9. Brittle Fracture and the Yield-Point Phenomenon

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ABSTRACT

A modified theory of Cottrell locking is further developed and compared qualitatively and, where possible, quantitatively with experiment. A simple inversion of the equation relating yield point to testing temperature leads to a condition for brittle fracture which is examined in the light of recent experimental work, particularly on iron and steel, molybdenum, and chromium. Constants evaluated from a consideration of experimental evidence on yielding and brittle fracture are consistent. The predictions of the theory with respect to the effect of grain size, loading rate, work hardening, and variations in yield strength on the ductile-brittle transition temperature are shown to be in accord with experiment.

Introduction

Various dislocation theories of fracture have recently been proposed.[1–13] Most of the arguments have been based on Stroh's concept that the stresses from a pile-up of dislocations at obstacles such as grain boundaries can crack the material adjacent to the pile-up. Cracking will not occur if the applied stress is too small or if the stress from the pile-up is relieved by the operation of a dislocation source or sources near the head of the pile-up. Such a crack, once formed, will grow in a brittle manner, provided the Griffith condition for crack propagation is satisfied and the growth of the crack is not halted by plastic deformation around its apex. Thus fracture requires that dislocations near the pile-up be locked at the time the pile-up forms. In the case of brittle fracture, the most probable mechanism for such locking is that postulated by Cottrell.[14–16] In ductile fracture, locking is provided by the stresses developed as a result of plastic deformation. The well-known ductile-brittle transition is in accord with

Cottrell's model, since the stress at which dislocations pull free from their solute atmospheres depends strongly on temperature.[15]

If the above views are substantially correct, the incidence of brittle fracture could well be closely related to the question of the temperature dependence of the yield point. This problem was originally investigated by Cottrell and Bilby,[15] who predicted a continuous increase in yield stress as the temperature is lowered to absolute zero. A recent modified treatment [17] indicates that the yield stress should approach a limiting value asymptotically at low temperatures, and an extension of this analysis [18] can explain the phenomena of microstrain and delayed yielding. In the present chapter, these new ideas are summarized and extended, and the theoretical predictions are compared with experimental results for a number of metals, both body-centered and face-centered cubic, in which yield points occur. Implications of the theory regarding brittleness and the ductile-brittle transition are discussed.

Theoretical Analysis

Louat [17] has analyzed Cottrell locking while taking into account the following facts: (1) The Maxwell-Boltzman distribution of solute atoms around dislocations breaks down near the dislocation center, and (2) the Peierls-Nabarro width of the dislocation decreases, and hence its elastic energy increases, as a solute atom approaches the dislocation center. On this basis, it was proposed that solute atoms near the center of the dislocation may be located in either of two (for simplicity) energy states having almost equal energy but markedly different restraining effects on the dislocation. The distribution of solute atoms between these levels was assumed to reach equilibrium rapidly even at low temperature because of the extreme dilatation of the lattice in such regions. This assumption is supported by evidence of low activation energies for diffusion along dislocation lines which has recently been provided by measurements of the diffusion of zinc along low-angle boundaries in silver.[19] From these considerations and an assumption that yielding follows when a sufficient length of dislocation exists with all its solute atoms in higher energy (least restraint) levels, a relation between yield strength and temperature was derived.

The analysis of the model described above was approximate, and in two particulars the approximation can be improved. First, the use of direct summation rather than integration in deriving the condition for yielding leads to the more accurate result

$$MN\alpha(1-\alpha)^{L/b} = 1 \qquad (1)$$

where MN is the number of atomic lengths of dislocation that exist in the region of the crystal in which maximum shear stresses occur, α is the occupation probability of solute atoms in the lower energy level, L is the length of dislocation in which the solute atoms are either in a higher energy level or are absent, and b is the magnitude of the Burgers vector. Again, it was considered that, in the presence of a sufficiently large stress, this length L would tear away from the restraining forces primarily at its ends. This gave the following expression for the yield stress:

$$L\sigma b = (\theta L\sigma_0 b/n) + 2\sigma_0 b^2 \qquad (2)$$

where θ is the over-all atmosphere density, σ_0 is the shear stress equivalent of the maximum force exerted by the solute atoms, and n is the ratio of the restraining effect of solute atoms in the lower levels to that of those in the higher levels. This approximation may be improved by including the effect of the stress on the "holding" atoms, namely, those that provide the force $2\sigma_0 b^2$. We then have

$$(L + 2b)\sigma = (\theta L\sigma_0/n) + 2\sigma_0 b \qquad (3)$$

Combining Eqs. 1 and 3 and incorporating a constant C representing the effect of internal stresses arising from precipitate particles and the like, we obtain

$$\sigma = 2\sigma_0 \frac{\ln[1/(1-\alpha)]}{\ln[MN\alpha/(1-\alpha)^2]} + (\theta\sigma_0/n) + C \qquad (4)$$

This result takes no account of the time for which the load has been applied, but this can be done in the following way. In a time t_0, of the order of that required for a particular solute atom to move from a lower to a higher energy level, the whole atmosphere arrangement involving MN atomic lengths of dislocation will have changed. It follows that, in a time t, the effective number involved in the sense of Eq. 1 is $\alpha MNt/t_0$. Applying this concept to the determination of yield stress where the load is applied at a constant rate K, one obtains the relation

$$\begin{aligned} dp(t)/dt &\approx [-MN\alpha(1-\alpha)^{L/b}]p(t)/t_0 \\ &\approx [-MN\alpha(1-\alpha)^{(2\sigma_0/Kt)-2}]p(t)/t_0 \end{aligned} \qquad (5)$$

where $p(t)$ is the probability that yielding will not have occurred up to time t. Solving this equation for $p(t) = \frac{1}{2}$, we have

$$\sigma = (2\sigma_0/A)\ln[1/(1-\alpha)] + D \qquad (6)$$

where $D = (\theta\sigma_0/n) + C$ is virtually independent of temperature and $A = \ln[\alpha MN\sigma t/2(1-\alpha)^2\sigma_0 t_0]$ can be treated as a constant at normal rates of loading if t/t_0 is so large that the variations in time needed to reach the yield point have no appreciable effect.

An additional refinement [18] to allow for the bending of the source under the action of the applied stress can be introduced here. Writing $\bar{c} = \mu b/l$ as the stress necessary to operate a Frank-Read source of length l in the absence of a solute atmosphere and ϕ as the semiangle subtended by the bent length of dislocation, Eq. 6 becomes

$$\sigma = (2\sigma_0/A) \ln \{1/[1 - \alpha(\sin \phi/\phi)]\} + \bar{c} \sin \phi + D$$

This can be written in the form

$$\sigma = (2\sigma_0/A) \ln \left\{\frac{\beta + \exp (W/kT)}{\beta + [1 - \theta(\sin \phi/\phi)] \exp W/kT}\right\} + \bar{c} \sin \phi + D$$

where k is the Boltzmann constant and β is the ratio of the number of available sites in the higher level to those in the lower energy level and is related to α through the expression

$$\alpha = \frac{\theta \exp (W/kT)}{\beta + \exp (W/kT)}$$

in which W is the energy difference between the two levels.

Over most of the temperature range we shall consider, the rate of bending of the source will be sufficiently slow for its effect to be neglected, and we can write

$$\sigma = (2\sigma_0/A) \ln \left[\frac{\beta + \exp (W/kT)}{\beta + (1 - \theta) \exp (W/kT)}\right] + D \qquad (7)$$

In this analysis no account has been taken of the solute atoms lying outside the central region of the dislocation. This portion of the atmosphere (secondary atmosphere) will reduce the stress fields, and hence the interaction energy, at the center of the dislocation. In general terms we expect, therefore, that the presence of this additional atmosphere will reduce the value of σ_0 (the maximum slope of the potential well in which the central atoms are located) relative to the case in which the secondary atmosphere is absent.

Provided $(1 - \theta) \ll 1$ (which is normally to be expected), a certain temperature range can exist (the "middle" range) over which

$$\exp (W/kT) \gg \beta \gg (1 - \theta) \exp (W/kT)$$

Under these conditions, Eq. 7 reduces to

$$\sigma = (2\sigma_0/A)[(W/kT) - \ln \beta] + D \qquad (8)$$

or in other words, in this range σ is proportional to $1/T$. Also, at very low temperature, that is, when

$$(1 - \theta) \exp (W/kT) \gg \beta$$

Eq. 7 becomes

$$\sigma = (2\sigma_0/A) \ln [1/(1 - \theta)] + D \qquad (9)$$

so that σ is approximately invariant with T since θ, which is determined by macroscopic diffusion processes, must be virtually constant at these temperatures. The temperature T_θ at which the yield stress becomes invariant is given by

$$\beta = (1 - \theta) \exp (W/kT_\theta) \qquad (10)$$

or

$$T_\theta = W/\{k \ln [\beta/(1 - \theta)]\} \qquad (11)$$

Finally, at high temperatures, one expects from Eq. 6 that the yield stress would again be comparatively insensitive to changes in temperature.

Therefore, the above theory predicts as follows regarding the temperature dependence of the yield stress in metals where strong Cottrell locking occurs. A range of temperature exists over which the yield stress is proportional to $1/T$: below (at lower temperatures) and, to a lesser extent, above this "middle" range, the yield stress is relatively constant. The atmosphere density θ has no influence on the stress values in the middle range, but determines the temperature T_θ, which is the low-temperature limit of the linear portion and also the value of the (constant) yield stress at temperatures below T_θ. A high value of θ should produce a low value of T_θ and a high value of stress below T_θ. Equation 8 predicts that the slope of the curve in the middle range should be proportional to W, σ_0, and A, of which W and σ_0 are the more important and might be expected to vary with V, the interaction energy between solute atoms and dislocations. Therefore, the slope should be defined principally by the elastic constants of the material and also, to a lesser extent, by the amount of secondary atmosphere present. Finally, ln β and D can produce a temperature-insensitive change in the position of the curve. The predicted curve for yield stress plotted against $1/T$ where $(1 - \theta) \ll 1$ is given in Fig. 1*a*; smaller values of θ would be expected to restrict or eliminate the linear part as in Fig. 1*b*. It is clear that the foregoing analysis is relevant where deformation occurs solely by slip; it would also seem that the analysis should remain valid when deformation twinning or brittle fracture occur. For example, a later section of this chapter is devoted to an extension of the Mott-Stroh theory of brittle fracture in metals, from which one concludes fairly confidently that the initial step must be the formation of an embryonic crack by a dislocation process. Also, it has been proposed by Cottrell and Bilby [20] that mechanical twinning is a dislocation process involving the operation of a Frank-Read source, and it has been indicated by Louat [21] that its temperature dependence should differ only in detail from that of slip. Hence, irrespective of whether slip, twinning, or brittle fracture occurs, the fundamental process is probably the release of dislocations from Cottrell atmospheres.

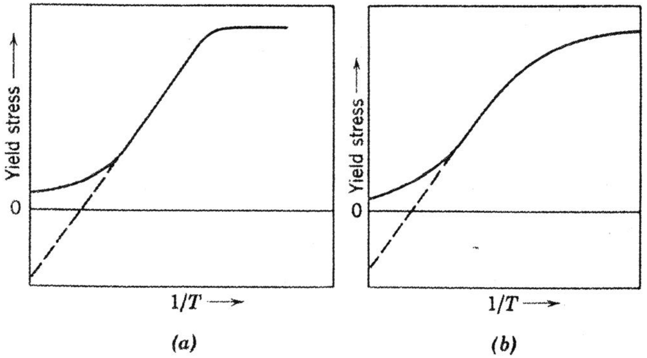

Fig. 1. Theoretical yield stress-temperature relationships according to Eq. 7.

Comparison of Theory and Experimental Results

Figures 2 to 8 plot yield stress and brittle-fracture stress, as appropriate, against the reciprocal of the test temperature for those metals in which evidence of Cottrell locking has been observed. Experimental details are given in Table 1. The arrows on the curves indicate the lowest temperature at which ductility was detected under the test conditions used. If no arrow was used, it means that the specimens were ductile,

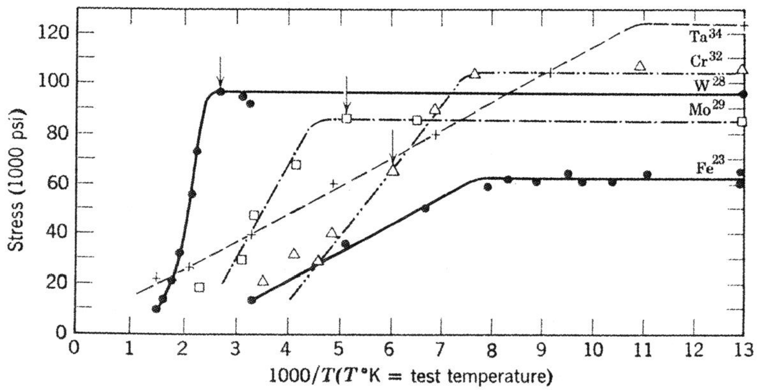

Fig. 2. Curves of stress plotted against $1/T$ for some body-centered cubic metals.

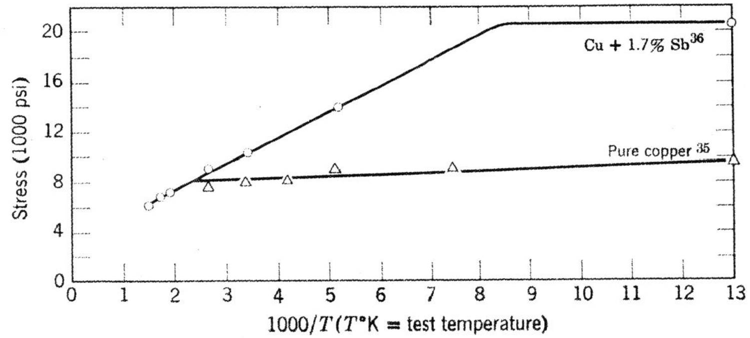

Fig. 3. Curves of stress plotted against $1/T$ for some face-centered cubic metals.

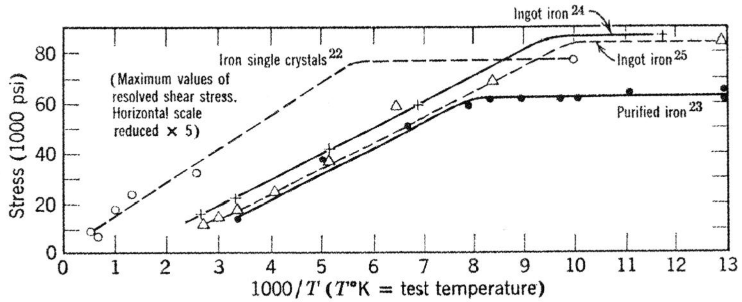

Fig. 4. Curves of stress plotted against $1/T$ for iron.

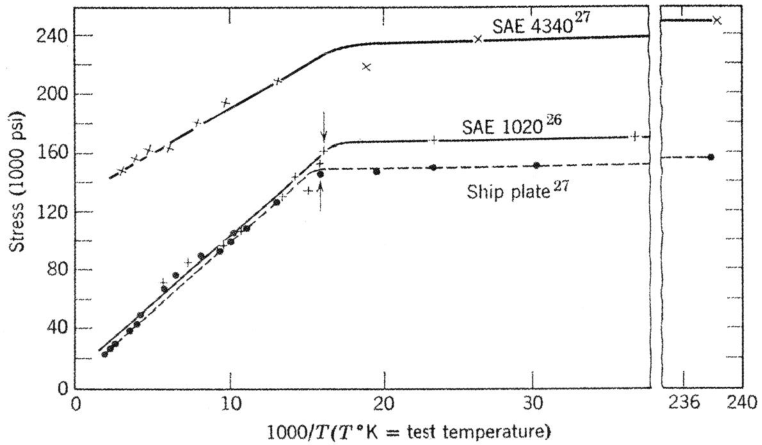

Fig. 5. Curves of stress plotted against $1/T$ for three steels.

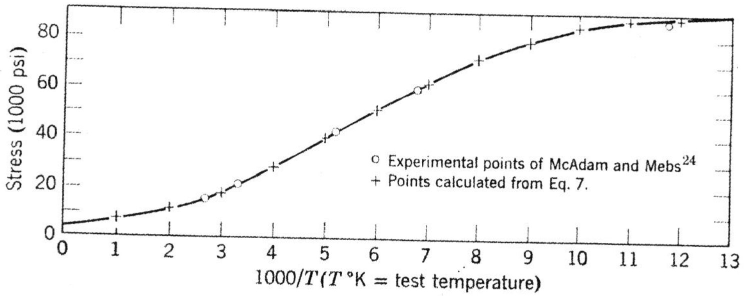

Fig. 6. Curves of stress plotted against $1/T$ for ingot iron.

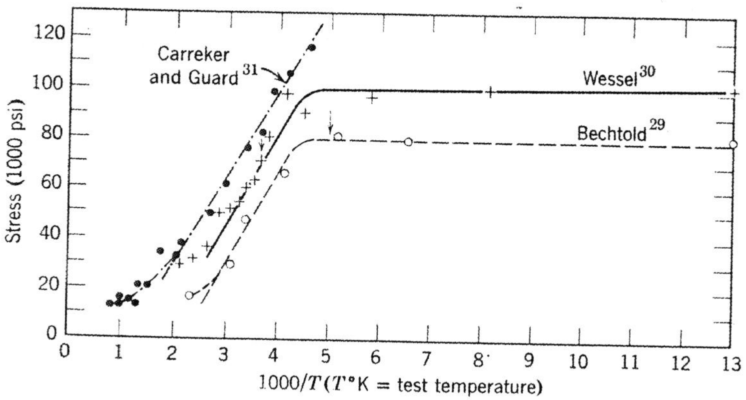

Fig. 7. Curves of stress plotted against $1/T$ for molybdenum.

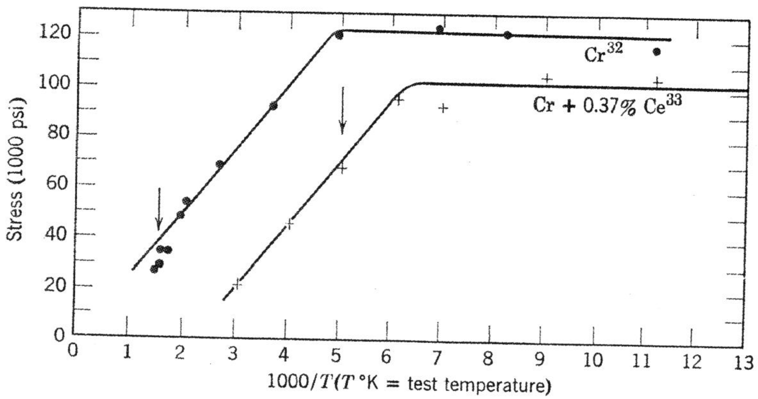

Fig. 8. Curves of stress plotted against $1/T$ for chromium.

TABLE 1. Metals in Which Cottrell Locking Is Observed

Material	Preparation *	Test Condition	Important Impurities
Iron single crystals [22]	Strain-anneal (specimens 0.062-in. diameter)	As-received	C 0.0027 O_2 0.001_6 N_2 0.001_6
Purified iron [23]	Cold-rolled and swaged (specimens 1-mm diameter)	Wet-hydrogen treated	C 0.002 † N_2 0.0005 O_2 0.0034
Ingot iron [24]	Cold-drawn	Recrystallized at 950°C	C 0.0015
Ingot iron [25]	Hot-rolled and cold-drawn	Recrystallized	C 0.02 O_2 0.058 N_2 0.002 H_2 0.0005
Steel [26] (SAE 1020)	Hot-rolled	As-received	C 0.2 (nominal)
Steel [27] (SAE 4340)	—	Quenched and tempered 2 hr at ~600°C	C 0.3 Ni 2.64
Steel [27] (Ship plate)	—	—	—
W [28]	Rod swaged from sintered powder	Recrystallized at 1950°C	C 0.02 N_2 0.008
Mo [29]	Bar rolled from sintered powder	Recrystallized at 1150°C	C 0.014 O_2 0.0017 N_2 0.0056
Mo [30]	—	Recrystallized at 1250°C	C 0.007 N_2 0.12 O_2 0.002
Mo [31]	Arc cast, forged, and swaged (specimens 0.03 and 0.06 in. diameter)	Recrystallized at 1100°C	C 0.04 O_2 0.003 (nominal)
Cr [32]	Arc cast, extruded, and rolled (bend tests on strip 0.04 in. thick)	Recrystallized at 1100°C	O_2 0.02 N_2 0.002
Cr-Ce alloy [33]	Arc cast, forged, and rolled (bend tests on strip 0.04 in. thick)	Recrystallized at 1100°C	O_2 0.02 N_2 0.002 Ce 0.37
Ta [34]	Rod swaged from sintered powder	Recrystallized at 1700°C	C 0.01 N_2 0.01
Cu (O.F.H.C.) [35]	Hot-worked and annealed	As-received	—
Cu-Sb alloy [36]	Hot-worked (specimens $\frac{1}{8}$-in. diameter)	Annealed at 700°C, slow-cooled	Sb 1.7

Notes: * Specimens were conventional test specimens unless otherwise indicated.
† Analysis before wet-hydrogen treatment.

at least to a limited extent, throughout the range of temperature investigated.

Figure 2 shows that the shape of the stress against $1/T$ curves for body-centered cubic metals is as predicted by theory. In all cases, stress varies linearly with $1/T$ over the "middle" range of temperature, while at higher and lower temperatures the stress is approximately invariant. Again, in agreement with theory, the slopes of the linear parts of the stress-$1/T$ curves increase with increasing values of Young's modulus which are 27, 28.5, 36, 42, and 52×10^6 psi for Ta, Fe, Cr, Mo, and W, respectively.[37] Similar behavior is found in a face-centered cubic alloy (Cu + 1.7% Sb) which shows yield-point effects,[36] whereas the yield stress of pure copper is relatively insensitive to changes in temperature over the same range (Fig. 3).

Curves for a number of iron and steel specimens are shown in Figs. 4 and 5. Equation 7 was fitted to the experimental results of Eldin and Collins for SAE 1020 steel, and the values of the parameters that gave the best fit are listed in Table 2.

TABLE 2. Parameters for Best Fit

(In every case, $W/k = 600$, $\beta = 7$)

Material and Investigators	$(1 - \theta)$	D (1000 psi)	$2\sigma_0/A$ (1000 psi)
SAE 1020 Eldin and Collins	exp (−8.4)	45	18
Ship plate Wessel	exp (−7)	41	18
SAE 4340 Wessel	exp (−8.8)	155	10
Ingot iron McAdam and Mebs	0.05	0	31.6
Ingot iron Geil and Carwile	0.04	0	26
Purified iron Gibbons	0.21	0	40

With use of the same values of W and β, which must be constant for a particular strain-aging system (for example, carbon in iron), Eq. 7 was then fitted to the other results in Figs. 4 and 5; the parameters derived are again listed in Table 2. Figure 6 illustrates the agreement obtained between theory and experiment in the case of the results of McAdam and Mebs. Agreement in other cases was as good as this, with the

exception that the stresses at high testing temperatures for ship plate and SAE 4340 were lower than predicted. From this analysis it appears that the theory not only predicts the correct form of the stress-temperature relationship for Cottrell locking systems, but that quantitative agreement can be obtained in different materials by adjusting only those parameters that one would expect to vary because of such factors as composition and thermal or mechanical history.

From Table 2 and Figs. 4 and 5, it is apparent that the slopes of the linear parts of the stress versus $1/T$ curves ($2\sigma_0/A$) for the group of iron specimens and the group of steel specimens are relatively constant within each group but vary considerably from group to group. Any variation in slope must enter through A or σ_0 or be attributed to stress concentrations. Of these factors, neither A nor stress concentration appears relevant. Both theory and experiment indicate that A should be of the order of 50, and it is unlikely, in view of its logarithmic character, that it would change significantly under normal circumstances with its variable parameters, namely, testing rate and specimen size, entering through MN. Table 2 shows that, as a group, the steel specimens have smaller slopes and higher D values than the iron specimens, which would not be anticipated if stress concentrations were particularly important. However, as stated previously, the shape of the potential well in which the innermost solute atoms are located, and hence σ_0, would be expected to vary with the number of solute atoms in the secondary atmosphere. Furthermore, the existence of a strong secondary atmosphere would introduce an additional temperature-insensitive restraint on dislocation motion and thus lead to increased values for D. It therefore seems likely that the above variation in slope results because secondary atmospheres have an enhanced effect on the steel as compared with the iron specimens.

Insufficient experimental evidence is available to enable a quantitative comparison between theory and experiment to be made for the other materials that show Cottrell locking. However, there is one further general point of interest. Some of the materials (Ta, Cu), for which results are presented in Figs. 2 to 8, deformed entirely by slip over the range of temperature investigated, while others (purified iron, ingot iron, SAE 4340 Cu-Sb), although ductile throughout, showed evidence of twinning as well as slip below T_θ. Of those specimens which were brittle, one group [SAE 1020, ship plate, W, Mo (Bechtold), Mo (Carreker and Guard)] was brittle at and below T_θ, while the remainder [Mo (Wessel), Cr, Cr-Ce] became brittle at some temperature on the linear part of the curve of stress versus $1/T$. Thus the theoretical curve of stress versus $1/T$ can be obtained without a change in the mode of deformation, and even when such a change occurs or brittle fracture appears, the stress can still follow the predicted pattern. The best example of this latter behavior is the recrystallized chromium in Fig. 8, which gave the predicted type of curve even though it was brittle over the greater part of the temperature range investigated.

It is clear from this section that the main predictions of the theory are borne out for a wide variety of body-centered cubic metals and for a face-centered cubic copper alloy. The implications of the theory as regards brittleness and the ductile-brittle transition are discussed in the next section.

Brittleness and the Ductile-Brittle Transition

Stroh [1–4] has examined the conditions necessary for a crack to form from a pile-up of n edge dislocations at a boundary. He gives two requirements: (1) that $nb\sigma = 12\gamma$ (σ is the applied stress, b the Burgers vector, and γ the surface energy), and (2) that secondary dislocation sources near the head of the pile-up do not operate before condition (1) is satisfied. On this basis, he obtains an expression for the ductile-brittle transition temperature in terms of the activation energy for the operation of a secondary source. Since Stroh's analysis is based on the Cottrell-Bilby [15] model of the release of a dislocation from its atmosphere, it is pertinent to repeat and extend it by using the present model, which appears to be in better agreement with the experimental evidence.

Theory

For a particular source, Eq. 8 may be written in the form

$$\ln K = \ln \left[\alpha N\sigma^2\beta^{2\sigma_0/(\sigma-D)}\Big/2(1-\alpha)^2\sigma_0 t_0\right] - 2\sigma_0 W/[kT(\sigma - D)] \quad (12)$$

where $K = d\sigma/dt$ is the loading rate.

Consider a (secondary) dislocation source near the head of a pile-up. Following Stroh, we assume that, to prevent cracking, this secondary source must operate before the stress on it reaches a characteristic value $\bar{\sigma}$ ($\bar{\sigma} \approx 10^{10}$ dynes/cm^2 for iron). From Eq. 12, we then have as the approximate condition for the formation of a crack

$$\ln K_b \geq \ln \left[\alpha N\bar{\sigma}^2\beta^{2\sigma_0/(\bar{\sigma}-D)}\Big/2(1-\alpha)^2\sigma_0 t_0\right] - 2\sigma_0(W/kT)(\bar{\sigma} - D) \quad (13)$$

Here $K_b = K(l/S)^{1/2}$, $2l$ is the grain diameter, and S is the distance of the secondary source from the head of the pile-up. K_b is thus the rate of increase of stress on the secondary source as the pile-up reaches equilibrium. The rate of increase of stress on the secondary sources is not, in fact, constant. However, since the main contribution to the integral

(Eq. 5) involved in deriving this expression is obtained when the shear stress is greatest, only a minor error will be introduced by adopting a value for K_b which is consistent with the rate of growth of stress as the pile-up approaches equilibrium.

Alternatively, Eq. 13 may be written to define a temperature T for a particular loading rate K; thus

$$T \leq \frac{2\sigma_0 W/[k(\bar{\sigma} - D)]}{\ln\{2N\bar{\sigma}^2\beta^{2\sigma_0/(\bar{\sigma}-D)}/[2(1-\alpha)^2\sigma_0 t_0 K_b]\}} \quad (14)$$

so that the probability of a crack being initiated as the result of the formation of a particular pile-up is greater than or equal to $\frac{1}{2}$ at all temperatures below T. The propagation of a crack requires (1) the successive satisfaction of a number of similar conditions with the stresses arising in these cases from the expanding crack itself, and (2) applied tensile stresses that satisfy the Griffith criterion. Provided the tensile stresses are sufficiently large, it follows that the condition for brittle (elastic) fracture will be of the same form as that for the initiation of a crack.

Before proceeding to a comparison of theory and the available experimental data, it must be emphasized that the theory is very idealized. In particular, the assumption that brittle fracture cannot occur if a secondary source operates requires further examination. We note that the glide plane of the secondary source will not in general coincide with that of the pile-up in the plane of the grain boundary. It follows that the stress relief achieved by the action of a secondary source will vary along the length of the grain boundary, so that a number of sources must operate if the stress relief is to be reasonably uniform. Therefore, our model should be modified to allow for the operation of a multiplicity of sources. Mathematically, this is achieved by dividing the right-hand side of Eq. 5 by a factor R which we shall treat as constant.

With this amendment and provided condition (2) above is satisfied, Eq. 14 should represent the behavior of material in which grain boundaries are few, for example, bicrystals. It cannot be applied to normal polycrystalline aggregates without modification. As a preliminary we note the following experimental facts:

(*a*) The ductile-brittle transition temperature is defined as that temperature at which the energy absorbed in fracture has a particular value for a given rate of loading and specimen geometry.

(*b*) Measurements (employing X-ray techniques) and the appearance of brittle-fracture surfaces do not in general indicate any progressive change in the energy absorbed through plastic deformation across a fracture surface.[38–40] (*c*) The surface of a brittle fracture in a polycrystalline metal is not planar and only approximates to this condition over individual grains.

(*d*) The velocity of a crack varies only within comparatively narrow limits.[41]

It follows from (*a*) that the ductile-brittle transition must involve a crack propagation condition and is not merely a matter of initiation. This conclusion is supported by other work such as the experiments of Robertson [42] and of Felbeck and Orowan.[39] From (*b*) and (*d*), we infer that the stresses in front of the moving crack tend to remain constant and do not increase with the square root of the crack length, as would be inferred if the applied load near the crack remained constant. Such a variation has been assumed in the analysis of the stresses around stationary cracks in infinite media and has been taken over by Mott [43] in his calculation of crack velocity. As has been pointed out by Mott himself, this analysis will be accurate only when the velocity is small compared with that of sound waves. However, both theory and experiment indicate that this condition is not satisfied. We conclude, therefore, that the inference drawn from (*b*) and (*d*) is tenable, and, in the absence of a complete analysis of the problem, which appears intractable, we shall assume that these stresses are constant.

We now consider the process by which a crack propagates. We suppose in the light of (*c*) that fracturing is a step-by-step process involving the cleavage of individual grains. Furthermore, a crack in a particular grain will in general be bounded by the grain boundaries and will not be coplanar with the main crack. It follows that, as a stress raiser, each small crack will act separately from the main crack and produce stresses proportional to $\bar{\bar{\sigma}} l^{1/2}$ where $\bar{\bar{\sigma}}$ is the constant stress produced by the main crack and $2l$ the grain diameter. Again, a crack will not, in general, propagate through a grain boundary without some change in direction, so that the crack must either move more slowly as it approaches a grain boundary or pause as it reaches it. Either situation leads to an increased probability of deformation occurring near the tip of the crack, thus preventing the formation of a crack in the next grain. Alternatively, we may suppose that, at considerable distances from the head of the main crack, dislocation sources are activated which later produce pile-ups and microcracks, as previously described, which join up with the main crack. Either description leads us to a consideration of shear stresses acting either on cracks or on dislocation pile-ups. In both cases, the stress concentration is proportional to $l^{1/2}$. However, the stresses produced by the cracks are likely to be the greater because in this case there can be no stress such as the "frictional" stress that opposes the motion of dislocations in pile-ups.[10] The essential feature of this model is that the process envisaged by Stroh is repeated at every grain boundary as the crack propagates. If we suppose, as was inferred previously, that at the transition temperature the energy absorbed in plastic deformation per unit area of fracture surface is constant, we may now take account of grain size by introducing another factor P similar to R. This factor is proportional to the number of grains per unit area and thus to $1/l^2$. Equation 13 then becomes

$$\ln K = \ln \left\{ \mathfrak{L}^{5/2} \alpha N \bar{\sigma}^2 \beta^{2\sigma_0/(\bar{\sigma}-D)} \Big/ [2l^{5/2}(1-\alpha)^2 \sigma_0 t_0] \right\} - 2\sigma_0 W/[kT_b(\bar{\sigma}-D)] \quad (15)$$

where for convenience we have written $S^{1/2}P/R = \mathfrak{L}^{5/2}/l^2$ and T_b is the ductile-brittle transition temperature. Similarly, Eq. 14 becomes

$$T_b = \frac{2\sigma_0 W[k(\bar{\sigma}-D)]}{\ln \left\{ \mathfrak{L}^{5/2} \alpha N \bar{\sigma}^2 \beta^{2\sigma_0/(\bar{\sigma}-D)} / [2Kl^{5/2}(1-\alpha)^2 \sigma_0 t_0] \right\}} \quad (16)$$

We have thus far established a temperature T_b for a given testing rate, and the material is fully brittle at all temperatures below T_b. On raising the temperature above T_b, some plastic deformation and consequent work hardening will occur. To set some value to this, we suppose that a plastic strain ϵ produces work hardening to an extent $f(\epsilon)$. This may be taken to imply that dislocation sources are then subjected to an internally induced stress that tends to impede their motion, while the dislocation pile-up continues to behave much as before, since the average internally induced stress acting over a slip plane should be small. Under these conditions, Eq. 15 becomes

$$\ln K = \ln \left\{ \mathfrak{L}^{5/2} \alpha N [\bar{\sigma} - f(\epsilon)]^2 \beta^{2\sigma_0/[\bar{\sigma}-f(\epsilon)-D]} \Big/ [2l^{5/2}(1-\alpha)^2 \sigma_0 t_0] \right\} - 2\sigma_0 W/\{kT_2[\bar{\sigma} - f(\epsilon) - D]\} \quad (17)$$

Since the first term on the right-hand side of Eq. 17 is not much changed by these considerations, we may infer that fracture at a given loading rate will ensue after a plastic strain ϵ when T lies between T_b and T_2 as defined by the relation

$$(\bar{\sigma} - D)/[\bar{\sigma} - D - f(\epsilon)] = T_2/T_b \quad (18)$$

Thus the width of a transition from ductile to brittle behavior will be influenced by the rate of work hardening. It might therefore be expected that a material that exhibits a marked yield-point elongation and hence a low rate of work hardening should have a narrow transition range, and vice versa. This is in accord with the observations of Rinebolt and Harris,[44] who found that the transition zone in steel became broader as the carbon content was increased from 0.01 to 0.67% and thus also as the yield-point elongation was reduced.

We have finally to consider the effect of dilatational stresses. Many authorities are inclined to the view that the increase in brittleness produced by notches is due to hydrostatic stresses generated at the root of the notch. However, on Stroh's model for crack initiation, this would seem unimportant; it also seems unlikely that these stresses would be important as the crack propagates, since the displacements following the motion of the crack would tend to eliminate them. Furthermore, the effect appears to be reasonably well explained on the basis of the consideration of rate of loading. Thus Hollomon [45] gives the rate of strain (elastic) in a standard Charpy impact test as 10^3/sec or 10^7 times as fast as that in a conventional tension test. Vitman and Stepanov [46] have shown that such a change in loading rate would give an increase of ~100°K in the ductile-brittle transition temperature, which is the order of the increase produced in steel specimens by notches.

In the case of experiments in which the dilatational stresses are more general and stress relief less likely, these stresses must be expected to be important. They are not important in the analysis as it stands, but this is approximate. In particular, Stroh [4] has assumed in his investigation of the initial growth of a crack that all n dislocations of the pile-up group are free to enter the crack. This will be reasonable only when the width of the pile-up is very much greater than its length. In fact, for polycrystalline materials, this width is about the same as the length of the dislocation array; as a consequence, only a maximum of about $n/2$ dislocations at the center can enter the crack. Stroh,[4] however, has concluded that at initiation about $0.9n$ dislocations must enter the crack if it is to propagate. It is likely therefore that Stroh's condition must be modified from one of initiation to one of propagation. It is difficult in view of the complexity to assess the change involved, but it is clear that normal stresses, including hydrostatic, would be significant.

Comparison of Theory and Experiment

In the following we compare the theoretical predictions with the experimental results for the case of low-carbon steel. The values taken for the various parameters are as follows:

$$W = 600k, \quad \beta = 7, \quad \sigma_0 = 2 \times 10^{10}\ \text{dynes/cm}^2,$$
$$\bar{\sigma} = 10^{10}\ \text{dynes/cm}^2, \quad D = 0, \quad N = 10^3, \quad t_0 = \exp(2W/kT)/\nu$$

where $\nu = 10^{12}$/sec is the frequency of atomic vibrations. We shall leave $S^{1/2}P/R$ as an unknown factor to be determined from a consideration of experimental data.

Effect of grain size on the ductile-brittle transition. Hodge, Manning, and Reichold,[47] in their work on an iron containing 0.02% carbon tested at a constant (unspecified) loading rate, have reported a relation between transition temperature T and ASTM grain-size number n_g as follows:

$$n_g = G - HT$$

where G and H are constants for a particular set of experiments. Stroh [4] has pointed out, however, that the experimental results are equally well described by a relation

$$n_g = -G' + H'/T$$

and comparison with experiment gives

$$n_g = -10 + 3150/T \tag{19}$$

From Eq. 15, we obtain

$$n_g = \bar{N} - \ln \{\alpha N \bar{\sigma}^2 \beta^{2\sigma_0/(\bar{\sigma}-D)} / [2(1-\alpha)^2 \sigma_0 t_0 K]\} + 2.2\sigma_0 W/[kT(\bar{\sigma} - D)]$$

where $\bar{N}$ is the ASTM number appropriate for a grain size $\mathcal{L}$. Using the values for the various parameters quoted above and assuming a value of loading rate $K = 10^{18}$ dynes/cm² sec (as indicated from the results of Witman and Stepanov [46]), we have

$$n_g = \bar{N} - 20.2 + 2640/T \tag{20}$$

Equation 20 is thus of the right form, and quantitative agreement between theory and experiment is fair if $\bar{N}$ is taken as ~10.

Effect of loading rate on the ductile-brittle transition. Substituting the values given above for the various parameters into Eq. 15 gives

$$\ln (d\sigma/dt) = 60 + \ln (\mathcal{L}^{5/2}/l^{5/2}) - 2400/T$$

which becomes

$$\ln (d\sigma/dt) = 54.8 - 2400/T$$

when $\mathcal{L}$ is given the value appropriate for $\bar{N} = 10$, as determined above, and an ASTM grain-size number (n_g) of 4 is assumed. This result is in very fair agreement with that found from an examination of the published data of Witman and Stepanov [46] on the effect of loading rate on transition temperature for unnotched mild-steel specimens tested in tension:

$$\ln (d\sigma/dt) = 53 - 2760/T$$

Effect of *D*. In the foregoing analysis, particular significance attaches to the factor D, which represents the restraint on a dislocation source owing to factors other than primary Cottrell locking. Equation 15 shows that variations in D will lead to apparent variations in "activation energy" for fracture, remembering that this equation can be rewritten in the generic form $A = B \exp(-U/kT)$. Comparatively wide variations in this energy are therefore to be expected from one material to another, and in fact, the seven values quoted by Stroh [4] show a variation of 2.3/1.

As stated above, such a restraint on a dislocation source can arise from the presence of a secondary Cottrell atmosphere. It may also be due to solid solution hardening or to frictional forces as postulated by Petch [10] and others. Among these factors, we distinguish three classes: (1) restraints localized at the dislocation sources, (2) restraints effective everywhere on the slip planes, and (3) restraints effective only after the dislocations leave the sources. Referring to Eq. 16, we note that large values of D will imply (other things being equal) that the material is relatively brittle. This conclusion is now examined in the light of the experimental results presented in Figs. 5, 7, and 8, most of which show a ductile-brittle transition within the temperature range investigated. From Fig. 5 and Table 2, we observe that SAE 1020 steel and ship plate, which have roughly the same slope and only slightly different values for D, have the same transition temperature. On the other hand, SAE 4340 has a much larger value of D and a smaller slope, but it remains ductile at temperatures as low as 4.2°K. In Fig. 7, which relates to various experiments on molybdenum, the slopes of the curves are relatively constant; two sets of results agree with our prediction regarding D and brittleness, while the other set does not. In the latter case, the results of Carreker and Guard,[31] wire specimens were used, whereas the other workers employed conventional tension specimens. It is not clear at present how this difference could influence the result, and further work would be necessary to establish whether or not it is actually in opposition to the theory. The results in Fig. 8 appear to provide good support for the theory. The addition of 0.37% cerium to recrystallized chromium does not alter the slope of the linear part of the curve, but it does reduce the value of D appreciably, and the transition temperature is correspondingly much lower.

Returning now to a consideration of the brittleness of the materials whose yield strength behavior is given in Fig. 5, we note that both SAE 1020 and ship plate become brittle at temperatures very close to T_θ. On the basis of our model, this implies that these materials are just brittle at the temperatures under the conditions of testing involved, since the atmosphere behavior is virtually independent of temperature below T_θ. It follows that the SAE 4340, which is not brittle in this range, need only be slightly more ductile than the other materials. This may be expressed as the requirement that the factor $2\sigma_0/(\bar{\sigma} - D)$ for SAE 4340 need be only somewhat less than the values for the other materials. Substitution of appropriate values given in Table 2 and the assumption as before that $\bar{\sigma} = 10^{10}$ dynes/cm^2 indicate that this factor is essentially constant, a result which is satisfactory in view of the inaccuracies involved in this analysis.

Generally, then, it seems clear that the experimental evidence supports the view taken here of the importance of D and the restraints which give rise to it on the ductile-brittle transition.

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