The case when plastic flow accompanies cleavage-step formation will be discussed later.

Anelastic Effects

Anelasticity may have a large influence on cleavage under some circumstances. For solid solutions of oxygen in Ta,⁴⁰ for example, as much as 20% of the elastic energy may be absorbed anelastically and, hence, would increase γ_c significantly. However, since the maximum anelastic absorption occurs in a narrow frequency band, it would be effective only for a narrow range of crack velocities.

Electrical Effects

When crystals that are insulators are cleaved, patches of charge often form on their cleavage surfaces.² These charge patches subsequently dis-

sipate their energy by (a) gaseous discharge across the gap of the crack, (b) surface conduction, or (c) field emission of electrons. The amounts

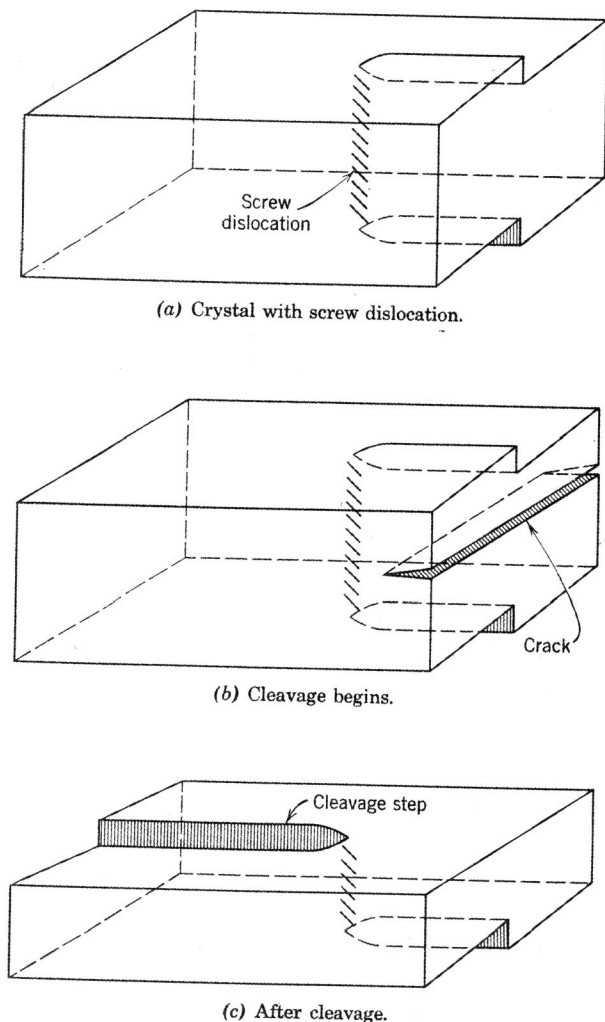


Fig. 4. Creation of cleavage step by screw dislocations.

of energy that can be dissipated in this way may be quite large, as much as ten or more times the true surface energy.⁴¹ When this effect is large, the electrical discharges result in the release of visible light, and the phenomenon is known as "triboluminescence." Mica and zinc sulfide (sphalerite) are examples of triboluminescent crystals.

One way in which patches of charge can arise is exhibited by LiF. Here, charges are associated with the formation of dislocations during cleavage. If a crystal of LiF is cleaved slowly so that groups of dislocations are formed at the tip of the crack,³⁶ patches of charge are found to be associated with the groups of dislocations. This has been demonstrated by sprinkling fine aluminum oxide particles ($\sim 1\text{-}\mu$ diam) onto the cleavage surfaces. The particles tend to collect at the places where dislocations formed during cleavage. The effect is probably a direct result of the dislocations and not a result of any interaction between the crack and the dislocations. This is indicated by the fact that insulating powder tends to collect at dislocations in freshly bent LiF crystals where no crack is involved. The electrical effects of dislocations in ionic crystals have been discussed in some detail by Fishbach and Nowick.⁴²

Plastic Deformation

There are three modes of plastic deformation that may accompany cleavage. First, there is the possibility of deformation owing to the concentrated stresses at the tip of the crack; second, the nominal bending stresses in the "arms" may cause plastic bending in them; and third, when parts of a crack lie on different planes, they may join together by plastically shearing the material that lies between them.

For certain crystals, plastic bending of the "arms" is difficult to avoid unless the "arms" are very thick. The necessary thickness may be estimated as follows. The maximum tensile stress in a beam that is subjected to a bending moment M is

$$\sigma_m = \frac{6M}{wt^2}$$

Then, since the maximum bending moment is FL during cleavage, and the resolved shear stress is $\tau_m = \sigma_m \sin \chi \cos \lambda = K\sigma_m$ (where χ and λ are the angles between the direction of tension and the glide plane and the direction of tension and the glide direction, respectively), the maximum force that can be applied to a cleavage "arm," without causing it to bend plastically, is

$$F = \frac{wt^2\tau_y}{6KL}$$

where τ_y is the field stress of the crystal. However, in order to cause a cleavage crack to propagate, a force, $F = (1/L)\sqrt{6\gamma wEI}$, must be applied to a crystal (see section on cleavage mechanics). Therefore, if cleavage is to occur before plastic bending, we have the condition

$$t > \frac{18K^2\gamma E}{\tau_y^2} \quad (17)$$

Thus, for a crystal like rock salt, where $\gamma \simeq 200$ dynes/cm, $E \simeq 5 \times 10^{11}$ dynes/cm², $K = \frac{1}{2}$, and $\tau_y \simeq 5 \times 10^6$ dynes/cm², it is necessary for $\tau \gtrsim 18$ cm in order to avoid plastic bending. Since this is larger than most laboratory specimens, rock salt is usually bent during cleavage.

The concentrated stresses near the tip of a cleavage crack are localized to a region that is of the order of t in size (Fig. 2). Therefore, the stress concentration factor is $\sim \sqrt{t/b}$, where b is of the order of atomic dimensions. Since, according to Eq. 15, the maximum tensile stress that can be supported is $Ea/\pi y_0$, the maximum shear stress is $\sim G/\pi$; therefore, the concentrated stresses near the crack tip (caused by an applied stress τ_A in the direction y perpendicular to the crack) vary as

$$\begin{aligned} \tau &\simeq \tau_A \sqrt{t/y}, & y \geq y_m = t(\pi\tau_A/G)^2 \\ \tau &= \frac{G}{\pi}, & y \leq y_m \end{aligned} \quad (18)$$

Here y_m is determined by the condition that $\tau \leq G/\pi$.

If plastic flow occurs freely wherever the local stress exceeds the yield stress τ_y of the crystal, then the flow will absorb at least as much energy as is stored in the elastic field. This energy is

$$\begin{aligned} \gamma_p &= \int \frac{\tau^2}{2G} dy \\ &= \frac{G}{2\pi^2} \int_0^{y_m} dy + \frac{t\tau_A^2}{2G} \int_{y_m}^{y^*} \frac{dy}{y} \end{aligned} \quad (19)$$

where the upper limit of integration occurs when $\tau = \tau_y$, that is, at $y^* = t(\tau_A/\tau_y)^2$. Integrating, we have

$$\gamma_p = \frac{t\tau_A^2}{2G} + \frac{t\tau_A^2}{2G} \ln \left(\frac{G}{\pi\tau_y} \right)^2$$

Neglecting the first term, we obtain

$$\gamma_p \simeq \frac{t\tau_A^2}{G} \ln \left(\frac{G}{\pi\tau_y} \right)$$

The applied stress is given by Eq. 17 at the beginning of plastic flow, so we substitute this equation, letting $E \simeq 2G$, and $K \simeq \frac{1}{2}$:

$$\gamma_p = 9\gamma \ln (G/\pi\tau_y) \quad (20)$$

Thus we see that the energy absorbed by plastic deformation is proportional to the true surface energy multiplied by a factor that depends on the yield stress.

Equation 20 is an underestimate for many crystals because the assumed stress distribution will be considerably altered by small amounts of plastic

flow. However, this estimate does show that energies that exceed the true surface energy by at least an order of magnitude can be absorbed by plastic flow at the tip of a cleavage crack. Also, when the amount of plastic flow is small, this equation is approximately correct.

Since cleavage is a kinetic process, Eqs. 19 and 20 for the plastic energy absorption should include the *strain rate* if a detailed analysis is to be made. Some discussion of this has been given elsewhere.⁴³ Many crystals flow very rapidly at stresses above their yield stresses, however, and an estimate of γ_p can be based on their yield stresses. The flow is not limited by the problem of dislocation nucleation because dislocations can be nucleated readily by the concentrated stresses at a crack tip.⁴⁴ On the other hand, since the velocities with which dislocations move are a function of the applied stress,⁴⁵ this must be taken into account. If the dislocation velocity depends strongly on the applied stress (say, as τ^n where $n > 10$), then the yield stress is a good criterion of plastic flow because it does not change much during large variations of the strain rate.

In some crystals, dislocations do not move fast unless very high stresses are applied to them. This means that Eqs. 19 and 20 do not hold because a relatively long time is required for the concentrated stresses to relax, and γ_p will depend on the velocity of the crack.

Crystals that contain crystal boundaries or large numbers of dislocations will tend to absorb much larger amounts of plastic energy than Eq. 20 predicts. This results from the tearing and complicated stress distributions that arise when crack propagation is not continuous and coplanar. Under these adverse conditions, γ_p commonly attains values as high as 10^6 ergs/cm²,⁴⁶ and under extreme conditions, as much as 10^8 ergs/cm².⁷

Energy Release During Cleavage

By exposing the interior of a crystal, cleavage provides a means for the release of energy. The freshly exposed surfaces can react readily with their new environments to form physically or chemisorbed layers on themselves. Also, various forms of internal stresses can be relieved at the new surfaces.

Chemical Effects

The shapes of the tips of some two-dimensional cleavage cracks are shown in Fig. 5.⁴⁷ These are cracks in an infinite simple-cubic crystal of lattice constant a . The applied stress is a uniform tension in the z -direction equal to 1% of the Young's modulus of the material, and the cracks have various lengths ranging from $10a$ to 10^4a . It may be seen that the bonds between the atoms are very vulnerable to attack in the

immediate vicinity of the crack tip where they are stretched almost to the breaking point. If molecules from the environment attack them at this vicinity, the energy of the chemical reaction can contribute to the

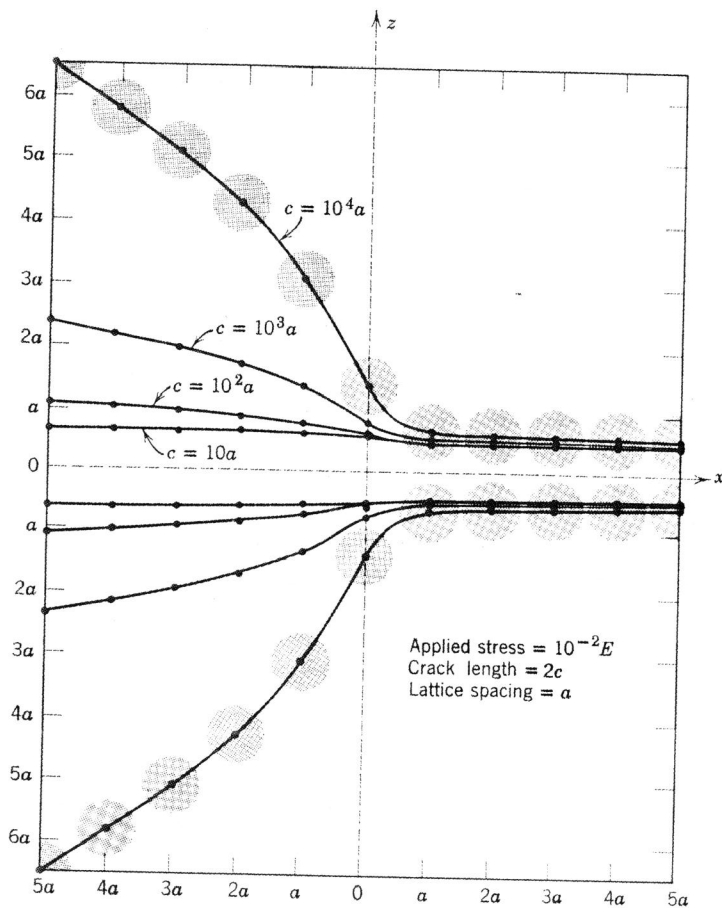


Fig. 5. Atomic displacements near the tips of cracks of various lengths.

propagation of the crack. If the energy of the reaction is γ_R per unit surface area, then Eq. 10 becomes

$$F = \frac{1}{L} \sqrt{6wEI(\gamma - \gamma_R)}$$

Thus if γ_R is large, the force required to advance the crack may become very small. In special cases, the fracture surface energy may be increased by chemical reactions if the reacting molecules "bridge" the crack surfaces so as to bind them together.

Since the atomic bonds near the tip of a crack go through all stages of being stretched until they are broken, it seems unlikely that the rate of chemical attack is limited at that point. In order to get to the reactive sites, the reactive molecules must diffuse within the crack gap by either gaseous or surface diffusion. There is not much room for gaseous diffusion in small cracks, so surface diffusion probably limits the rate at which the reactive molecules get to the crack tip and, hence, influences the rate of crack propagation.

The difference that Obreimov² found between the cleavage surface energy of mica in air as compared with its value in vacuum is often cited as a "chemical effect." It seems likely, however, that this difference was entirely due to electrical effects.⁴¹ A more convincing demonstration of chemical effects was made by Berdennikov²⁵ for glass. He measured the stresses required to propagate cracks in glass and found that the fracture surface energy of soda-lime glass is about 1200 ergs/cm² in vacuum and is decreased by various liquids (alcohols and water) in inverse proportion to the logarithm of their dielectric constants. The lowest value equaled 290 ergs/cm² for the case of water.

In special cases, a foreign chemical species may be contained within the structure of a crystal. This may increase or decrease the cleavage surface energy, depending on how tightly the foreign molecules are bonded to the structure. In one case,⁴⁸ it has been observed that dye molecules that become incorporated in K_2SO_4 during its growth cause it to cleave readily. If K_2SO_4 is grown in the absence of the dye, it does not cleave at all.

Internal Stresses

When a cleavage crack passes near or through a center of internal stress such as a dislocation or point defect, some elastic strain energy can be released. In the case of point defects, only those that lie within one or two atom distances of the cleavage surface can be affected. The amount of energy that can be released at each defect must be less than $\sim \gamma A_p$, where A_p is the cross-sectional area of the defect. Therefore the concentration of defects must be quite large in order to change γ_C significantly. Clusters of defects in the form of small gas pockets could have a considerably greater effect.

Dislocations that are intersected by a crack can make rather appreciable contributions to γ_C . Thus, if a crack passes along the glide plane of a dislocation loop, the loop will be annihilated and its energy released to the crack. If the energy of a dislocation line is H_d /cm and N /cm² of them are cut by the crack, their contribution to the cleavage surface energy will be $\gamma_d = -NH_d$. Then, since $H_d \simeq 2 \times 10^{-4}$ ergs/cm,

if $\gamma = 200$ ergs/cm², the cleavage surface energy will be nullified if $N \simeq 10^6$ dislocations/cm² are cut by the crack.

If a crack intersects edge dislocations perpendicular to their axes, little elastic energy will be released because only relatively small stresses act in the direction perpendicular to the surface. However, for a screw dislocation, the entire elastic field of the dislocation can be relaxed at the surface. This will liberate about $Gb^2/4\pi$ ergs/dislocation. (See Cottrell⁴⁹ for a discussion of the energies of dislocations near free surfaces.) At the same time, a cleavage step will be created with energy $\simeq \gamma bL \simeq Gb^2L$ if the step is L cm long. Thus, there will be a net energy release unless $L \gtrsim 0.1$ cm.

Ductility as Cleavage Resistance or Tenacity

The word "ductility" has been defined by various people in different ways. To some, it means the amount of plastic strain that a material sustains before it fractures; to others, it means reduction in area prior to fracture; to still others, it means impact strength as measured in such tests as the Charpy or Izod test. The dictionary says that a ductile material is one that is "capable of being drawn out or hammered thin." These various definitions seem to suggest that what is really meant by a "ductile" material is one that has a high resistance to cracking or cleavage. To think of materials in this way, that is, in terms of "cleavage resistance," has the advantage that this quantity can be defined in terms of distinct physical parameters, whereas the "ductility" of a material seems to be a very vague concept.

As a measure of cleavage resistance, a dimensionless parameter can be defined which we shall call the "tenacity parameter" T . This parameter is defined through Eq. 17 and is

$$\text{Tenacity} = T = \frac{18K^2\gamma_c E}{t\tau_y^2} \quad (21)$$

If $T > 1$, a material cannot be cleaved and is said to be *tenacious* (ductile); on the other hand, if $T < 1$, the material can be cleaved and is said to be *brittle*.

The quantities that make up the tenacity parameter require some interpretation.* E , K , and t are simply the elastic modulus, the orientation

* Tenacity is not a material constant but depends on some characteristic length of the body that is being tested. In the case of a cleavage specimen, the characteristic length is t , but in other cases, it will depend on the particular geometry. For a crack in an infinite body, it is the crack length, and for other bodies, it may be the length or thickness, the grain size, or the size of some stress raiser such as a notch. It will always be the length associated with the size of the region of high elastic strain which can be relaxed by cleavage.

factor $\sin \chi \cos \lambda$, and the thicknesses of the cleavage arms, respectively, but the other quantities are more complex. The yield stress τ_y is complicated by the fact that it depends on the loading rate. As yield stress usually increases with the loading rate, the tenacity parameter shows a corresponding decrease with increasing loading rate. The most complex quantity of all is γ_c . It is the true surface energy γ modified by the various processes that absorb and release energy at the surface.

It is important to recognize that the surface energy γ enters into the tenacity parameter as a factor that may be increased or decreased by multiplying factors. It is not an *additive term* that may be neglected if other processes absorb large amounts of energy. Thus, the energy absorbed by plastic flow, for example, is not large just because a material is soft. No matter how small the yield stress, some intrinsic surface energy is needed in order to create stresses that will make the plastic energy absorption high.

Comparative Tenacities of Materials

It is well known that the ease with which crystals can be cleaved varies markedly among materials. Materials like mica flake apart easily, whereas metals, especially gold, are virtually impossible to cleave. In other words, some materials are very *tenacious* and others are not. Some of the reasons for these differences in tenacity will be discussed in this section in terms of the quantities that make up the tenacity parameter (Eq. 21).

Although the true surface energy γ and the elastic modulus E play a role in determining tenacity, it is apparent from Table 1 that their role is not a dominant one. For example, copper, germanium, and CaF₂ have roughly the same surface energies but quite different degrees of tenacity. Similarly, LiF and copper have roughly the same elastic moduli but quite different tenacities.

Some authors tend to use the yield stress of a material, as compared with its fracture stress, as a measure of tenacity. This cannot be correct, however, because crystals such as KI and AgCl, for example, yield at similar stress levels (at the same temperatures relative to their melting points), and yet they have quite different tenacities. On the other hand, if one considers the yield stress of a *given material*, then yield stress and tenacity correlate quite well. An example is the case of steel shown in Fig. 6,⁵⁰ where a rapid rise in the square of the yield stress is accompanied by a rapid decrease in the reduction of area prior to fracture (Eq. 21).

The remaining quantity that can account for variations in tenacity is the energy dissipated by various processes near the tip of a crack. The

most important dissipative process by far is plastic deformation, and it will be discussed in some detail. Since cleavage is a dynamic process, the rate of energy absorption by plastic flow is the quantity that is of primary interest, and it depends on the plastic strain rate. In terms of dislocations, the plastic strain rate is:⁴⁵

$$\frac{d\epsilon}{dt} = b\rho\bar{v}$$

where b is the Burgers vector, ρ is the dislocation density, and \bar{v} is the average dislocation velocity. There is not much variation in b from one

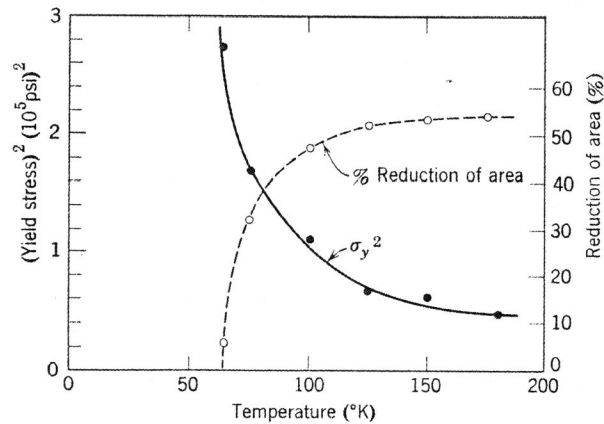


Fig. 6. Relation of reduction of area (tenacity) to yield stress in a 1020 steel. (Data from Eldin and Collins.⁵⁰)

crystal to another, but \bar{v} and ρ may vary considerably. If either ρ or v is very small, then $d\epsilon/dt$ will be small, and little energy will be absorbed.

Role of Dislocation Density

The average dislocation velocity depends on the applied stress. The only crystal for which measurements exist is LiF, but the general behavior of other crystals can be inferred from the LiF data. Figure 7 shows a set of data for LiF which follow the relation $\bar{v} = v_0 \exp(-A/\tau)$ quite well. Here, v_0 is a velocity that is nearly equal to sound velocity, and τ is the applied shear stress. Two additional curves are shown for hypothetical crystals. For covalent crystals, it is expected that the velocity may be zero until a rather high stress level is reached; then the velocity will increase slowly with increasing stress up to sound velocity. On the other hand, for a metal crystal, the dislocation velocity may be large even for small applied stresses, but again, it never exceeds the velocity of sound in the crystal.

In the case of covalent crystals (either hypothetical or real, such as silicon and germanium), the energy absorbed by plastic flow is small at low temperatures because dislocations do not move in these materials except at high stress levels. Therefore, near a crack tip, the region in which $d\epsilon/dt$ is appreciable is very small. Thus, one factor that can cause low tenacity in a crystal is low *dislocation mobility*. This is not the only important factor, though, because even if the dislocations in a crystal are mobile, if not many of them are present, the plastic strain rate will

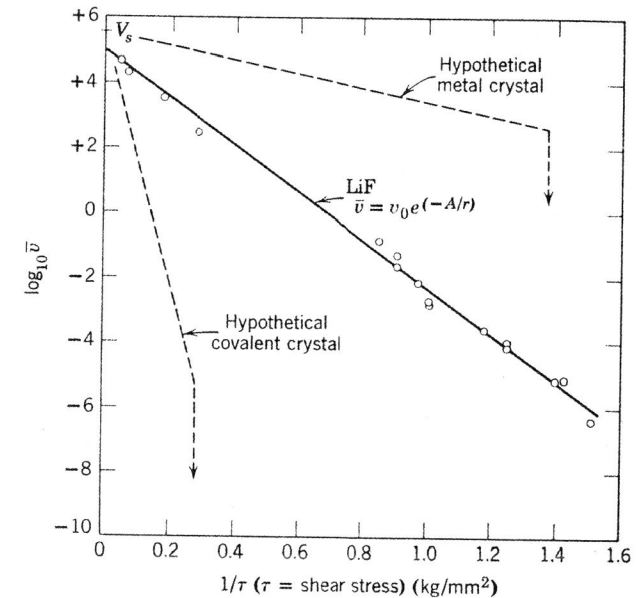


Fig. 7. Screw dislocation velocities versus stress in crystals

be small. Three factors might limit the dislocation density in a material: (1) low nucleation rate, (2) low multiplication rate, and (3) high rate of strain hardening (leaves small *effective* dislocation density). The first of these factors is not believed to be important because dislocations can be nucleated easily near the tips of cracks.⁴⁴ Also, the rate of multiplication seems adequate in most crystals.⁴⁵ However, there do appear to be systematic differences in the dislocation densities of strained crystals, so these must be attributed to strain-hardening effects.

Table 2 compares strain-hardening rates of typical crystals and the dislocation densities in them after some strain (50% elongation). Although the hardening rates of the materials do not show a systematic behavior, it may be seen that the dislocation density is very large for metals, considerably smaller for ionic crystals, and still smaller for

germanium. The reason for this variation is not clear at the present time, but it could play an important role in the fracture behavior of the crystals. We see that the variation in dislocation density is such that it acts to reinforce the variation of dislocation mobilities shown in Fig. 6 and to produce high strain rates in metals at a given stress level and low strain rates in ionic and covalent crystals at the same stress level.

TABLE 2. Comparison of Strain-Hardening Rates and Dislocation Densities of Various Crystals^{45,51-53}

Crystal	Strain-Hardening Rate $d\tau/d\gamma$ (kg/mm ²)	Dislocation Density at 50% Strain (dislocations/cm ²)
Cu	12-24	10 ¹¹
Ag	7-15	—
LiF	14.0	5 × 10 ⁸
KCl	5.6	—
Ge	≫ 100	~10 ⁸

Effect of Crystal Structure on Crack Propagation

Figure 8 shows a schematic rock-salt-type crystal. A (100) surface is shown with a crack on the (001) plane along the line *AB* in Fig. 8*a*. To relieve the elastic strains near point *B*, the crack might advance along its own plane, or alternatively, some plastic flow might occur as in Fig. 8*b*. The smallest unit of plastic flow that can occur is one unit Burgers vector, and, when this occurs, the ions on opposite sides of the crack are brought into repulsive positions. Therefore, at the same time that some elastic energy is relieved by the unit of plastic flow, the electrostatic energy of the crystal increases. This electrostatic energy causes an effective back stress on the motion of the dislocation. The back stress can be estimated in the following way.

The configuration at *C* in Fig. 8*b* may be seen to be almost that of an ion vacancy, except that one of the nearest-neighbor sites is unfilled and there is some energy attributable to the unbalanced repulsive forces. The same situation exists at the next layer parallel to the plane of the paper, except that the ions are reversed in sign. Thus one has approximately a row of ion vacancy pairs. Now Mott and Littleton⁵⁴ have shown that the binding energy of an ion vacancy pair is approximately $\frac{3}{2}Gb^3$ per ion, where G = shear modulus and b = Burgers vector. Therefore, we take this to be the energy per ion length of our pseudovacancy row. If a second unit of glide occurs, the configuration of ions becomes attractive again, so the energy owing to the gliding starts at zero, rises

to a maximum when the displacement $x = b$, and drops to zero again when $x = 2b$. We approximate the variation by a sine function: $E = E_m \sin(\pi x/2b)$, where $E_m = \frac{3}{2}Gb^3$. For small x , the sine can be replaced by its argument, and $\pi \simeq 3$, so we have $E \simeq \frac{9}{4}Gb^2x$ ($x \ll b$). Thus, the force that resists the gliding is $\simeq 2Gb^2$ per ion. The force on a dislocation line owing to an applied stress τ is τb^2 per atom, so unless $\tau \simeq 2G$, a dislocation cannot move away from a crack in the rock-salt

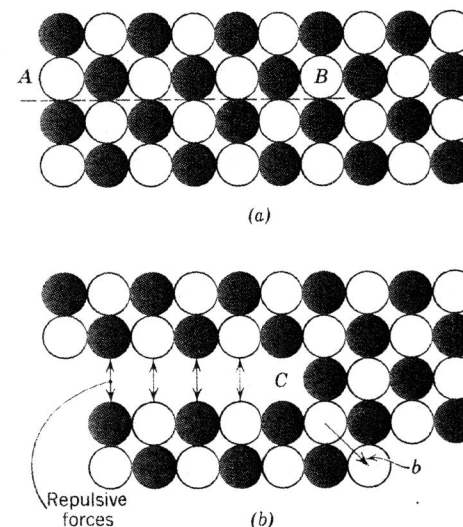


Fig. 8. Effect of plastic flow at the tip of a crack in the rock-salt structure.

structure. The stresses are very large near a crack tip, of course, but they do not get as large as $2G$.

It is believed that the afore-mentioned effect may account for the low tenacities of ionic crystals as compared with metallic crystals. One piece of evidence that seems to support this idea is that, when dislocation loops are nucleated at the tips of cracks in LiF crystals,⁴⁴ the types of loops that would cause the electrostatic faulting effect do not appear. Imagine a crack that lies on the (100) plane with its edge along the [010] direction. It will be observed that loops with $\langle 011 \rangle$ Burgers vectors are nucleated, but those with $\langle 101 \rangle$ Burgers vectors are not. The latter produce faulting, whereas the former do not.

The fact that the silver halides have high tenacities is also consistent with this idea because a large fraction of their binding is due to the van der Waals forces between their highly polarizable ions. Therefore, it is expected that electrostatic faults would not have a strong effect in them.

Furthermore, this phenomenon may account for the Joffe effect. If a faulted crack were filled with water, the energy of the fault would be greatly reduced because of the high dielectric constant of water. This would make it much more difficult to nucleate and propagate cracks in wet ionic crystals because plastic flow would stop the cracks from growing.

Summary

This paper discusses the cleavage of crystals from the point of view of principles. First, the practical art is considered, and a set of rules is given for obtaining good cleavages. Then the energy-balance method is used to develop a mechanics of cleavage. An equation of motion for cleavage cracks is derived which concisely expresses the kinetics of cleavage crack propagation. The stress distributions around cleavage cracks are described on the basis of a photoelastic study.

Relations that can be used in experiments to measure cleavage surface energies are also derived. Next, a simple theory that can be used to estimate the anisotropic surface energies of crystals is presented, and it is used to predict the cleavage planes of crystals.

Measurements of crack velocities are discussed, and it is pointed out that measurements of terminal crack velocities agree with theory.

Extended discussion is given of the processes that absorb and release energy during cleavage. Energy is *absorbed* by the formation of cleavage steps, by electrical discharges, by anelastic effects, and by plastic deformation. For the latter case, a relation is derived which shows that the energy absorbed by plasticity is equal to the true surface energy multiplied by a simple factor that contains the yield stress. Energy is *released* by chemical effects and by intersections of cracks with centers of internal stress such as dislocations or point defects.

Ductility is discussed in terms of resistance to cleavage. A parameter, called the *tenacity*, is defined. It is a measure of cleavage resistance. The tenacities of various materials are compared, and it is pointed out that dislocation motion is the most important factor in determining the tenacity of a crystal. Materials with low dislocation mobilities show low tenacities. Also, materials appear to differ as to their dislocation densities after a given plastic strain, and this can affect their tenacity.

Finally, it is pointed out that electrostatic faults may be produced in ionic crystals when plastic flow occurs at the tips of cracks in them. These faults tend to suppress plastic flow and, hence, may have a large influence on the fracture of ionic crystals.

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DISCUSSION

E. F. PONCELET, *Stanford Research Institute*. The method given in this chapter for finding the equation of motion of a crack is not accurate enough to lead to useful results. It may be of interest to recall that there exist two solutions of the problem that give results that check with experimental data.

The first solution was obtained by Craggs,¹ in which, using the Griffith approach, the decrease in strain energy induced by the propagation of a crack is equated to both the new surface energies and the elastic wave propagations generated. In this manner and by using very rigorous mathematical methods, Craggs arrived at a limiting crack velocity equal to the Rayleigh wave velocities. This is a little high, because of the simplifications required to solve the equations.

The second solution was obtained by the writer, using a statistical rather than a thermodynamical model and relating the rate at which bonds at the crack tip broke with the stress at the crack tip itself. This treatment² yields values for static fatigue that are in agreement with experiments. It gives for the limiting crack velocity the correct value of one-half the transverse wave velocity.³

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J. C. SURTS, *Harvard University*. The following treatment re-derives some of Gilman's equations in his section, "The Mechanics of Cleavage," with slightly different results, and it solves the equation of motion for crack propagation. The point of departure is Eq. 7, which is correct but which contains the additional feature that $H = \text{constant} = 0$. That H is constant arises essentially from the fact that this is a conservative system with the kinetic energy a homogeneous quadratic in the velocities, and therefore, the total energy is conserved. The constant may be evaluated at L equals zero, where S , T , and U are all zero, giving $H = 0$. Therefore we have

$$AL - BL^3 + Cv_c^2L^5 = 0 \quad (\text{D.1})$$

Solving Eq. D.1 for v_c gives

$$v_c = \frac{1}{2} \sqrt{\frac{5}{6}} \frac{t_1 v_s}{L} \sqrt{1 - \frac{S}{U}}$$

where $2t_1 = \text{specimen thickness}$. Therefore the condition for the initiation of crack motion is $S = U$, and the corresponding critical crack length is $L^{*2} = A/B$. Therefore

$$v_c = \frac{1}{2} \sqrt{\frac{5}{6}} \frac{t_1 v_s}{L} \sqrt{1 - \left(\frac{L^*}{L}\right)^2} \quad (\text{D.2})$$

Equation D.2 is an equation of the motion. However, one may derive an expression similar to Gilman's equation of motion by differentiating Eq. D.1 with respect to time, obtaining

$$2Ca_cL^5 + 5Cv_c^2L^4 + A - 3BL^2 = 0$$

Or, rewriting in terms of the critical crack length,

$$a_c + \frac{5v_c^2}{2L} - \frac{15t_1^2v_s^2}{48L^3} \left[1 - \frac{1}{3} \left(\frac{L^*}{L}\right)^2 \right] \quad (\text{D.3})$$

Now, from Eq. D.3, when $L = L^*$ and $v_c = 0$, then,

$$a_c = \frac{5t_1^2v_s^2}{24L^{*3}} \quad (\text{D.4})$$

or, in other words, the crack has an initial acceleration. This is not unreasonable. First, one cannot discuss the case of $L < L^*$, since, by Eq. D.2, this gives an imaginary velocity. We must start, therefore, with a crack of length L^* with zero initial velocity, apply a force F , and

then calculate the resulting motion. At $t = 0$, we must have an initial acceleration that corresponds to this applied force F ; this initial acceleration is given by Eq. D.4. Now $a_c = v_c dv_c/dL$ ($t = \text{time}$), and therefore, a_c equals zero when dv_c/dL equals zero (for $v_c \neq 0$). From Eq. D.2, then, $a_c = 0$ when $L = \sqrt{2} L^*$. One may see the general character of the motion from Eq. D.2, which gives a curve of the form shown in Fig. D.1.

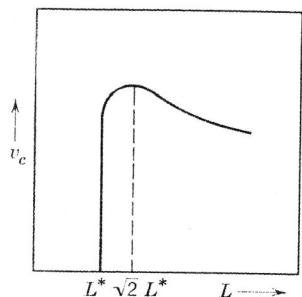


Fig. D.1. Form of Eq. D.2.

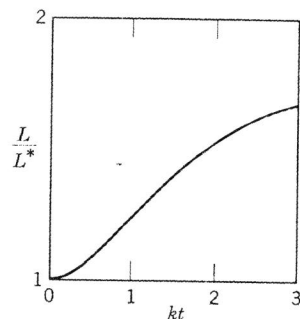


Fig. D.2. Form of Eq. D.5.

One may derive an explicit solution to Eq. D.2 or D.3 as follows: For $\alpha = L/L^*$ ($L^{*2} = A/B$), then Eq. D.2 may be written as

$$\frac{d\alpha}{dt} = \frac{k}{\alpha} \sqrt{1 - \alpha^{-2}} \quad \text{where} \quad k = \frac{1}{2} \sqrt{\frac{5}{6}} \frac{t_1 v_s}{L^{*2}}$$

therefore

$$k dt = \frac{\alpha d\alpha}{\sqrt{1 - \alpha^{-2}}}$$

and

$$kt = \int \frac{\alpha d\alpha}{\sqrt{1 - \alpha^{-2}}} = \frac{\alpha}{2} \sqrt{\alpha^2 - 1} + \frac{1}{2} \log(\alpha + \sqrt{\alpha^2 - 1}) \quad (D.5)$$

Equation D.5 explicitly gives the relation between crack length and time in generic form. All parameters affect the motion through the two general parameters k and L^* . Equation D.5 is shown graphically in Fig. D.2. For $L > \sim 3L^*$, the solution is parabolic, or

$$L(t) = (0.96 \sqrt{t_1 v_s}) \sqrt{t}$$