8. Classical and Dislocation Theories of Brittle Fracture

E. OROWAN

Department of Mechanical Engineering
Massachusetts Institute of Technology

ABSTRACT

The classical theory of notch brittleness has not been superseded by recent developments. It points out the conditions under which brittle fracture can occur in a potentially ductile material. The dislocation theories complement the classical theory by dealing with the physical nature of the cleavage strength, in particular, with the role of slip and twinning in crack propagation or nucleation.

The circumstance that cleavage fracture in steel seems to be preceded by some plastic deformation does not mean that the relevant cracks must be *produced* by a reaction between dislocations: Observations in steel and in MgO suggest the action of a plastic crack *propagation* mechanism capable of extending cracks that are subcritical in the Griffith sense by an interaction between dislocations and the crack. This idea was first considered in 1934 with reference to the fracture of NaCl.

The observation that stationary cleavage cracks in steel cannot propagate directly as cleavage cracks except at very low temperatures shows that brittle fracture under such conditions requires a high crack velocity which can be attained only if the work of crack propagation is supplied from released elastic energy. Consequently, a necessary condition of *brittle* (that is, low-energy) cleavage fracture in steels under essentially static load is the fulfillment of the equation $\sigma \approx \sqrt{Ep/c}$, where p is the work (largely plastic) of crack-wall formation. This equation is a condition of transcrystalline crack acceleration; microcracks within individual grains are observed if this condition is not fulfilled while crack propagation within the grain is possible. A suggestion is made for explaining the Greene effect (the occurrence in welded structures of brittle fractures not introduced by substantial plastic deformation at the tip of a crack or notch).

Cleavage Fracture and Plastic Deformation

The prominent theme of the Conference has been the relationship between cleavage fracture and plastic deformation: It is the question whether, and in what way, stresses resulting from dislocations play an essential role in cleavage fracture. The origin of this problem goes back almost thirty years;[1] in connection with the brittle fracture of steels, it has received additional emphasis by recent work of Eldin and Collins,[2] Low,[3] Wessel,[4] and Hahn, Averbach, Owen, and Cohen.[5] The point itself is illustrated in Fig. 1. Curve Y is the yield stress in uniaxial tension

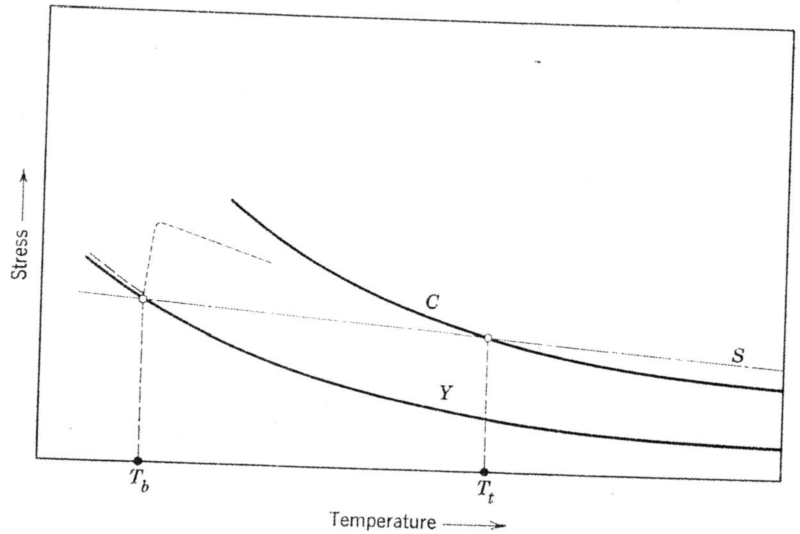

Fig. 1. Y is the uniaxial and C the notch-constrained yield stress, S the fracture stress for cleavage fracture. The dashed curve parallel to, and just above, Y is the curve of cleavage-fracture stress in uniaxial tension below T_b. The dotted curve shows the locus of ductile fractures of notchless specimens above T_b.

as a function of temperature; in ferritic steels, it rises rapidly with falling temperature. The classical theory of fracture in potentially ductile materials was based on the Mesnager-Ludwik principle,[6,7] according to which fracture would occur when the tensile stress reached a critical value not identical with the yield stress in tension. In Fig. 1, let S be this value (the fracture strength) as a function of the temperature. If the geometrical relationship of the curves Y and S is as assumed in the figure, the material is ductile above and brittle below the embrittlement temperature T_b; this is the basic idea of the Davidenkov-Wittman theory[8] of the transition temperature. Above the embrittlement temperature, cleavage fracture can still occur if the specimen contains a notch or crack sharp enough to give rise to "plastic constraint" superposing a triaxial tension and raising the greatest tensile stress during plastic yielding to the level given by the curve C, which intersects S at the "transition temperature" $T_t > T_b$. This is the Mesnager-Ludwik theory of notch brittleness. The highest constraint factor (ratio of C to Y) that can be produced by a crack or a notch is about 2.6 to 3.3;[9] therefore, a material cannot be notch-brittle if S is more than about three times Y.

The remarkable addition to this classical picture due to the work of Eldin and Collins and their successors is indicated by the dashed curve in Fig. 1. Below a critical temperature, which may be identified with that of embrittlement, the fracture stress is nearly identical with the yield stress. This suggests that plastic deformation is an essential factor in the cleavage-fracture process; it may nucleate cracks of the critical size required by the Griffith fracture condition, or it may be able to propagate smaller cracks by a mechanism different from that of the Griffith theory. Above the temperature of embrittlement (of an unnotched specimen), the fracture is substantially of the ductile type, and the fracture stress, as indicated by the dotted curve, is considerably higher than the yield stress Y of the undistorted material. The cleavage-fracture strength, of course, must be higher than the curve S; thus, a considerable amount of plastic deformation must raise the cleavage strength. This is a familiar phenomenon; its German name is "Reissverfestigung" ("strain strengthening").[10] In all probability, it is due to the dissection of the cleavage planes by plastic deformation,[11] for example, by the appearance of screw dislocations piercing the cleavage planes. The cleavage strength above the temperature T_b can be obtained from experiments with sharply notched specimens; the available experiments do not seem to give a conclusive picture, and a careful investigation of this point is desirable.

Plastic Crack Nucleation and Plastic Crack Propagation

In the contributions to this volume by Cottrell, by Hahn, Averbach, Owen, and Cohen, by Stroh, Gilman, and Parker, dislocation reactions have been discussed by which cleavage planes are forced apart until the resulting crack reaches the size at which it can propagate by the release of elastic strains, in the manner envisaged in the Griffith theory. Such processes are distinctly different from the first plastic mechanism of cleavage fracture suggested in 1934,[1] the starting point of which was the circumstance that crystals of NaCl, Zn, or Cd were observed to fracture by cleavage at tensile stresses so low that, in order to explain them by means of the Griffith formula, crack length many times greater than the diameter of the crystal would have been necessary. It was plausible to conclude that, in such cases, cleavage crack propagation occurred in a way fundamentally different from that assumed in the Griffith theory, by means of plastic deformation. The particular crack-propagation

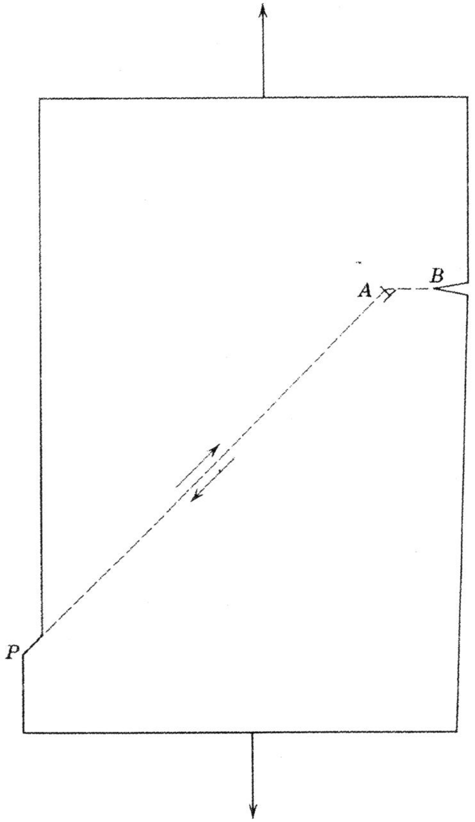

Fig. 2. Crack propagation by interaction with edge dislocation A.

mechanism suggested for NaCl, KCl, and similar crystals is illustrated in Fig. 2. A crystal of cubic orientation is under uniaxial tension in the [100] direction; at its surface there is a crack too small to satisfy the Griffith condition at the observed value of the tensile strength. If slip starts by the propagation of an edge dislocation from a point P towards the tip B of the crack, a high tensile stress arises when the dislocation A comes close to B. If the distance AB is small enough, cleavage may occur between A and B with a consequent increase in the crack length by AB. If a number of sufficiently closely spaced slip planes are active,

the process may repeat itself, and the crack length may increase until it reaches the Griffith value; propagation by elastic strain release then takes over.

The requirements for this type of plastic crack propagation are not as exacting as they may appear at first. In reality, the configuration is not two-dimensional as in Fig. 2; the depth of the crack will in general vary along the surface, and the slip plane will intersect the crack-tip contour; consequently, the propagation condition will certainly be satisfied along segments of the contour. In addition, the stress resulting from the dislocation is intensified by the stress concentration of the crack. The stress-concentration factor, of course, is lower than that of a crack of depth c in a uniformly stressed volume; its order of magnitude is probably that corresponding to a crack of depth $AB = d$, or

$$q \simeq 2\sqrt{d/\rho} \tag{1}$$

where ρ is the radius of curvature of the crack tip. The tensile stress produced by the dislocation A at B is of the order

$$\sigma \simeq \frac{Gb}{2\pi(1-\nu)}\,\frac{1}{d}\cos\theta \approx \frac{Gb}{2\pi}\,\frac{1}{d} \tag{2}$$

and since $\rho \approx b$, the mean tensile stress in the cleavage plane between A and B is of the order

$$\bar{\sigma} = G/\pi\,\sqrt{b/d} \tag{3}$$

Thus the effective dislocation-stress concentration factor of the crack is of the order of 10 for $d = 100b$.

The mechanism shown in Fig. 2 seems identical with that observed by Parker [12] in MgO crystals (see Figs. 3 and 4 of Chapter 10). As indicated in Fig. 3, two broad bands of slip meet along the surface crack AB. Parker attributes this to the action of the Cottrell mechanism of dislocation coalescence; however, this does not seem likely for the following reason. The crack could not have started in this way at the surface point B, because the volume in which the coalescing edge dislocations could have produced a tensile stress would have been outside the crystal. On the other hand, if the crack had been produced in the interior at point A by the coalescence of dislocations in the slip planes AC and AD followed by cleavage crack propagation outward, the broad bands of slip would not have appeared. Yet they are there, practically covering the length of the crack; this seems to be evidence of a crack-propagation mechanism of the type seen in Fig. 2.

Another interesting piece of evidence in favor of the operation of a non-Griffith type of plastic crack-propagation mechanism is seen in Fig. 4,

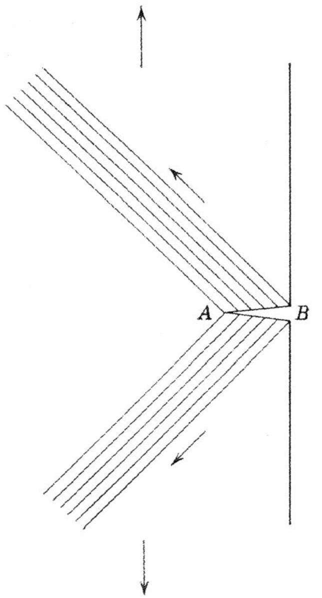

Fig. 3. Cleavage crack propagation in MgO according to Parker. The propagation mechanism seems to be that shown in Fig. 2.

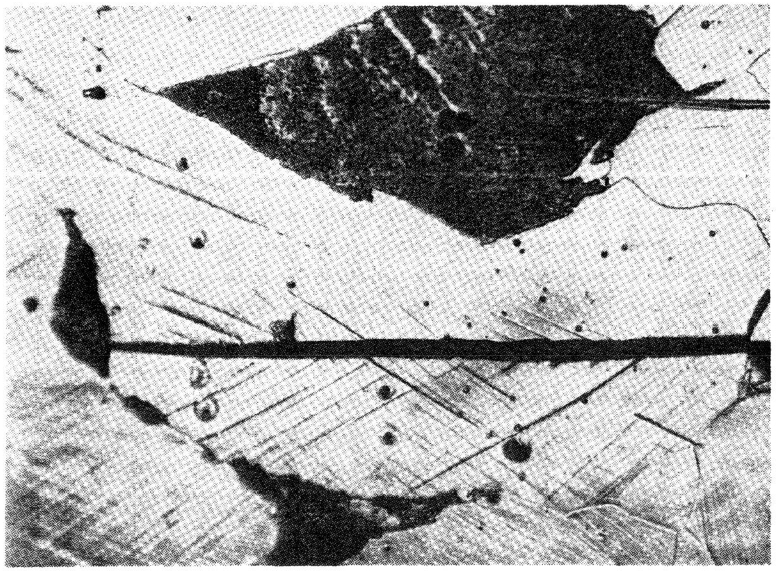

Fig. 4. A microcrack in Steel E strained 4% at −152°C. (From Hahn, Averbach, Owen, and Cohen.[5])

which is reproduced from the chapter of Hahn, Averbach, Owen, and Cohen, with kind permission of the authors. Whether the deformation markings along the crack (in a grain of polycrystalline iron) are due to slip or twinning, they seem to have arisen in the course of crack propagation, indicating that the crack has probably widened by plastic deformation near its tip as this progressed, rather than by a pure elastic cleavage process following the creation of a crack of critical length by dislocation coalescence. Evidently, the shear stresses around the crack tip *B* in Fig. 2 are such as to attract edge dislocations, hinder their movement beyond *B*, and thus give rise to pile-ups. The stress concentration of the crack itself, therefore, promotes dislocation movements that may give rise to propagation of the crack. This is of interest because it is usually assumed that plastic deformation is bound to cause a "blunting" of the crack and hinder its propagation. This is true only for simple Griffith-type crack propagation in a plastic continuum. In a crystal, as shown by Fig. 2, plastic deformation may increase the stress locally rather than leveling it.

Intracrystalline and Transcrystalline Crack Propagation

In view of the evidence of plastic deformation being a significant factor in the cleavage fracture of steel, at least in certain temperature ranges, and of the likelihood of a plastic crack-propagation mechanism fundamentally different from that of the Griffith type, crack propagation within an individual grain may not require the pre-existence, or the creation by some dislocation reaction, of a crack of the critical Griffith size. Extremely small cracks, of which there must be many in every grain, may grow by a plastic propagation mechanism until they reach the critical size, or until they are arrested by the nearest grain boundary. If the grain diameter is greater than the critical Griffith crack size, the crack may change over to a largely elastic propagation mechanism while running in the grain; however, when it reaches the grain boundary, it may stop because the work of propagation across the boundary is far greater than that within the grain. When a crack crosses a boundary, it splits up into many cracks running in parallel cleavage planes, a process that requires a considerable amount of plastic tearing and pulling. The average amount of plastic work in the surface of an apparently purely brittle fracture in a hot-rolled steel plate can be estimated by measuring the mean plastic work from the distortion of X-ray diffraction spots, obtaining the depth of the plastically distorted layer by etching off the surface in several steps, and repeating the X-ray photographs after each step. In this way, the work of creating a surface of brittle fracture per unit area in the transition range of a ship plate has been determined [13] and found to be roughly 2×10^6 ergs/cm^2; with decreasing temperature, its magnitude will probably decrease considerably.

Since cleavage-crack propagation within the grain is probably not a process of the Griffith type, no critical crack length being involved, it must be asked whether crack propagation across the grains is of the plastic or of the elastic type. In the latter case, the fracture stress may be given by an expression of the Griffith type with the mean plastic crack-wall formation work p taking the place of the surface energy in the Griffith formula. The answer to this question has been obtained in a series of experiments [13] in which a brittle crack was produced in the edge of a ship plate at the temperature of liquid nitrogen and then the plate fractured in tension at room temperature, which was within the transition range. The brittle crack was never observed to propagate directly as a brittle crack: First, intense plastic deformation developed around its root, followed by some ductile crack propagation, and then a new brittle crack arose in the center of the ductile crack front and spread rapidly across the plate. Occasionally the brittle crack stopped before fracture was complete, apparently because of the drop of load during propagation; in this case, an increase of the load again produced strong plastic deformation at its tip, followed by some ductile crack propagation which soon changed to brittle fracture. Once the brittle crack started to run fast, no visible plastic deformation occurred, except perhaps in a narrow strip at the surface of the plate. The plausible and so far only existing explanation of this observation was that the development of plastic constraint required for raising the tensile stress to the level of the cleavage strength (Fig. 1) demanded a considerable plastic deformation around the tip of the crack. Once the "constrained yield stress" was high enough to start a cleavage crack, the crack soon acquired a speed so high that the yield stress in front of it was raised to the value of the cleavage strength by the velocity effect alone, and plastic deformation for producing a strong triaxiality of tension became unnecessary. This means a considerable change in the classical theory of brittle fracture in steel. It seems that the triaxiality of tension, upon which the Mesnager-Ludwik theory of notch brittleness was based, is not the salient point; strong triaxiality may give rise to cleavage fracture, but only at the cost of so much plastic deformation that the fracture, while of the cleavage type, is far from being a brittle one. It seems that, in the transition range, cleavage fracture becomes brittle fracture only when the crack acquires a velocity high enough to raise the yield stress to the cleavage-fracture level through its velocity dependence.

This provides an answer to the question of whether transcrystalline crack propagation is of the elastic strain release or of the plastic type. In the transition range, a cleavage fracture can become a brittle fracture only if the tensile stress is raised to the fracture level mainly by the velocity effect. Under relatively slow loading, however, the crack can acquire high velocity only if the work of propagation is supplied by the release of elastic energy: that is, if the fracture is basically of the Griffith type. Under such circumstances, therefore, brittle fracture in ferritic steel is bound to the fulfillment of two conditions:

1. The stress must be high enough to initiate cleavage fracture.
2. It must be high enough to satisfy the condition

$$\sigma_t \approx \sqrt{Ep/c} \tag{4}$$

of transcrystalline crack propagation by elastic strain release, which is the condition of crack acceleration and thus of *brittle* cleavage fracture.

Very often, the first condition is satisfied but not the second; in this case, cleavage fracture occurs in isolated grains or small patches without producing cleavage fracture across the entire cross section. These are the cases investigated by Hahn, Averbach, Owen, and Cohen; the familiar occurrence of small cleavage patches in largely fibrous surfaces of fracture in Charpy specimens is another example.

If the values of the observed fracture stress, of the surface energy of the cleavage plane γ, and of Young's modulus are inserted in the Griffith formula $\sigma_t \approx \sqrt{E\gamma/c}$, crack lengths c of the order of a micron are obtained for low-carbon steels. Consequently, if the plastic crack-wall formation work p is used in Eq. 4, the necessary crack length will be of the order of 1 mm. Occasionally this has been regarded as a difficulty; actually, however, it is in agreement with the fact that brittle fracture cannot be obtained in the transition range in a low-carbon steel unless a notch or crack of a depth at least of the order of magnitude of a millimeter is present. The lengths of the microcracks observed by Hahn and associates are not related to the crack length demanded by Eq. 4, nor to any critical crack length that may be required for the initiation of a cleavage crack within a grain. A microcrack is simply a crack that cannot propagate further in a case where the intragranular crack-propagation condition is fulfilled but not the transgranular condition. Its length, as a rule, is of the order of the grain diameter and has no particular relevance to the fracture problem. With decreasing temperature the transcrystalline crack-formation work p decreases, and with it the necessary crack length. If the critical crack length demanded by Eq. 4 is reduced to the order of magnitude of the grain diameter, the first intracrystalline crack results in complete fracture, and microcracks are not observed; above the transition range (for a notched or unnotched specimen) cleavage fracture does not occur, and so microcracks arise only within the transition range, as has been observed by Hahn, Averbach, Owen, and Cohen.

Brittle Fracture and the Yield Phenomenon

The static upper yield point in low-carbon steels can rise as high as twice the lower yield point; X-ray evidence indicates that it may amount to two or three times the lower yield point if the stress is concentrated in a small volume. It was suggested, therefore,[9] that brittle fracture in steels may be a consequence of the yield phenomenon: The tensile stress may be raised to the cleavage stress level without any plastic constraint or velocity effect. More recently, Cottrell [14] has suggested that the yield phenomenon may be fundamentally connected with brittle fracture in steels.

The presence of a high enough upper yield point may in fact be an important factor in initiating cleavage fracture. In particular, it may account for the remarkable and extremely important phenomenon of the *Greene effect*, which will be discussed in the following section. A general and fundamental connection between brittle fracture and yield phenomenon in ferritic steels, on the other hand, seems very unlikely. Except at very low temperatures, cleavage fracture in a notch-impact specimen or in the precracked tensile specimens discussed previously is introduced by copious plastic deformation which must have wiped out the upper yield point. Moreover, it is known that the removal of the yield point by prestraining does not cause any fundamental change in the fracture behavior of steels. It seems permissible, therefore, to eliminate the yield phenomenon from the list of possible ultimate causes of the brittleness of ferritic steels.

The Greene Effect

As mentioned previously, a cleavage crack in a low-carbon steel plate does not propagate under tension directly as a cleavage crack except at very low temperatures; it gives rise to large plastic deformations and some fibrous crack propagation before the plastic constraint produced can start the propagation of a secondary cleavage crack. This is quite different from the very familiar case of catastrophic service fractures where a cleavage crack starts to run with no visible amount of preceding plastic deformation. Such entirely brittle fractures (apart from the plastic surface work p) have been produced in the laboratory first by Greene,[15] and subsequently by Weck.[16] In both cases, the specimens used were welded together from two steel plates after a notch was cut (for example, by a jeweler's saw) into an edge to be welded. Fracture occurred sometimes while the welded specimen was being cooled clamped to a rigid frame.

The Greene effect is of the greatest importance in engineering; at the same time, it presents one of the last major problems of brittle fracture in steels. It cannot be a consequence of a metallographic change due to heating, for heating alone cannot produce fully brittle crack propagation. It has been suggested [17] that the embrittlement might be a consequence of strain hardening at the tip of the crack: During welding, the edges of the plates suffer plastic compression by thermal expansion, which is followed during cooling by plastic extension (parallel to the weld seam). However, the plastic deformations and the strain hardening around a crack in a weldless plate broken in tension are far greater without causing comparable embrittlement.

Whether or not a steel plate is in the annealed (hot-rolled) state, it develops a sharp yield point in the neighborhood of the weld seam. In itself this cannot produce embrittlement of the Greene type; however, in a welded plate, two additional circumstances are present. The tip of a crack near the weld, as just mentioned, suffers some plastic deformation and strain hardening, and both upper and lower yield points after strain aging are higher in a strain-hardened than in a fully annealed steel.[18] Unless the cleavage strength increases with plastic strain faster than the upper yield point, therefore, the tendency to brittle fracture should increase.

Another possible cause of the Greene effect is the superposition of plastic constraint and the yield phenomenon. In a strain-aged plate containing a crack, the tensile stress rises to the value σ_u of the upper yield point before plastic deformation starts; afterward, plastic constraint develops, and the highest tensile stress rises to

$$\sigma_c = q\sigma_y \tag{5}$$

where q is the constraint factor of the crack and σ_y the effective mean uniaxial yield stress in the strain-hardened state. However, if tension is applied during the aging treatment, as in a welded specimen, the tensile stress rises to the value

$$\sigma_{\max} = q\sigma_u \tag{6}$$

when additional tension is exerted by an applied load or by further thermal contraction (for example, if, as in Weck's experiments, the specimen is cooled while clamped to a rigid frame). The upper yield point can be twice as high as the lower yield point, and q may be between 2 and 3. Consequently, the tensile stress can rise only to $2\sigma_y$ or $3\sigma_y$ in a strain-aged notched plate, but it can reach $4\sigma_y$ to $6\sigma_y$ in a notched plate strain-aged while under a tensile load capable of producing plastic deformation around the notch.

It may be mentioned that the Greene effect has been observed recently by Mylonas, Drucker, and Brunton [19] in a somewhat different arrangement. A square plate of steel was notched in the centers of two opposite sides and compressed in a direction parallel to its plane and perpendicular to the line connecting the notches; in this way, a residual tensile stress was produced at the notch tips. Subsequently the specimen was welded to steel gripping-plates along its unnotched sides and fractured in tension. The mean tensile stress at fracture was often far below the lower yield point, and the fracture was unusually brittle. Strain aging (during welding) under residual tensile stress would account for these observations.

Possible Connection with the Snoek Effect

When a ferritic carbon steel crystal is under tension in the [100] direction, carbon atoms migrate from positions between the two nearest Fe-neighbors in the [010] and [001] directions to positions near those of the [100] direction. The result is the development of a tensile stress in the Fe-lattice which does not contribute to plastic yielding; consequently, the effective disrupting stress is raised by the "internal" stress due to the carbon atoms, and the cleavage strength in the [100] direction is reduced. However, the effect seems to be too small to be of practical importance.

Classical and Dislocation Theories of Brittle Fracture

From the preceding considerations, it is seen that the classical theory of brittle fracture in ferritic and similar materials has not been superseded by recent developments. Most of the remarkable fracture properties of ferritic materials (the embrittling effect of notches, the transition phenomenon, the velocity effect, the Greene effect) can be understood as direct consequences of a strongly temperature- and velocity-dependent yield stress and of a cleavage strength that is only slightly higher than the uniaxial yield stress (in the case of a polycrystalline material). The ultimate cause of the strong temperature and velocity dependence is, of course, of great interest and is directly related to properties of dislocations. It is probably the result of the temperature and velocity dependence of the driving stress (frictional stress, Peierls-Nabarro stress), which in turn may be a consequence of the strong directional bonding forces resulting from the incomplete *d*-shells in the atoms of the transition metals. A clearer picture is obtained, however, by means of simple macroscopic concepts such as the yield stress and the cleavage strength when their

mutual relationship is sufficient for understanding a fracture phenomenon. The problem of the molecular background of the macroscopic quantities may then be treated separately.

Of course, the near-identity of yield stress and fracture stress in α-iron at low temperatures is a phenomenon outside the classical framework. According to the view given above, it is also probably outside the semi-classical scheme described in the papers of Cottrell and of Hahn, Averbach, Owen, and Cohen. Instead of a Griffith-type propagation of cracks, nucleated by a sharply localized dislocation reaction, it seems more likely that cracks of subcritical size (in the sense of the Griffith theory) are propagated by a plastic mechanism of the type shown in Fig. 2, based on the interaction of the crack with dislocations. However, this does not seem to invalidate the basic features of the classical theory: It merely adds the interesting point that plastic deformation makes a significant contribution to the applied stress as far as overcoming the cleavage strength is concerned.

Possible Causes of Ferritic Brittleness

The fracture behavior of ferritic steels is not merely an intermediate case between full brittleness and unlimited ductility. It represents a strange duplicity, an instability that can change the highly ductile behavior seen in the common tensile test to almost complete brittleness merely by the effect of a notch. The fundamental cause of this behavior has been known for several years: [13] It is the circumstance that, as soon as a small patch of cleavage has arisen and the energy release condition of Eq. 4 is satisfied, the crack accelerates until the velocity-increased yield stress rises to the level of the cleavage strength, so that plastic constraint becomes dispensable and the work of fracture drops to a very low value. The physical basis of this process is the high velocity and temperature dependence of the yield stress, resulting probably from the strong temperature dependence of the dislocation driving stress, which in turn is probably connected with the body-centered lattice and the *d*-electron bonds.

The striking difference between ferritic and austenitic steels is mainly a consequence of the far smaller velocity and temperature dependence of the yield stress of γ-iron, possibly caused by the presence of half-dislocations with less tendency to degenerate by "cleaving" on the tensile side at lower temperatures and thus to require a higher driving stress. That a body-centered lattice in itself need not have a particularly low cleavage strength is indicated by the behavior of sodium and potassium. For these metals (particularly for Na), the temperature of liquid nitrogen corresponds roughly to room temperature in the case of iron. Yet in liquid nitrogen, Na and K do not show any sign of a cleavage fracture, not even if the specimen has been provided with a very sharp notch by indentation with a razor blade. The possibility that the proneness of α-Fe to cleavage fracture may be due to a characteristic dislocation reaction cannot be excluded; it seems more likely, however, that its cause lies directly in the cohesive properties of a body-centered lattice of transition-metal atoms.

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