



A progressive damage fatigue model for unidirectional laminated composites based on finite element analysis: theory and practice

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ABSTRACT. The simulation of the fatigue damage of laminated composites under multi-axial and variable amplitude loadings has to deal with several new challenges and several methods of damage modelling. In this paper we present how to account for the complex loading by using the damage hysteresis operator approach for fatigue. It is applied to a fatigue model for intra-laminar damage based on stiffness degradation laws from van Paepegem [1] and has been extended to deal with unidirectional carbon fibres. The parameter identification method is presented here and parameter sensitivities are discussed. The initial static damage of the material is accounted for by using the Ladevèze damage model and the permanent shear strain accumulation based on Van Paepegem's formulation. This approach has been implemented into commercial software. The intra-laminar fatigue damage model combines efficient methods with a low number of tests to identify the parameters of the stiffness degradation law, this overall procedure for fatigue life prediction is demonstrated to be cost efficient at industrial level.

KEYWORDS. Composite; Fatigue; Variable Amplitude; Stiffness degradation.



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INTRODUCTION

The increase of lightweight material in transportation industries is today facing greater scrutiny on fatigue life prediction of composite structures based on realistic load situations. The critical step towards accurate prediction is to reproduce the loading conditions undergone by the composite component. In automotive applications, the



challenge is related to the variability of those conditions: non proportional and variable amplitude loading leading to multi-axial local stress states and long duration fatigue loading. This is why Siemens PLM software [2] has developed an innovative composite fatigue CAE methodology (patent pending) keeping track of the material degradation under such conditions. Sevenois and van Paepegem [3] reviewed and compared the state of the art for fatigue model techniques of woven and UD composite. The study concluded that out of the four modelling methodology

- fatigue life (SN curve based),
- residual strength,
- residual stiffness
- (micro-)mechanics model

the residual stiffness models are most suitable for mechanical performance using experimental data and can also be combined with a residual strength approach.

The presented methodology is based on residual stiffness fatigue laws combined with an efficient damage operator approach to calculate the progressive damage and residual stiffness. This approach is able to perform fatigue simulations for variable amplitude loads and allows ply-stacking optimization without additional testing or material characterizations.

INTRA-LAMINAR FATIGUE SOLUTION: THE MODEL

In the following sections, the fatigue model strategy will be presented from the load definition, stiffness degradation theory, calculation optimisation algorithm to the parameter identification procedure.

Fatigue damage laws

The damage evolution law is based on the work of van Paepegem [1] for woven glass fibres that has been verified in several projects (e.g. [4][5]). Three intra-laminar damage variables D_{11} , D_{22} and D_{12} are defined at ply level and linked to the stress tensor by the following behaviour law, Eqn.(1)

$$\underline{\sigma} = HCH(\underline{\varepsilon} - \underline{\varepsilon}^p) \quad (1)$$

where C is the stiffness tensor, ε^p is a permanent strain tensor and H is defined as

$$H = \begin{bmatrix} \sqrt{1-D_{11}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{1-D_{22}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1-D_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

In [1] the damage variables D_{ij} were split into a positive and a negative part (d_{ij}^+ and d_{ij}^-), where the positive part increases when the stress is positive, and the negative one when it is negative. At the end the two parts were added, including a crack closure coefficient for the combination of tension and compression. In this work, only positive stress ratios are used, which means that there is no switch between tension and compression over a cycle and simplifies the problem. Either the stress is always positive and the damage D_{ij} is equal to d_{ij}^+ ; or it is always negative and it is equal to d_{ij}^- , Eqn.(3).

Furthermore, the formulations of Van Paepegem [1] must be adapted to unidirectional plies. First of all, to account for the high in-plane orthotropic behaviour of unidirectional plies, independent c_i parameters are defined for the three components of the damage. Therefore, fifteen parameters are used ($c_{i,jk}$) instead of five. For the same reason, the coupling between D_{11} and D_{22} which was implemented for woven is not included for UD: in woven fabrics, matrix de-cohesion clearly affects



the stiffness in longitudinal and transverse directions, whereas in unidirectional plies, the effect of matrix degradation on longitudinal behaviour can be neglected. However, the coupling between D_{22} and D_{12} remains mandatory and has been maintained. Finally, the deletion of this coupling imposes the addition of a propagation term in the formulation of D_{12} , so that a pure shear load in a ply remains able to lead to its collapse. With these assumptions applied to the formulations taken from [1], the evolution laws for the damage variables become:

$$\begin{aligned} \frac{d(d_{11}^+)}{dN} &= c_{1,11} \Sigma_{11}^+ \exp\left(-c_{2,11} \frac{d_{11}^+}{\sqrt{\Delta \Sigma_{11}^+}}\right) + c_{3,11} d_{11}^+ \Sigma_{11}^{+2} \exp\left(c_{5,11} \Sigma_{11}^+ - c_{4,11}\right) \\ \frac{d(d_{11}^-)}{dN} &= \left[c_{1,11} \Delta \Sigma_{11}^- \exp\left(-c_{2,11} \frac{d_{11}^-}{\sqrt{\Delta \Sigma_{11}^-}}\right) \right]^3 + c_{3,11} d_{11}^- \Sigma_{11}^{-2} \exp\left(\frac{c_{5,11}}{3} \Sigma_{11}^- - c_{4,11}\right) \\ \frac{d(d_{22}^+)}{dN} &= c_{1,22} (1 + D_{12}^{f2}) \Sigma_{22}^+ \exp\left(-c_{2,22} \frac{d_{22}^+}{(1 + D_{12}^{f2}) \sqrt{\Sigma_{22}^+}}\right) + c_{3,22} d_{22}^+ \Sigma_{22}^{+2} \exp\left(c_{5,22} \Delta \Sigma_{22}^+ - c_{4,22}\right) \\ \frac{d(d_{22}^-)}{dN} &= \left[c_{1,22} (1 + D_{12}^{f2}) \Sigma_{22}^- \exp\left(-c_{2,22} \frac{d_{22}^-}{(1 + D_{12}^{f2}) \sqrt{\Sigma_{22}^-}}\right) \right]^3 + c_{3,22} d_{22}^- \Sigma_{22}^{-2} \exp\left(\frac{c_{5,22}}{3} \Delta \Sigma_{22}^- - c_{4,22}\right) \quad (3) \\ \frac{d(d_{12}^+)}{dN} &= c_{1,12} (1 + d_{12}^{-2}) \Sigma_{12}^+ \exp\left(-c_{2,12} \frac{d_{12}^+}{2(1 + d_{12}^{-2}) \sqrt{\Sigma_{12}^+}}\right) + c_{3,12} d_{12}^+ \Sigma_{12}^{+2} \exp\left(c_{5,12} \Sigma_{12}^+ - c_{4,12}\right) \\ \frac{d(d_{12}^-)}{dN} &= c_{1,12} (1 + d_{12}^{+2}) \Sigma_{12}^- \exp\left(-c_{2,12} \frac{d_{12}^-}{2(1 + d_{12}^{+2}) \sqrt{\Sigma_{12}^-}}\right) + c_{3,12} d_{12}^- \Sigma_{12}^{-2} \exp\left(c_{5,12} \Sigma_{12}^- - c_{4,12}\right) \end{aligned}$$

where $c_{i,jk}$ are the 15 fatigue material coefficients that must be identified, the fatigue failure indices Σ_{ij} give the connection to residual strength as the ratio between the effective stress and the (actual) ultimate strength of the material in the ij component, and defined as

$$\Sigma_{ij}^+ = \max_{cycle} \left(\Sigma_{ij} \frac{\sigma_{ij}}{|\sigma_{ij}|} \right); \quad \Sigma_{ij}^- = \max_{cycle} \left(\Sigma_{ij} \frac{-\sigma_{ij}}{|\sigma_{ij}|} \right) \quad (4)$$

where $\sigma_{ij} = \begin{cases} 0, & \sigma_{ij} < 0 \\ \sigma_{ij}, & \sigma_{ij} \geq 0 \end{cases}$

Accumulated permanent strain

In addition to these damage evolution laws, the model takes into account the permanent strain which appears in the ply due to a cyclic shear loading. Some matrix debris formed by the shear stress is accumulated in the opening matrix cracks during tension stress [1], which leads to a non-reversible deformation of the ply.

The c_9 parameter drives the fatigue permanent strain accumulation following the formulation:



$$\frac{d\gamma_{12}^p}{dN} = c_9 \max_i \gamma_{12} \frac{dd_{12}^+}{dN} + c_9 \max_i - \gamma_{12} \frac{dd_{12}^-}{dN} \quad (5)$$

Initial degradation of the ply

It is also important to consider the effect of the first static loading of the ply on its fatigue damage and strain behaviour, as it is not included in the fatigue degradation. Therefore, the static damage and permanent strain are evaluated with a static damage model [1] by first running an initial non-linear analysis up to the peak load of the cyclic fatigue analysis. The resulting initial damage and permanent strain are imposed as initial state of the fatigue analysis and the first cycle can be computed with a correct stress distribution. The fatigue damage tensor D^f is then added to the initial damage tensor D^s

$$D = D^s + D^f \quad (6)$$

The permanent strain is handled in the same way

CALCULATION OPTIMISATION: FROM N-JUMP TO DAMAGE JUMP

In this section we introduce a new methodology of handling stiffness degradation methods efficiently with variable amplitude load situations

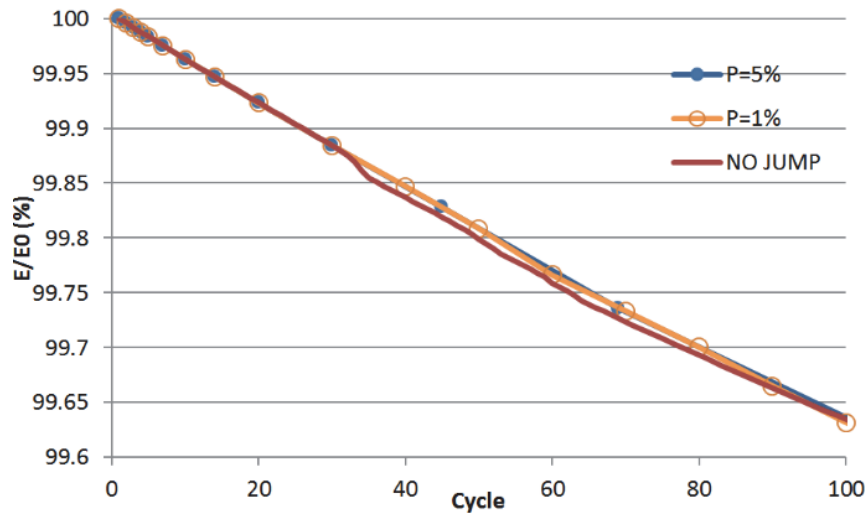


Figure 1: Effect of the N-Jump algorithm on stiffness degradation.

Block loading: N-Jump

For this we start from the methodology for block loads, the N-Jump algorithm. The purpose of the N-Jump algorithm as presented in [1] is to avoid running a full FE analysis at each load cycle and to deliberately choose a few relevant load cycles only. The cycles with no significant damage growth are “jumped”. From a first FE analysis, at each Gauss point of the FE model, the theoretical number of cycles to jump NJUMP1 is estimated by extrapolating the damage, Eqn. (7) and applying to Eqn. (3)

$$\Delta D = D_{N+NJUMP1} - D_N = \begin{cases} 10^{-20} \text{ if } D = 0 \\ 0.5D \text{ if } 0 < D \leq 0.2 \\ 0.1 \text{ if } D > 0.2 \end{cases} \quad (7)$$

A global cycle jump NJUMP is defined such that P% of Gauss points verify $N_{JUMP1} < N_{JUMP}$ for the three components. The value of P has been set to 5% in this study. The damage is finally extrapolated after NJUMP, using again the progressive damage formulations Eqn.(3). To validate this algorithm and the value of P, three similar fatigue analyses (three point bending) have been run on the first 100 loading cycles with a 45 degree ply layup; one without N-Jump, one with P=1% and one with P=5% See Fig. 1. [1] compares the stiffness degradation observed in the three analyses and validates the accuracy of the N-Jump algorithm.

Variable Amplitude Load

In the automotive industry, the synthesis of realistic fatigue loading involves complex load schedules for different roads with variable loading (see Fig. 2 and [15]). However, the common efficient way to simulate input fatigue loading is to consider block loading with a set of constant maximum stress amplitude sequences. Nevertheless, this method is not able to accurately predict the damage progression and stress redistribution under real life variable amplitude.

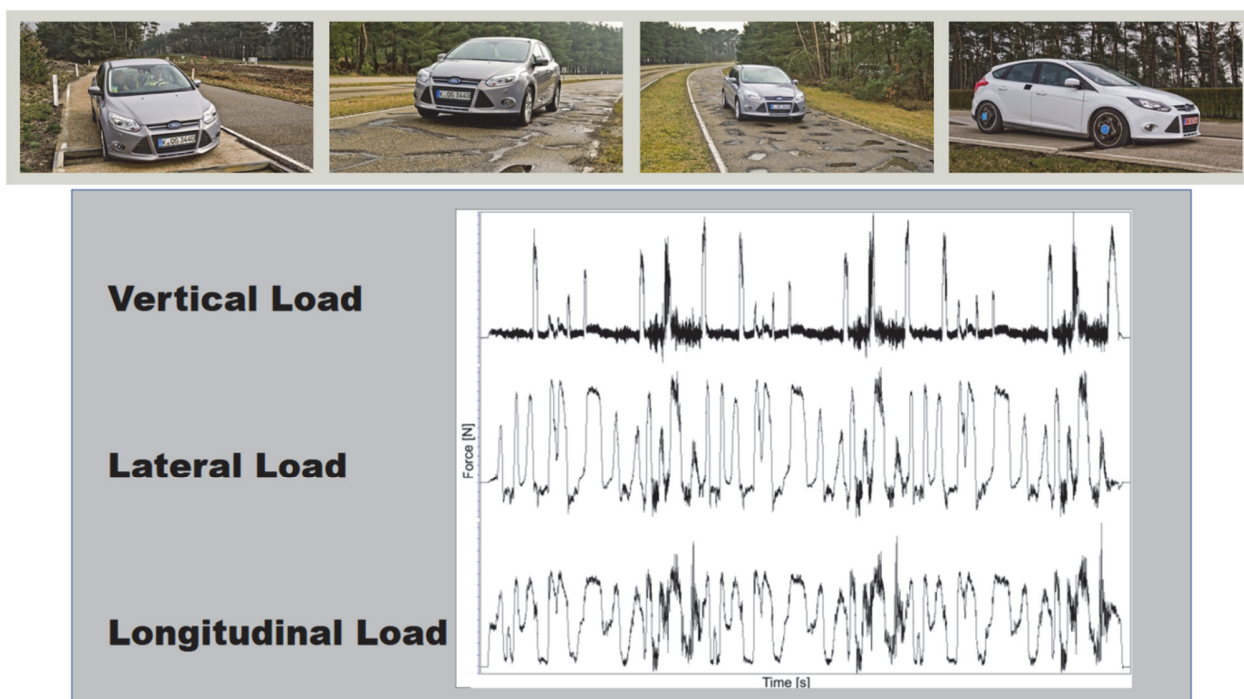


Figure 2. Variable Amplitude example in automotive application. Many different load events, many loads – shown here is just a short sequence of the wheel forces at one wheel

New Method: The Damage Accumulation Jump

In the case of variable amplitude, traditional fatigue approaches for metallic material use SN curves, linear Miner-Palmgren (see [6]) damage accumulation and cycle count (rainflow counted cycles [7]) based damage evaluations. In 1945, Miner developed a linear damage accumulation method, based on the work of Palmgren and added the contribution of various stress amplitude loading to the damage. However, as for SN curves, the loading history of the material is not accounted for. In rainflow counting methods the damage level depend on full closing hysteresis loop of load cycles (Fig. 3). In the case of composite materials, the fatigue behaviour is changing over time due to changes in the matrix damage state. When applying variable amplitude loading, the largest load cycles – that contribute to the larger amount of damage – commonly take a very long time to complete, due to the many nested cycles. In this case the approach to only consider cycles when they are completed can no longer be justified.

In the 1990ies Brokate and Krejci applied the mathematical toolset of hysteresis operators to fatigue theory (see [8, 9]) analysing the linear damage accumulation and analogies between damage accumulation and energy dissipation.

Based on this work it is possible to extend the rainflow based methodology to non-linear damage accumulation in a both mathematical and methodological sense: (see [10-13]) the damage hysteresis operator approach.



The idea is based on the hysteresis operators for kinematic hardening (i.e. to calculate elastic-plastic stress-strain behaviour from pseudo elastic stress histories) and how dissipated energy is calculated in these models. The new idea is to replace the constitutive laws of elasto-plastic stress-strain behaviour with constitutive laws for a “stress-damage potential” behaviour (see [8,10]).

The hysteresis operator approach is able to calculate damage at any time increment instead of ‘closed cycle increment’. The extensions explained in [10] and [11] allow the damage status and the damage behaviour to be updated depending on internal (i.e. pre-damage) and any external factors (i.e. temperature, humidity, etc.) Therefore this approach is also suited to follow the progressive damage curves and also including the damage history of the material.

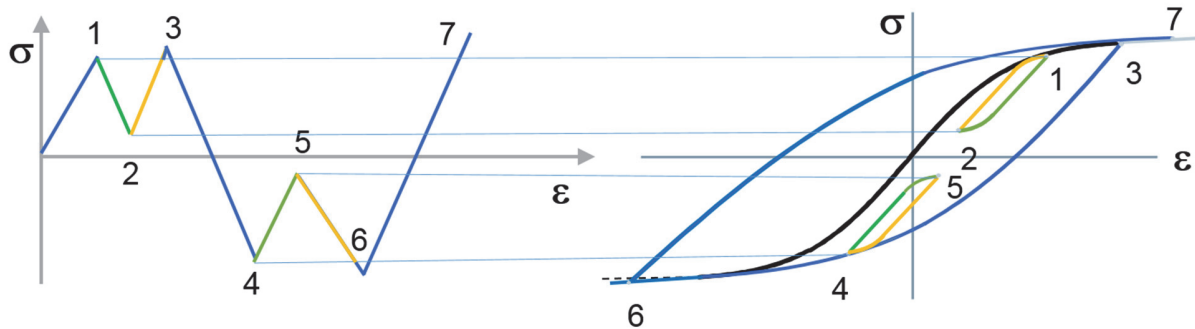


Figure 3. Illustration of rainflow method based on stresses/strains with nested cycles.

This approach was verified to give good prediction when applied to temperature dependent fatigue analysis with non-linear damage accumulation (see overview of results in [14]) and for full car structures with full load histories (e.g. [15]). It is also applied for modelling the stiffness degradation in short fibre reinforced plastics [16].

PARAMETER IDENTIFICATION PROCEDURE

Test protocol

In this study, two different test protocols are proposed. The first is a traditional approach with tensile tests on five layups and five load levels per layup capturing a representative span of fatigue life (based on test method [17]). This results in twenty-five configurations. The second is an application of a more innovative approach again based on ideas in [1] with one-sided bending tests on the same five layups, but with only one load level for each layup, so only five configurations. The idea is to assess if the load distribution on this kind of specimen can provide enough information to feed the damage model, as a three-points-bending test results in progressive load levels along the same specimen, both in tension and compression.

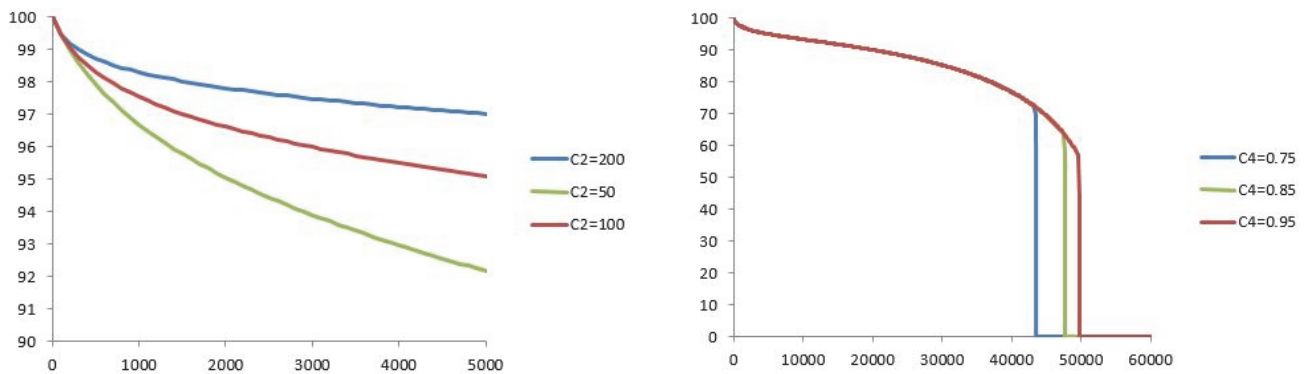


Figure 4: Effect of $c_{2,12}$ (left) and $c_{4,12}$ (right) on numerical stiffness degradation

Peak data such as load, displacement, strains from extensometers and gauges, temperature are measured from these tests at several representative cycles. The stiffness degradation of each specimen can be estimated from these evolutions. Additionally, running-in and unloading raw data are extracted to analyse the initial stiffness drop of the specimens and their final permanent strain.

Parameter identification protocol

The parameter identification consists of an FE-based optimization of the fifteen fatigue parameters to fit simulated stiffness degradation with experimental results. A step-by-step methodology identifying the parameters one by one from specific experimental data has been setup. As illustrated by two examples in [1], each c_i parameter has a specific contribution on the numerical stiffness degradation, and therefore can be adjusted to fit with experimental results.

Therefore, two volume finite element models reproducing the two testing procedures have been created, and the nonlinear fatigue solver with N-Jump algorithm is used to correlate the stiffness degradation of each testing configuration by adjusting the fifteen $c_{i,jk}$ parameters (note that the N-Jump is herein used as tests are carried out under constant amplitude loading).

An illustration of the resulting correlation between experimental and simulated stiffness evolutions is given in [17].

Finally, the parameter c_9 is estimated by crosschecking the raw data of the unloading of the specimens.

VALIDATION AND APPLICATION

The first validation has been conducted on a coupon under constant amplitude loading. The same three point bending analysis as above is run to 80 000 cycles at an imposed load level. A more complex quasi-isotropic layup [0/60/-60]4s is now studied. The resulting stiffness degradation is compared to experimental data [17]. The good predictability illustrates the main interest of a ply-level damage law: identification is performed on specific layups, and the resulting material data remains available for any layup without additional identification.

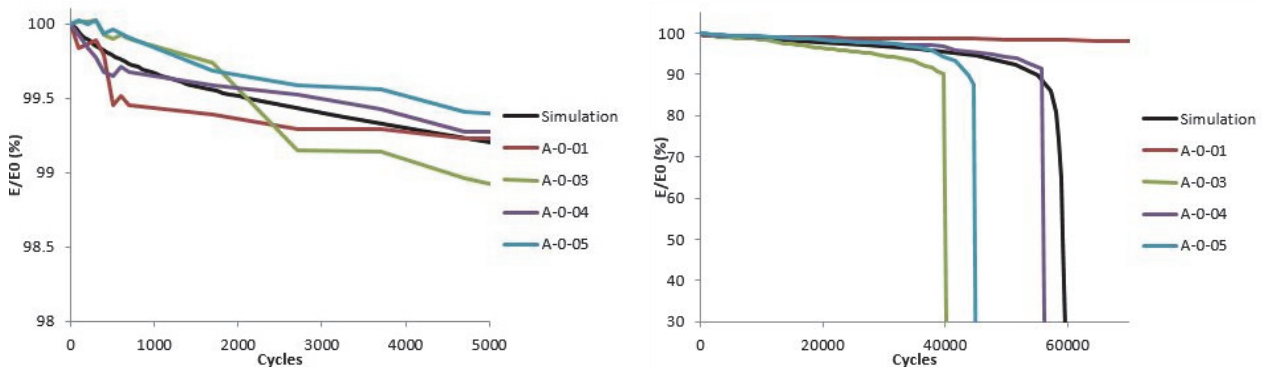


Figure 5: Correlation between experimental and simulated stiffness evolutions ([0]20 layup in 3 point bending).

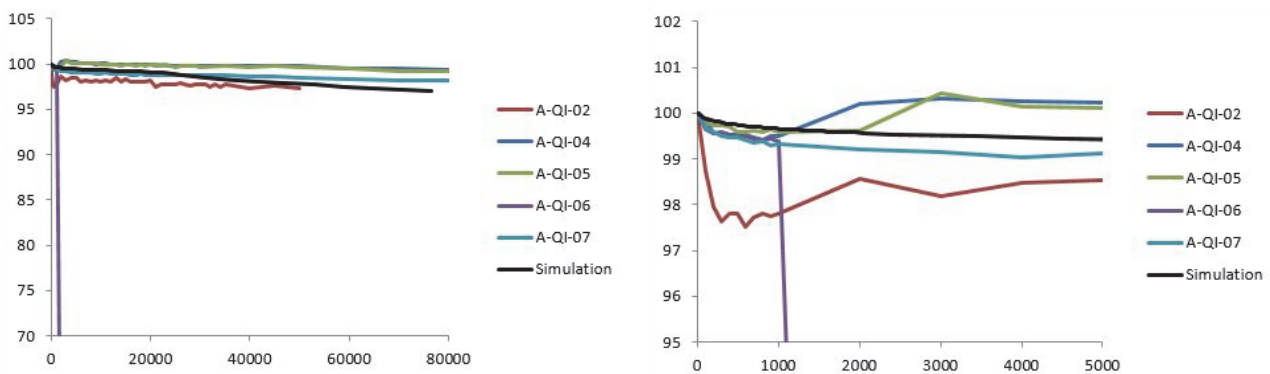


Figure 6: Predictability of stiffness reduction of a 3 point bending quasi-isotropic coupon.



Further validation cases on component like (i.e. flat and V-shaped components) have been conducted. In these cases the overall stiffness degradation contribution from the fatigue loading were quite small. The comparison of the predicted and measured stiffness reduction showed good coincidence.

As the failures in these examples were not initiated from fatigue failure due to intra-ply damage but due to the inter-laminar delamination from the edge, the examples were not sufficient to fully validate the methodology. So the authors decided that additional validation cases are needed and under investigation.

CONCLUSION

A new methodology to efficiently analyse the fatigue damage by stiffness reduction under variable amplitude was introduced. It combines the efficiency of the N-Jump algorithm with complex load scenarios under variable and non-proportional loading.

First validations show good coincidence between simulated and measured global stiffness behaviour. Further investigation are needed that also include the inter-ply behaviour and the initiation of delamination.

This introduces new challenges to extend this methodology to a complete intra-laminar and inter-laminar fatigue damage solution for variable amplitude and multi-axial loadings. The first steps have been already done but further validation of these extensions is still needed.

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